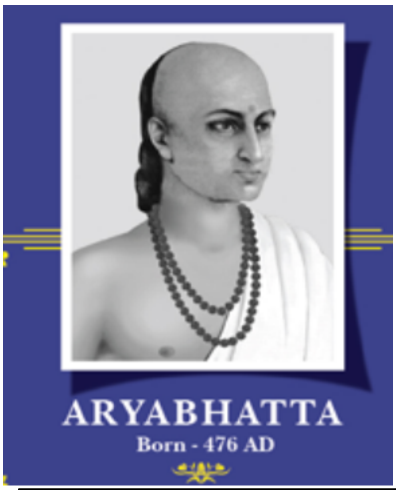
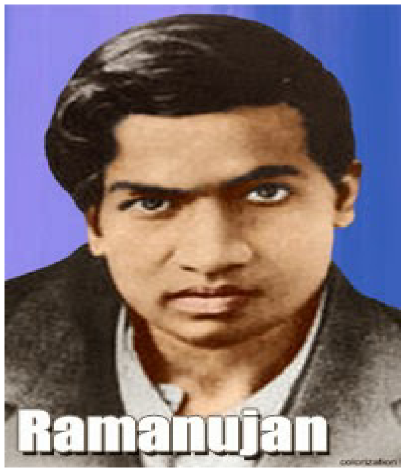
Mathematics All lessons Thinking corner and Progress Check Answers

(Based On New Syllabus 2019-20)





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10th Maths – Chapter 1 to 3 (Book in One Marks & Activities)

Green indicates Thinking Corner, Blue indicates Progress Check

Chapter – 1 RELATIONS AND FUNCTIONS

1. If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that aϵA, bϵB is called the Cartesian Product of A and B, and is denoted by A×B .

2. The "cartesian product" is also referred as "cross product".

3. In general A×B ≠ B×A, but n(A×B) = n(B×A).

4. When will A×B be equal to B×A? When A = B.

5. A×B = Ø if and only if A = Ø or B = Ø.

6. If n(A) = p and n(B) =q then n(A×B) = pq.

7. For any two non-empty sets A and B, A×B is called as cartesian product.

8. If n(A×B) = 20 and n(A)= 5 then n(B) is 4.

9. If A = {−1,1} and B = {−1,1} then geometrically describe the set of points of A×´B.

Square {(-1,-1), (-1,1), (1,-1), (1,1)}.

10. If A, B are the line segments given by the intervals (–4, 3) and (–2, 3) respectively, represent the cartesian product of A and B. {(-4,-2), (-4,3), (3,-2), (3,3)}. (Rectangle)

11. If A, B, C are three non-empty sets then the cartesian product of three sets is the set of all possible ordered triplets.

12. In general, cartesian product of two non-empty sets provides a shape in two dimensions and cartesian product of three non-empty sets provide an object in three dimensions.

Progress Check

Let A = {1, 2, 3, 4} and B = {a, b, c}.

1. Which of the following are relations from A to B?	2. Which of the following are relations from B to A?
✓ (i) { (1, b), (1, c), (3, a), (4, b) }	(i) { (c, a), (c, b), (c, 1) } X
X (ii) { (1, a), (b, 4), (c, 3) }	(ii) { (c, 1), (c, 2), (c, 3), (c, 4) } ✓
X (iii) { (1, a), (a, 1), (2, b), (b, 2) }	(iii) { (a, 4), (b, 3), (c, 2) } ✓

A×B = {(1,a), (1,b), (1,c), (2,a), (2,b), (2,c), (3,a), (3,b), (3,c), (4,a), (4,b), (4,c)}

$B \times A = \{(a,1), (a,2), (a,3), (a,4), (b,1), (b,2), (b,3), (b,4), (c,1), (c,2), (c,3), (c,4)\}$

13. If n(A)=p, n(B)=q, then the total number of relations that exist between A and B is 2^{pq}

14. A relation which contains no element is called a "Null relation".

15. A function is also called as a mapping or transformation.

16. Relations are subsets of Cartesian products. Functions are subsets of Relations.

17. True or False: All the elements of a relation should have images. False

18. True or False: All the elements of a function should have images. True

19. True or False: If R : A→ B is a relation then the domain of R = A. False

20. If f : $\mathbb{N} \to \mathbb{N}$ is defined as f (x) = x2, the pre-image(s) of 1 and 2 are 1 and ---.

21. The difference between relation and function is Image.

(Relation : One pre-image – one or more images) (Function : One pre-image – One image)

22. Let A and B be two non-empty finite sets. Then which one among the following two collection is large?

(i) The number of relations between A and B.

(ii) The number of functions between A and B.

23. The range of a function is a subset of its co-domain.

24. Is the relation representing the association between planets and their respective moons a function? No (Because Different Planets have different numbers of Moons)

25. In vertical line test if every vertical line intersects the curve in at most one point then the curve drawn in a graph represents a function.

25. Any equation represented in a graph is usually called a 'curve'.

26. A one-one function is also called an injection.

27. An onto function is also called a surjection.

28. A one–one and onto function is also called a one – one Correspondence or bijection.

29. Can there be a one to many function? No.

30. Is an identity function one – one function? Yes.

31. All one – one functions are onto functions. False.

32. There will be no one – one function from A to B when n(A)= 4 , n(B)= 3 . True.

33. All onto functions are one – one functions. False.

34. There will be no onto function from A to B when n(A)= 4 , n(B) = 5 . True.

35. If f is a bijection from A to B, then n(A) = n(B) . True.

36. If n(A) = n(B) , then f is a bijection from A to B. False.

37. All constant functions are bijections. False.

38. If f(x) = x^m = and g(x) = x^n, does f o g = g o f ? Yes [∵ f o g = g o f = x^{mn}]

39. The Composition g o f (x) exists only when range of f is a subset of domain of g.

40. State your answer for the following questions by selecting the correct option.

1. Composition of functions is commutative

(a) Always true (b) Never true (c) Sometimes true

2. Composition of functions is associative

(a)Always true (b) Never true (c) Sometimes true

41. Is a constant function a linear function? Yes

42. Is quadratic function a one – one function? No

43. Is cubic function a one – one function? Yes

44. Is the reciprocal function a bijection? Yes

45. If f : A→ B is a constant function, then the range of f will have one element.

46. If A and B are finite sets such that n(A) = p , n(B) = q then the total number of functions that exist between A and B is q^p.

Chapter – 2 NUMBERS AND SEQUENCES

1. Euclid, one of the most important mathematicians wrote an important book named "Elements" in 13 volumes. The first six volumes were devoted to Geometry and for this reason, Euclid is called the "Father of Geometry".

2. The remainder is always less than the divisor. If r = 0 then a = bq so b divides a. Similarly, if b divides a then a = bq.

3. Euclid's Division Lemma can be generalised to any two integers.

4. When a positive integer is divided by 3

1. What are the possible remainders? 0, 1, 2

2. In which form can it be written? a = 3q + r, 0 ≤ r < 3

5. Find q and r for the following pairs of integers a and b satisfying a =bq +r .

1). a = 13, b = 3 (q = 4, r = 1) 2). a = 18, b = 4 (q = 4, r = 2)
3). a = 21, b = –4 (q = -5, r = 1) 4). a = –32, b = –12 (q = 3, r = 4)
5). a = –31, b = 7 (q = -5, r = 4)

6. Euclid's division algorithm is a repeated application of division lemma until we get remainder as Zero.

7. The HCF of two equal positive integers k, k is k.

8. Two positive integers are said to be relatively prime or co prime if their Highest Common Factor is 1.

9. Is 1 a prime number? 1 is neither a prime number nor a composite Number.

10. Every natural number except 1 can be expressed as Prime numbers.

11. In how many ways a composite number can be written as product of power of primes? Only one way

12. The number of divisors of any prime number is 2.

13. Let m divides n. Then GCD and LCM of m, n are m and n.

14. The HCF of numbers of the form 2^m and 3^n is 1.

15. Can you think of positive integers a, b such that $a^b = b^a$?
Yes, There is. If a = 2, b = 4; then $2^4 = 4^2 = 16$

16. How many integers exist which leave a remainder of 2 when divided by 3? Many.

17. Two integers a and b are congruent modulo n if a and b leave same remainder when divided by n.

18. The set of all positive integers whichleave remainder 5 when divided by 7 are { 5,12,19,26,…}.

19. The positive values of k such that (k−3) -5 (mod11) are 19, 30, 41, 52, ….

20. If 59 ≡ 3 (mod 7), 46 ≡ 4 (mod 7) then 105 ≡ 0 (mod 7), 13 ≡ 6 (mod 7), 413 ≡ 0 (mod 7), 368 ≡ 4 (mod 7).

21. The remainder when 7x13x19x23x29x31 is divided by 6 is 1.
1 x 1 x 1x (-1)x(-1)x1

22. If the number of elements in a sequence is finite then it is called a Finite sequence.

23. If the number of elements in a sequence is infinite then it is called an Infinite sequence.

24. Fill in the blanks for the following sequences
(i) 7, 13, 19, 25 , ... (ii) 2, 5, 10, 17, 26,… (iii) 1000, 100, 10, 1, 0.1, ...

25. A sequence is a function defined on the set of Natural numbers.

26. The nth term of the sequence 0,2,6,12,20,... can be expressed as n(n-1).

26. Say True or False

(i) All sequences are functions – True. (ii) All functions are sequences. – False.

27. Though all the Sequences are Functions, not all the functions are sequences.

28. The difference between any two consecutive terms of an A.P. is a constant.

29. If a and d are the first term and common difference of an A.P. then the 8th term is $a + 7d$.

30. If t_n is the n_{th} term of an A.P., then $t_{2n} - t_n$ is nd.

31. The common difference of an A.P. can be positive, negative or zero.

32. If t_n is the n_{th} term of an A.P. then the value of $t_{n+1} – t_{n-1}$ is $2d$.

33. An Arithmetic progression having a common difference of zero is called a constant arithmetic progression.

34. The common difference of a constant A.P. is zero.

35. If a and l are first and last terms of an A.P. then the number of terms is $n = (l – a)/d + 1$.

36. If every term of an A.P. is multiplied by 3, then the common difference of the new A.P. is $3d$.

37. Three numbers a, b and c will be in A.P. if and only if $2b = a + c$.

38. The sum of terms of a sequence is called Series.

39. If a series have finite number of terms then it is called Finite series.

40. A series whose terms are in Arithmetic progression is called Arithmetic series.

41. If the first and last terms of an A.P. are given, then the formula to find the sum is $S_n = n(a + l)/2$.

42. The value of n must be positive. Why? Because it is defined on Natural numbers.

43. State True or False. Justify it.

1). The nth term of any A.P. is of the form pn+q where p and q are some constants.
True. $t_n = a + (n – 1)d = a + nd – d = dn + (a – d) = pn + q$; Where $p = d$; $q = a - d$

2). The sum to nth term of any A.P. is of the form $pn^2+qn + r$ where p, q, r are some constants. True. $Sn = n(2a + (n – 1)d)/2 = n(2a + nd –d)/2 = (2an + n^2 d –dn)/2$
$= an + n^2 d/2 –dn/ = n^2d/2 + (2a – d)n/2 = pn^2 + qn + r$
Where $p = d/2$; $q = (2a – d)/2$; $r = 0$

44. What is the sum of first n odd natural numbers? $S_n = n^2$

45. What is the sum of first n even natural numbers. $S_n = n(n +1)$

46. A G.P. is obtained by multiplying a fixed non-zero number to the preceding term.

47. The ratio between any two consecutive terms of the G.P. is a constant and it is called Common ratio.

48. Fill in the blanks if the following are in G.P.

(i) $1/8$, 3/4, 9/2, 27   (ii) 7, 7/2, 7/4   (iii) 2, $2\sqrt{2}$, 4 ,…

Thinking Corner

Is the sequence $2, 2^2, 2^{2^2}, 2^{2^{2^2}}, \ldots$ is a G.P. ?

No. (Since Common ratio is not a constant one.)

49. If first term = a, common ratio = r, then find the value of t_9 and t_{27}.

$t_9 = ar^8$, $t_{27} = ar^{26}$

50. In a G.P. if $t_1 = 1/5$ and $t_2 = 1/25$ then the common ratio is 1/5.

51. Three non-zero numbers a,b,c are in G.P. if and only if $b^2 = ac$.

52. Split 64 into three part such that the numbers are G.P. 2 x 4 x 8 = 64.

53. If a, b, c, … are in G.P. then 2a, 2b, 2c, … are also in G.P.

54. If 3, x, 6.75 are in G.P. then x is 4.5

55. The above formula for sum of first n terms of a G.P. is not applicable when r = 1.

56. If $r = 1$, then $S_n = a + a + a + \ldots + a = na$.

57. A series whose terms are in Geometric progression is called Geometric Series.

58. When r = 1 , the formula for finding sum to n terms of a G.P. is na.

59. When $r \neq 1$, the formula for finding sum to n terms of a G.P. is $a(r^n - 1)/(r - 1)$.

60. Sum to infinite number of termsof a G.P. is $a/(r – 1)$.

61. For what values of r, does the formula for infinite G.P. valid? $-1 < r < 1$

62. Is the series 3 + 33 + 333 + ... a Geometric series?

No. (Since the common Ratio is not a constant one).

63. The value of r, such that $1 + r + r_2 + r_3 \ldots = ¾$ is -1/3.

64. How many squares are there in a standard chess board? 204

65. How many rectangles are there in a standard chess board? 1296

66. The sum of cubes of first n natural numbers is the square of the sum of the first n natural numbers.

67. The average of first 100 natural numbers is 50.5.

68. Say True or False. Justify them.

1. The sum of first n odd natural numbers is always an odd number. – False.
2. The sum of consecutive even numbers is always an even number. – True.
3. The difference between the sum of squares of first n natural numbers and the sum of first n natural numbers is always divisible by 2. – True.
4. The sum of cubes of the first n natural numbers is always a square number. – True.

Chapter – 3 ALZEBRA.

1. Al-Khwarizmi is hailed as "Father of Algebra".

2. The term "Algebra" has evolved as a misspelling of the word 'al-jabr'.

3. Linear equations are the first degree equations.

4. Quadratic equations are the second degree equations.

5. Cubic equations are the third degree equations.

6. xy – 7 = 3 is not a linear equation in two variables since the term xy is of degree 2.

7. A linear equation with two variables represent a straight line in xy plane.

8. The number of possible solutions when solving system of linear equations in three variables are 3.

9. If three planes are parallel then the number of possible point(s) of intersection is/are zero

10. For a system of linear equations in three variables the minimum number of equations required to get unique solution is three.

11. A system with 0 = 0 will reduce to identity.

12. A system with 1 = 0 will provide absurd equation.

13. Greatest Common Divisor of two given polynomials f (x) and g (x) is find out using Euclidean Algorithm.

14. If f (x) and g(x) are two polynomials of same degree then the polynomial carrying the highest coefficient will be the dividend.

15. In case, if both have the same coefficient then compare the next least degree's coefficient and proceed with the division.

16. When two polynomials of same degree has to be divided, coefficient should be considered to fix the dividend and divisor.

17. If r(x) = 0 when f(x) is divided by g(x) then g(x) is called GCD of the polynomials.

18. If f (x) = g(x)q(x)+r(x), - r(x) must be added to f(x) to make f(x) completely divisible by g(x).

19. If f (x) = g(x)q(x)+r(x), r(x) must be subtracted to f(x) to make f(x) completely divisible by g(x).

20. Complete the factor tree for the given polynomials f(x) and g(x). Hence find their GCD and LCM. f(x) = 2x3 – 9x2 – 32x –21; g(x) = 2x3 – 7x2 – 43x – 42

    f(x) = 2x3 – 9x2 – 32x –21 = (2x + 3)(x – 7)(x + 1)

    g(x) = 2x3 – 7x2 – 43x –42 = (2x + 3)(x – 7)(x + 2)

    GCD [f (x) and g (x)] = (2x + 3)(x – 7) ;LCM [f (x) and g (x)] = (2x + 3)(x – 7)(x + 1)(x + 2)

21. Is f(x) x g(x) x r(x) = LCM [f (x),g(x),r(x)] × GCD [f (x),g(x),r(x)] ? Not always equal. (But it is equal only when the three polynomials are co-prime to each others).

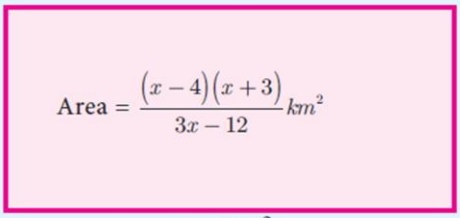
22. Are x^2 -1 and $tanx = \frac{sinx}{cosx}$ rational expressions? 1- No; 2 – Yes.

23. The number of excluded values of $(x^3 + x^2 – 10x – 8)/(x^4 + 8x^2 – 9)$ is 2 (1, -1)
[∵ $(x^4 + 8x^2 – 9) = (x^2 + 9)(x + 1)(x – 1)$]

Progress Check

Find the unknown expression in the following figures.

1.



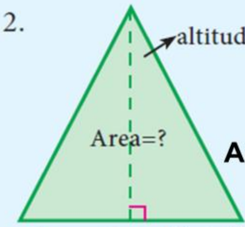
breadth = ?

$\frac{X + 3}{X - 3}$

length = $\frac{x-3}{3}$ km

Fig. 3.5

2.



altitude = $\frac{2(x+y)}{x-y} m$

$(x > y)$

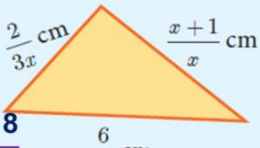
Area = $\frac{(x + y)^3}{(x – y)}$

base = $(x+y)(x+y)m$

Fig. 3.6

Progress Check

1. Write an expression that represents the perimeter of the figure and simplify.



$\frac{3x^2 + 5x + 18}{3x^2}$

2. Find the base of the given parallelogram whose perimeter is $\frac{4x^2 + 10x - 50}{(x-3)(x+5)}$

$\frac{2(x – 5)}{(x – 3)}$

24. The sum of two rational expressions is always a rational expression. False.

Example : $(x^2 + y^2)/(x + y) + 2xy/(x + y) = (x + y)$

25. The product of two rational expressions is always a rational expression. False.

Example : $(x – y)/(x + y) \times (x + y)/ (x – y) = 1$

26. Is $x^2 + 4x + 4$ a perfect square? Yes. Since $x^2 + 4x + 4 = (x + 2)^2$

27. What is the value of x in $3\sqrt{x} = 9$? $x = 9$

28. The square root of $361x^4y^2$ is $19x^2y$.

29. $\sqrt{(a^2x^2 +2abx +b^2)} = (ax + b)$ [∵ $(a^2x^2 +2abx +b^2) = (ax + b)^2$]

30. If a polynomial is a perfect square then, its factors will be repeated even number of times (odd / even).

31. The long division method in finding the square root of a polynomial is useful when the degree of the polynomial is higher.

32. To find the square root of a polynomial, the degrees of the variables are either in descending order or in ascending order.

33. The values of x such that the expression ax2 +bx +c becomes zero are called roots of the quadratic equation.

34. Fill up the empty box in each of the given expression so that the resulting quadratic polynomial becomes a perfect square.

(i) $x^2 + 14x +$ 49 [$(14/2)^2$] (ii) $x^2 - 24x +$ 144 [$(-24/2)^2$] (iii) $p^2 + 2qp +$ q^2 [$(2q/2)^2$]

35. If the constant term of $ax^2+bx+c=0$ is zero, then the sum and product of roots are -b/a and zero.

Progress Check

Quadratic equation	Roots of quadratic equation α and β	co-efficients of x^2, x and constants	Sum of Roots $\alpha+\beta$	Product of roots $\alpha\beta$	$-\frac{b}{a}$	$\frac{c}{a}$	Conclusion
$4x^2-9x+2=0$	2, 1/4	4, -9 , 2	9/4	1/2	9/4	1/2	Equal
$\left(x-\frac{4}{5}\right)^2=0$	4/5, 4/5	25, - 40, 16	8/5	16/25	8/5	16/25	Equal
$2x^2-15x-27=0$	9, -3/2	2, -15 , -27	15/2	-27/4	15/2	-27/4	Equal

36. If the graph of the given quadratic equation intersect the X axis at two distinct points, then the given equation has two real and unequal roots.

37. If the graph of the given quadratic equation touch the X axis at only one point, then the given equation has only one root which is same as saying two real and equal roots.

38. If the graph of the given equation does not intersect the X axis at any point then the given equation has no real root.

39. A matrix is a rectangular array of elements. The horizontal arrangements are called rows and vertical arrangements are called columns.

Progress Check

1. Find is the element in the second row and third column of the matrix $\begin{pmatrix} 1 & -2 & 3 \\ 2 & 1 & 5 \end{pmatrix}$ 5

2. Find is the order of the matrix $\begin{pmatrix} \sin\theta \\ \cos\theta \\ \tan\theta \end{pmatrix}$ 3 x 1

3. Determine the entries denoted by $a_{11}, a_{22}, a_{33}, a_{44}$ from the matrix

$$\begin{pmatrix} 2 & 1 & 3 & 4 \\ 5 & 9 & -4 & \sqrt{7} \\ 3 & \frac{5}{2} & 8 & 9 \\ 7 & 0 & 1 & 4 \end{pmatrix}$$

$a_{11} = 2$
$a_{22} = 9$
$a_{33} = 8$
$a_{44} = 4$

40. When giving the order of a matrix, you should always mention the number of rows first, followed by the number of columns.

41. The number of column(s) in a column matrix is one.

42. The number of row(s) in a row matrix is one.

43. The non-diagonal elements in any unit matrix are zeros.

44. Does there exist a square matrix with 32 elements? No.
[∵ 32 is not a perpect square nuber].

45. If A and B are any two non zero matrices, then $(A+B)^2 \neq A^2 + 2AB + B^2$.

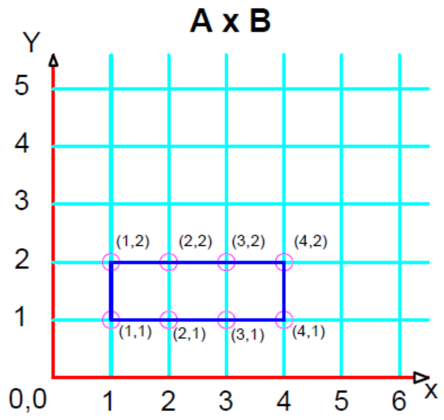
46. However if $AB = BA$, then $(A+B)^2 = A^2 + 2AB + B^2$

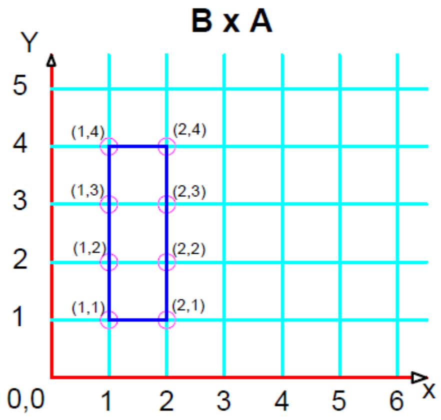
ACTIVITIES Chapter – 1

Activtiy – 1

Let A = {x | x ∈ ℕ, x ≤ 4}, B = {y | y ∈ ℕ, y < 3} Represent AxB and BxA in a graph sheet. Can you see the difference between AxB and BxA?

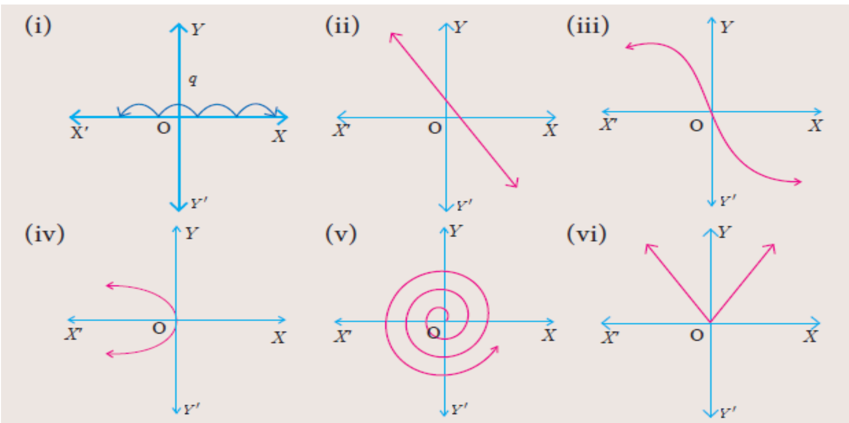
Solution : A = {1,2,3,4}; B = {1,2}
AxB = {(1,1), (1,2), (2,1), (2,2), (3,1), (3,2), (4,1), (4,2)}
AxB = {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4)}





Activtiy – 3

Check whether the following curves represent a function. In the case of a function, check whether it is one-one? (Hint: Use the vertical and the horizontal line tests)



(i). It is a fuction, but not an one-one.
(ii). It is an one-one function.
(iii). It is an one-one function.
(iv). It is not at all a function.
(v). It is not at all a function.
(vi). It is a fuction, but not an one-one.

Activtiy – 4

Given that h(x) = f o g(x) , fill in the table for h(x)						
x	f(x)	x	g(x)	x	h(x)	How to find h(1) ?
1	2	1	2	1	3	h(x) = f o g(x)
2	3	2	4	2	4	h(1) = f o g(1)
3	1	3	3	3	1	= f (2) = 3
4	4	4	1	4	2	Therefore, h(1) = 3

h(2) = f o g(2) = f (4) = 4; h(3) = f o g(3) = f (3) = 1; h(4) = f o g(4) = f (1) = 2

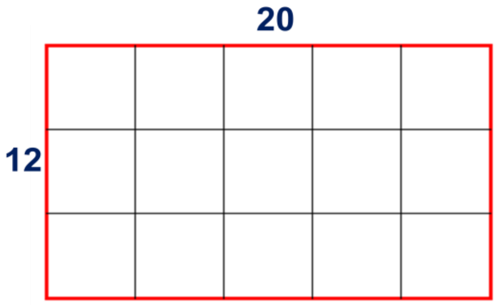
ACTIVITIES Chapter – 2

Activtiy – 1

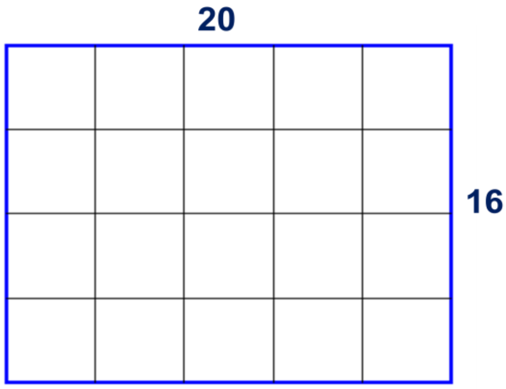
This activity helps you to find HCF of two positive numbers.

Find the HCF of (a) 12,20 (b) 16,24 (c) 11,9.

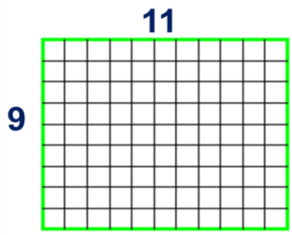
Trying with 1×1 square; 2×2 square; 3´3 square and so on.



4 x 4 square fill this rectangle without any gap. ∴ HCF = 4



4 x 4 square fill this rectangle without any gap. ∴ HCF = 4



1 x 1 square fill this rectangle without any gap. ∴ HCF = 1

Activtiy – 2

This is another activity to determine HCF of two given positive integers.
(i) From the given numbers, subtract the smaller from the larger number.
(ii) From the remaining numbers, subtract smaller from the larger.
Using this Activity, find the HCF of
(i) 90,15 (ii) 80,25 (iii) 40,16 (iv) 23,12 (v) 93,13

(i). 90-15=75; 75-15= 60; 60-15=45; 45-15=30; 30-15=15; ∴HCF = 15.
(ii). 80-25=55; 55-25= 30; 30-25=5; 25-5=20; 20-5=15; 15-5=10; 10-5=5; ∴HCF = 5.
(iii). 40-16=24; 24-16=8; 16-8=8; ∴HCF = 8.
(iv). 23-12=11; 12-11=1; ∴HCF = 1.
(v). 93-13=80; 80-13= 67; 67-13=56; 56-13=43; 43-13=30; 30-13=17; 17-13=4; 13-4=9; 9-4=5; 5-4=1; ∴HCF = 1.

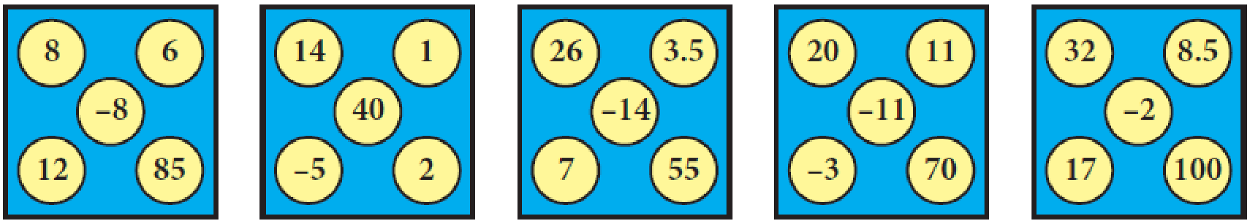
Activtiy – 3

Can you find the 4-digit pin number ‘pqrs’ of an ATM card such that $p^2 \times q^1 \times r^4 \times s^3$ = 3,15, 000 ?

315000 = 63x5x1000 = 9x7x 5x 8x125 = 3x3x7x 5x2x2x2x5x5x5 = $3^2 x 7^1 x 5^4 x 2^3$

Activtiy – 4

There are five boxes here. You have to pick one number from each box and form five Arithmetic Progressions.

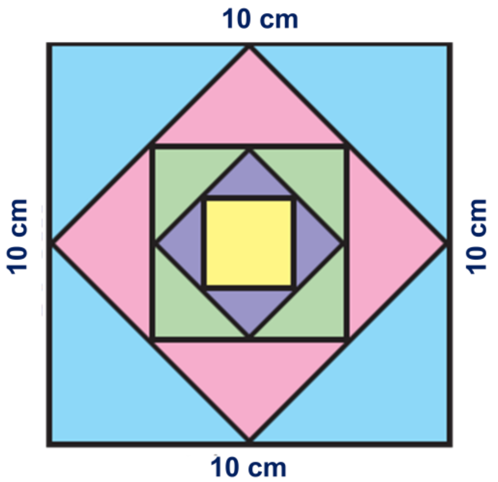


AP1 : –2, –5, –8, –11, –14. **AP2 : –3, 2, 7, 12, 17.** **AP3 : 1, 3.5, 6, 8.5, 11.**

AP4 : 8, 14, 20, 26, 32. **AP5 : 40, 55, 70, 85, 100.**

Activtiy – 5

The sides of a given square is 10 cm. The mid points of its sides are joined to form a new square. Again, the mid points of the sides of this new square are joined to form another square. This process is continued indefinitely. Find the sum of the areas and the sum of the perimeters of the squares formed through this process.



	Area (cm2)	Perimeter (cm)
Square 1	100	40
Square 2	50	$20\sqrt{2}$
Square 3	25	20
Square 4	12.5	$10\sqrt{2}$
Square 5	6.25	10

ACTIVITIES Chapter – 3

Activtiy – 1

(i) The length of a rectangular garden is the sum of a number and its reciprocal. The breadth is the difference of the square of the same number and its reciprocal. Find the length, breadth and the ratio of the length to the breadth of the rectangle.

Length = $x + 1/x = (x^2 + 1)/x$; Breadth = $x^2 - (1/x)^2 = (x^4 - 1)/x^2$

$L/B = [(x^2 + 1)/x] \div [(x^4 - 1)/x^2] = [(x^2 + 1)/x] \times [x^2/(x^4 - 1)] = x/(x^2 - 1)$

$L : B = x : x^2 - 1$

(ii) Find the ratio of the perimeter to the area of the given triangle.

Perimeter = $(13x + 37x + 40x) = 90x$;

Area = $1/2 \times 40x \times 12x = 240x^2$; $P/A = 90x \div 240x^2$; **$P : A = 3 : 8x$**

Activtiy – 2

Consider a rectangular garden in front of a house, whose dimensions are (2k + 6) metre and k metre. A smaller rectangular portion of the garden of dimensions k metre and 3 metres is leveled. Find the area of the garden, not leveled.

Area of the Garden = $(2k + 6)k = 2k^2 + 6k$

Area of Levelled Portion = $3k$

Unlevelled Garden Area = $2k^2 + 6k - 3k = 2k^2 + 3k = k(2k + 3)$

Activtiy – 3

Serve the fishes (Equations) with its appropriate food (roots). Identify a fish which cannot be served?

(i). $4x^2 + 12x + 9 = 0$; $4x^2 + 12x + 9 = (2x + 3)^2$; $x = -3/2, -3/2$; ∴It has solution.

(ii). $x^2 + 6x + 9 = 0$; $x^2 + 6x + 9 = (x + 3)^2$; $x = -3, -3$; ∴It has solution.

(iii). $x^2 - x - 20 = 0$; $x^2 - x - 20 = (x - 5)(x + 4)$; $x = 5, -4$; ∴It has solution.

(iv). $2x^2 - 5x - 12 = 0$; $2x^2 - 5x - 12 = (2x + 3)(x - 4)$; $x = 4, -3/2$; ∴It has solution.

(v). $x^2 - 1 = 0$; $x^2 - 1 = (x - 1)(x + 1)$; $x = 1, -1$; ∴It has solution.

(vi). $x^2 + 16 = 0$; $x^2 = -16$; No square root for negative value ∴It has no solution.

Activtiy – 4

(i). Take calendar sheets of a particular month in a particular year.

(ii). Construct matrices from the dates of the calendar sheet.

(iii). Write down the number of possible matrices of orders 2x2, 3x2, 2x3, 3x3, 4x3, etc.

(iv). Find the maximum possible order of a matrix that you can create from the given calendar sheet.

(v). Mention the use of matrices to organize information from daily life situations.

July - 2019

Sun	Mon	Tue	Wed	Thu	Fri	Sat
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

$$A_{2x2} = \begin{vmatrix} 1 & 2 \\ 8 & 9 \end{vmatrix} \begin{vmatrix} 2 & 3 \\ 9 & 10 \end{vmatrix} \begin{vmatrix} 3 & 4 \\ 10 & 11 \end{vmatrix} \begin{vmatrix} 4 & 5 \\ 11 & 12 \end{vmatrix} \begin{vmatrix} 5 & 6 \\ 12 & 13 \end{vmatrix} \begin{vmatrix} 7 & 8 \\ 14 & 15 \end{vmatrix}, \ldots, \begin{vmatrix} 23 & 24 \\ 30 & 31 \end{vmatrix}$$

$$B_{3x2} = \begin{vmatrix} 1 & 2 \\ 8 & 9 \\ 15 & 16 \end{vmatrix} \begin{vmatrix} 2 & 3 \\ 10 & 11 \\ 17 & 18 \end{vmatrix} \begin{vmatrix} 3 & 4 \\ 10 & 11 \\ 18 & 19 \end{vmatrix} \begin{vmatrix} 4 & 5 \\ 11 & 12 \\ 18 & 19 \end{vmatrix} \begin{vmatrix} 5 & 6 \\ 12 & 13 \\ 19 & 20 \end{vmatrix} \begin{vmatrix} 7 & 8 \\ 14 & 15 \\ 21 & 22 \end{vmatrix}, \ldots, \begin{vmatrix} 16 & 17 \\ 23 & 24 \\ 30 & 31 \end{vmatrix}$$

$$C_{2x3} = \begin{vmatrix} 1 & 2 & 3 \\ 8 & 9 & 10 \end{vmatrix} \begin{vmatrix} 2 & 3 & 4 \\ 9 & 10 & 11 \end{vmatrix} \begin{vmatrix} 3 & 4 & 5 \\ 10 & 11 & 12 \end{vmatrix} \begin{vmatrix} 4 & 5 & 6 \\ 11 & 12 & 13 \end{vmatrix}, \ldots, \begin{vmatrix} 22 & 23 & 24 \\ 29 & 30 & 31 \end{vmatrix}$$

$$D_{3x3} = \begin{vmatrix} 1 & 2 & 3 \\ 8 & 9 & 10 \\ 15 & 16 & 17 \end{vmatrix} \begin{vmatrix} 2 & 3 & 4 \\ 9 & 10 & 11 \\ 16 & 17 & 18 \end{vmatrix} \begin{vmatrix} 3 & 4 & 5 \\ 10 & 11 & 12 \\ 17 & 18 & 19 \end{vmatrix} \begin{vmatrix} 4 & 5 & 6 \\ 11 & 12 & 13 \\ 18 & 19 & 20 \end{vmatrix}, \ldots, \begin{vmatrix} 15 & 16 & 17 \\ 22 & 23 & 24 \\ 29 & 30 & 31 \end{vmatrix}$$

$$E_{4x3} = \begin{vmatrix} 1 & 2 & 3 \\ 8 & 9 & 10 \\ 15 & 16 & 17 \\ 22 & 23 & 24 \end{vmatrix}, \ldots, \begin{vmatrix} 8 & 9 & 10 \\ 15 & 16 & 17 \\ 22 & 23 & 24 \\ 29 & 30 & 31 \end{vmatrix} \quad F_{3x4} = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 8 & 9 & 10 & 11 \\ 15 & 16 & 17 & 18 \end{vmatrix}, \ldots, \begin{vmatrix} 14 & 14 & 16 & 17 \\ 21 & 22 & 23 & 24 \\ 28 & 29 & 30 & 31 \end{vmatrix}$$

$$G_{4x4} = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 8 & 9 & 10 & 11 \\ 15 & 16 & 17 & 18 \\ 22 & 23 & 24 & 25 \end{vmatrix}, \ldots, \begin{vmatrix} 7 & 8 & 9 & 10 \\ 14 & 15 & 16 & 17 \\ 21 & 22 & 23 & 24 \\ 28 & 29 & 30 & 31 \end{vmatrix}$$

And Similarly we can make 2x4, 2x5, 2x6, 3x5, 3x6, 3x7, 4x2, 4x5, 4x6, 5x2, 5x3 matrices.

The Highest order of matrix is 4x6 $= \begin{vmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 8 & 9 & 10 & 11 & 12 & 13 \\ 15 & 16 & 17 & 18 & 19 & 20 \\ 22 & 23 & 24 & 25 & 26 & 27 \end{vmatrix}$

Activity 5

No.	Elements	Possible orders	Number of possible Orders
1.	4	1 x4, 4 x1, 2 x2	3
2.	9	1×9, 9×1, 3×3	3
3.	20	1x20, 20x1, 2x10, 10x2, 4x5, 5x4	6
4.	8	1 x8, 8 x1, 2 x 4, 4 x 2	4
5.	1	1 x 1	1
6.	100	1x100, 100x1, 2x50, 50x2, 4x25, 25x4, 5x20, 20x5, 10x10	9
7.	10	1×10, 10×1, 2×5, 5×2	4

Do you find any relationship between number of elements (second column) and number of possible orders (fourth column)? If so, what is it?

Yes. The number of possible order is equal to the number of factors of element's number.

Elements	Factors	No. of Factors	Possible order
4	1, 2, 4	3	3
9	1, 3, 9	3	3
20	1, 2,4,5,10,20	6	6
8	1, 2, 4, 8	4	4
1	1	1	1
100	1, 2, 4, 5, 10, 20,25,50,100	9	9
10	1, 2, 5, 10	4	4

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மிக மிக எளிய பாடம் கணக்கே!

1. கணக்கு மிக மிக எளிய பாடமே. மன பயத்தால் தான் கடினமாக தோன்றுகிறது.

2. நம்முடைய மனபயத்தை நீக்கினால் மிக எளிதாக புரிந்து விடும்.

3. சிந்தனையும் புரிதலும் தான் நம்மை அடுத்த படிக்கு முன்னேற வைக்கும்.

4. குருட்டு மனப்பாடம் இருட்டறைக்குள் தள்ளிவிடும்.

5. புதிதாக ஓர் ஊருக்கு அல்லது ஓர் இடத்திற்குச் செல்லும் போது அதன் வழியை எவ்வாறு நினைவில் வைத்துக் கொள்வோமோ அதுபோல கணக்கில் அதன் வழிமுறையை நினைவில் கொள்ள வேண்டும்.

6. எந்த ஒரு கணக்கிற்கும் அடிப்படையான ஒன்றும் பொதுவான ஒன்றும் சிறப்பான ஒன்றும் இருக்கும். அதை நன்கறிந்து பின் அதிலிருந்து பிரிந்து விரிவாக்கம் செய்வதே நல்லது.

7. அடிப்படைச் செயலான (+ — x ÷) ஆகியவற்றில் சுருக்க முறைகள், வாய்ப்பாடு, சூத்திரங்கள், வழிமுறைகள் ஆகிய இந்த நான்கையும் நன்கறிதல் அவசியம்.

8. தினமும் இரவு படுக்கச் செல்லுமுன் 7ல் குறிப்பிட்டவற்றை மனதில் நினைவு கூர்வது நல்ல பலனைக் கொடுக்கும்.

9. சந்தேகங்கள் நிறைய எழுதல் அவசியம். அதே சமயம் அதை உடனுக்குடன் கூச்சப்படாமல் நன்கறிந்த யாரிடமாவது கேட்டுத் தீர்த்துக் கொள்ள வேண்டும்.

10. நன்கு படிக்கும் சில நண்பர்களுடன் கூட்டுப் பயிற்சி செய்தல் வேண்டும்.

11. ஒவ்வொரு வகுப்பு கணக்கும் அதற்கு முன் வகுப்பை அடிப்படையாகக் கொண்டும் அதேசமயம் அடுத்த வகுப்பிற்கு அடித்தளமாகவும் கொண்ட ஒரு நீண்ட தொடர் பாடமாகும்.

12. எனவே அந்தந்த வகுப்பு கணக்கை சந்தேகமின்றி கற்றல் மிக அவசியம்.

13. ஒரு நீண்ட நாடகத் தொடர் பார்ப்பவர் ஏதாவது ஒரு நாள் பார்க்காவிடின் அடுத்தவரிடம் அதை ஆவலுடன் கேட்டுத் தெரிந்து கொள்வதைப் போல் கணிதம் என்ற தொடர் பாடத்தை ஆவலுடன் கேட்டு தொடர்பு படுத்திப் படிக்க வேண்டும்.

14. ஞாபகசக்தி, நற்சிந்தனைக்கு பச்சைக் காய்கறிகள், ஏதேனும் ஒரு கீரை மற்றும் சைவம் மிகச் சிறந்தது.

15. ஆதவன் ஒளியைக் குவித்தால் நெருப்பு எளிது. ஆசான் ஒலியைக் கூர்ந்து கவனித்தால் கணக்கும் எளிதே.

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10^{th} Maths Solution to Exercise 2.1 and 2.2

Numbers (A short Remainder)

Introduction to numbers (It is nothing new to us)

1. Yes. In our earlier classes, to checkup the division we have used one formula i.e. $Dividend(a) = Divisor(b) \times (q)uotient + (r)emainder$. i.e. $a = b \times q + r$. This basic concept is applied here.
2. Before going into the subject, first we have to know the numbers like $\mathbb{N}$, $\mathbb{W}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, Prime, Composite, odd , even etc very clear.
3. We can split every composite numbers into prime factors like $100 = 2^2 + 5^2$.
4. But we can not split the prime numbers into prime factors. ∵ It itself a prime one.
5. 0 (zero) is neither a positive integer nor a negative integer.
6. Similary 1 is neither a prime number nor a composite number.
7. Knowing of divisibility check for 2, 3, 4, 5, 6, 8, 9, 10, 11, etc is much useful here.

Euclid's Division Lemma

1. Theorem 1 : Let a and b (a >b) be any two positive integers. Then, there exist unique integers q and r such that $a = bq + r,\ 0 \le r < b$. (Here $a > b$ and $r = 0\ to\ b - 1$. If r = 0, then 'a' is divisible by 'b' or 'b' is a divisor to 'a'.)
2. Theorem 2 : If a and b are positive integers such that $a = bq$ +r , then every common divisor of a and b is a common divisor of b and r and vice-versa. (We can find the HCF by using this Euclid's Division Algorithm continuously till the remainder gets o.)
3. Theorem 3 : If a, b are two positive integers with a > b, then GCD of (a, b) = GCD of $(a - b,\ b)$. (We can find the HCF by this subtraction method also.)
4. HCF (Highest Common Factor) is also known as GCD (Greatest Common Factor).
5. To find the HCF of 3 numbers, first find the HCF of the lowest two. And from this HCF and the 3rd number find new HCF which will be the HCF of the given 3 numbers.

Fundamental Theorem of Arithmetic (Throem – 4)

1. Every natural number except 1 can be factorized as a product of primes and this factorization is unique except for the order in which the prime factors are written.
2. Every composite number N, can spilit uniquely into the product of the power of primes. $N = p_1^{q_1} \times p_2^{q_2} \times p_3^{q_3} \times p_4^{q_4} \times \dots \times p_n^{q_n}$; For example : $18900 = 2^2 \times 3^3 \times 5^2 \times 7^1$.

Modulo Operations

1. Similar to basic arithmetic operations like addition, subtraction and multiplication performed on numbers we can think of performing same operations in modulo arithmetic.
2. Theorem 5 : If a, b, c and d are integers and m is a positive integer such that if $a \equiv b\ (mod\ m)$ and $c \equiv d\ (mod\ m)$ then,

$(i). (a+c) \equiv (b+d)(mod\ m)$ $(ii). (a-c) \equiv (b-d)(mod\ m)$ $(iii). (a \times c) \equiv (b \times d)(mod\ m)$

It means, like modulo can be added, subtracted, multiplied and divided.

3. **Theorem 6 : Any modulo can be added , subtracted, multiplied and divided by a constant integer c or it may be squared, cubed etc.**

   Suppose If $a \equiv b\ (mod\ m)$ **, then**

   $(i)\ a + c \equiv b + c\ (mod\ m)$; $(ii)\ a - c \equiv b - c\ (mod\ m)$;$(iii)\ a \times c \equiv b \times c\ (mod\ m)$

   If $a \equiv b\ (mod\ m)$ **; then squaring on both sides,** $a^2 \equiv b^2\ (mod\ m)$ **and likewise.**

To find the time after/before certain hour from the given time.

1. **Convert the given present time into 24 hr format. Then add or subtract the time according to after/before.**
2. **Divide it by 24 and find the remainder. Now the remainder is the required time in 24 hr format. Convert it to 12 hr format accordingly.**
3. **If the given present time is alredy in 24 hr format , then there is no need for conversion.**

   An example :The time 122 hours after 9 a.m **(To divide by 24, use 25th table)**

   The given time = 9 a.m = 9 + 12 = 21 hr (24 hr format) $(1 \times 24 = 25 - 1 = 24)$

   The required time 112 hr after. ∴21 + 122 =133 hr $(2 \times 24 = 50 - 2 = 48)$

   By Dividing 133 by 24 , 133 = 5 x 24 + 13 $(a = bq + r)$ $(5 \times 24 = 125 - 5 = 120)$

   The remainder is 13. The required time is 13 hr (24 format) or 1 p.m (12 hr format).

To find the day after/before certain days from the given day.

1. **Note the day given.**
2. **Divide the number of days given by 7 (∵A week has 7days). Find the remaider.**
3. **If the remainder is 0, the required is the day what is given initially. If it is 1, the required day is the next day of the given day and so on.**

   An example : Today is Friday. What will be day after 100 days?.

   The given day is Friday ; The required day is after 100 day

   By Dividing 100 by 7 , 100 = 14 x 7 + 2 $(a = bq + r)$

   The remainder is 2. ∴The required day is the 2nd from Friday i.e. Sunday.

Exercise 2.1

1. **All the positive integers are : 1, 2, 3, 4, 5, 6 ,7, 8, 9,10, 11, ……**

   When subtracting 1 from multiples of 3 , it leaves a remainder of 2

   i.e. 3n – 1 always gives a remainder by dividing 3 where n is a positive integer

   $3 \times 1 - 1,\ 3 \times 2 - 1,\ 3 \times 3 - 1, 3 \times 1 - 1,\ 3 \times 4 - 1,$ ...

   2, 5, 8, 11, ... are leaves a remainder by diving by 3.

2. **Total number flower pots = 532**

   Number flower pots in each row = 21.

   ∴ Number of required rows = $532 \div 21$

   By dividing , $532 = 25 \times 21 + 7$

   Number of rows = 25, Remaining flower pots = 7.

3. **Any two consecutive positive integers have an one odd number and an one even number. So it's product is always an even number.**

   ∴The product of any two consecutive positive integers is always divisible by 2.

4. It can be easily solved by modulo arithmetic method.

We know $a \equiv r \ (mod\ q)$

Here $a \equiv 9 \ (mod\ 13)$

$b \equiv 7 (mod\ 13)$

$c \equiv 10 \ (mod\ 13)$

Adding these three, $a + b + c \equiv (9 + 7 + 10) \ (mod\ 13)$

$a + b + c \equiv 26 \ (mod\ 13)$

$a + b + c \equiv 0 \ (mod\ 13)$ 〔∵ 26 is divisible by 13 , r = 0〕

∴ $a + b + c$ is always divisible by 13

5. Let x be an any integer.

(i) If it is an even, then $x = 2k$

Squaring both sides $x^2 = (2k)^2$

$x^2 = 4k^2 = 4Q$ where $Q = k^2$

$x^2 = 4Q$ (∵ It is divisible by 4, the remainder $r = 0$

(ii) If it is an odd, then $x = 2k + 1$

Squaring both sides $x^2 = (2k + 1)^2$

$x^2 = 4k^2 + 4k + 1$

$x^2 = 4(k^2 + k) + 1$

$x^2 = 4Q + 1$ where $Q = (k^2 + k)$

$= 4Q + 1$ (It leaves a remainder 1, when it is divided by 4)

∴ The Square of any integer is divided by 4, it leaves a remainder 0 or 1.

6. Let us solve (iii) and (iv) leaving first two for your practice.

(iii) <u>To find HCF of 10224 and 9648</u>

Using Euclid's division algorithm, $10224 = 9648 \times 1 + 576$; $(r \neq 0)$

Again applying, $9648 = 576 \times 16 + 432$; $(r \neq 0)$

Again applying, $576 = 432 \times 1 + 144$; $(r \neq 0)$

Again applying, $432 = 144 \times 3 + 0$ $(r = 0)$

∴The HCF of 10224 and 9648 is 144.

(iv) <u>To find HCF of 84, 90 and 120</u>

Beeing 3 numbers, first the HCF of the lowest two numbers is to be found.

(i) HCF of 84 and 90,

Using Euclid's division algorithm, $90 = 84 \times 1 + 6$; $(r \neq 0)$

Again applying, $84 = 6 \times 24 + 0$ $(r = 0)$

∴The HCF of 84 and 90 is 6.

(ii) HCF of 6 and 120,

Using Euclid's division algorithm, $120 = 6 \times 20 + 0$ $(r = 0)$

∴ The HCF of 6 and 120 is 6.

∴The HCF of 84, 90 and 120 is 6.

7. Let the largest divisor be x.

Using the x as a divisor, 1230 leaves a remainder 12.

∴(1230 – 12) = 1218 is divisible by x.

Using the x as a divisor, 1926 leaves a remainder 12.

∴(1926 – 12) = 1914 is divisible by x.

So x is the HCF of 1218 and 1914.

(iii) To find HCF of 1218 and 1914

Using Euclid's division algorithm, $1914 = 1218 \times 1 + 696$; $(r \neq 0)$

Again applying, $1218 = 696 \times 1 + 522$; $(r \neq 0)$

Again applying, $696 = 522 \times 1 + 174$; $(r \neq 0)$

Again applying, $522 = 174 \times 3 + 0$; $(r = 0)$

∴The HCF of 1218 and 1914 is 174.

∴The required largest divisor is 174.

8. To find HCF of 32 and 60

Using Euclid's division algorithm, $60 = 32 \times 1 + 28$; $(r \neq 0)$

Again applying, $32 = 28 \times 1 + 4$; $(r \neq 0)$

Again applying, $28 = 4 \times 7 + 0$ $(r = 0)$

∴The HCF of 32 and 60 is 4. ∴ d = 4

$d = 32x + 60y \rightarrow 32x = d - 60y$

$$x = \frac{d-60y}{32}$$

When y = 1, $x = \frac{4-60\times1}{32} = \frac{56}{32}$ (It is not an integer.)

When y = -1, $x = \frac{4-60\times(-1)}{32} = \frac{64}{32} = 2$ (It is an integer.)

$\therefore x = 2, \quad y = -1.$

9. Given : Divisor (q) = 88, Remainder(r) = 61

We know that, $a = b \times q + r$

$a = b \times 88 + 61$

When b = 1, $a = 1 \times 88 + 61 = 149$

Now , a = 149, q = 11, r = ?

Using Euclid's Division Lemma, $149 = 13 \times 11 + 6$

∴ The required remainder = 6.

10. Any two consecutive integers are in the form of $2k, \ 2k+1$.

<u>To find HCF of $2k, \ 2k+1$</u>

Using Euclid's division algorithm, $2k + 1 = 2k \times 1 + 1$; $(r \neq 0)$

Again applying, $2k = 1 \times 2k + 0$ $(r = 0)$

∴The HCF of any two consecutive positive integers is 1.

∵ The HCF is 1, any two consecutive positive integers are coprime to each other.

Exercise 2.2

1. Given : The unit digit of 4^n is to be 6

When $n = 1, \ 4^1 = 4$; [Ends with 4]

$n = 2, \ 4^2 = 16$; [Ends with 6]

$n = 3,\ 4^3 = 64$; [Ends with 4]

$n = 4,\ 4^4 = 256$; [Ends with 6]

∴ The unit digit of 4^n is 6 when n takes the even natural numbers.

2. **Given :** $2^n \times 5^m$

The 2^n is always an even number for all natural numbers n, i.e. $2^1 = 2,\ 2^2 = 4$, etc

The unit digit of 5^m **is always 5 for all natural numbers m, i.e.** $5^1 = 5,\ 5^2 = 25$, etc

The unit digit of the product of an even number with 5 is always 0.

For example $2^3 \times 5^2 = 8 \times 25 = 200$

∴ For no such value m with $2^n \times 5^m$ ends with 5.

3. **To find the HCF of 252525 and 363636**

This can be solved in two method for dull students

Method 1 : Factorization method.

$252525 = 25 \times 10101 = 5 \times 5 \times 10101$

$363636 = 36 \times 10101 = 4 \times 9 \times 10101 = 2 \times 2 \times 3 \times 3 \times 10101$

∵ 10101 is common in both, it need not necessary to factorize it further.

∴The common number of the above two is 10101, which is the LCM of the given number.

Method 2 : Subtraction method. (Subtract the smaller from the larger always)

(As given and practised in the Activity 2)

$363636 - 252525 = 111111 \quad (\neq 0)$

$252525 - 111111 = 141414 \quad ''$

$141414 - 111111 = 30303 \quad ''$

$111111 - 30303 = 80808 \quad ''$

$80808 - 30303 = 50505 \quad ''$

$50505 - 30303 = 20202 \quad ''$

$30303 - 20202 = 10101 \quad ''$

$20202 - 10101 = 10101 \quad ''$

$10101 - 10101 = 0$

∴ 10101 is HCF of 252525 and 363636.

4. **Given :** $13824 = 2^a \times 3^b$

By factorization of 13824, (In the L division, Start with the smallest prime number)

$13824 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$

$13824 = 2^9 \times 3^3 = 2^a \times 3^b$

Equating the powers, $a = 9,\ \ b = 3$

5. **Given :** $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$

By factorization of 113400,

$113400 = 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7$

$13824 = 2^3 \times 3^4 \times 5^2 \times 7^1 = p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4}$

Equating the bases, $p_1 = 2,\ \ p_2 = 3,\ p_3 = 5\ \ p_4 = 7$

Equating it's powers, $x_1 = 3,\ \ x_2 = 4,\ x_3 = 2\ ,\ x_4 = 1$

6. Given : To find LCM and HCF of 408 and 170

Fundamental Theorem of Arithmetic : Every composite number can be uniquely split into the product of powers of primes.

By factorization of 408 and 170, it becomes,

$408 = 2 \times 2 \times 2 \times 3 \times 17 = 2^3 \times 3^1 \times 17^1$

$170 = 2 \times 5 \times 17 = 2^1 \times 5^1 \times 17^1$

(LCM = All primes (Common one time) with its highest power.)

(HCF = Common primes only with its lowest power.)

LCM = $2^3 \times 3^1 \times 5^1 \times 17^1 = 8 \times 3 \times 5 \times 17 = 2040$

HCF = $2^1 \times 17^1 = 2 \times 17 = 34$

7. The greatest of 6 digit number = 999999

LCM of 24, 15 and 36 is

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$

$15 = 3 \times 5 = 3^1 \times 5^1$

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

LCM of $24, 15, 36 = 2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 360$

Using Euclid's Division Lemma, $a = b \times q + r$

Putting $a = 999999,\ q = 360$

$999999 = b \times 360 + r$

$999999 = 2777 \times 360 + 279$

$999999 - 279 = 2777 \times 360$

$999720 = 2777 \times 360$

∴ The 6 digit number of 999720 is exactly divisible by 24, 15 and 36.

8. LCM of 35, 56, 91 is

$35 = 5 \times 7 = 5^1 \times 7^1$

$56 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7^1$

$91 = 7 \times 13 = 7^1 \times 13^1$

LCM of $35, 56, 91 = 2^3 \times 5^1 \times 7^1 \times 13^1 = 8 \times 5 \times 7 \times 13 = 3640$

∴ The least number which leaves a remainder 7 by dividing 35, 56, 91 = $3640 + 7 = 3647$

9. The least number which is divisible from the first 10 natural numbers is the LCM of 1 to 10. (Here 1 is the divisor to all, 2 is the divisor to all even number, 5 is the divisor to 10, 3 is the divisor to 6 and 9, 4 is the divisor to 8, 6 is the divisor to 9x10.)

So It is enough to find the LCM for 7, 8, 9, 10 leaving 1, 2, 3, 4, 5, 6.

$7 = 7^1,\ 8 = 2^3,\ 9 = 3^3,\ 10 = 2^1 \times 5^1$

LCM of $1\ to\ 10 = 2^3 \times 3^3 \times 5^1 \times 7^1 = 8 \times 9 \times 5 \times 7 = 2520$

∴ The least number which is divisible from the first 10 natural numbers = 2520

சிந்திய வியர்வை உணவாகும்
சிந்தையிற் கல்வி அறிவாகும்
விந்தைகள் பலவும் எளிதாகும்
தந்தையுந் தாய்க்கும் மகிழ்வாகும்

Better Late Than Never

Exercise 2.3

① (i) $71 \equiv x \pmod 8$

$71 - x = 8n$ for some integer n.

[It means $(71-x)$ is divisible by 8]

If we put $x = 7$

$71 - 7 = 64$ which is divisible by 8.

$\therefore x = 7$ (Positive value only)

(ii) $78 + x \equiv 3 \pmod 5$

$78 + x - 3 = 5n$ for 'n' integer.

$75 + x = 5n$

$75 + 5 = 80$ which is div. by 5.

$\therefore x = 5$. (Positive value)

(iii) $89 \equiv (x+3) \pmod 4$

$89 - x - 3) = 4n$.

$86 - x = 4n$.

$86 - 2 = 84$ which is div. by 4.

$\therefore x = 2$

(iv) $96 \equiv \frac{x}{7} \pmod 5$

$96 - \frac{x}{7} = 5n$

$96 - \frac{7}{7} = 95$ which is div. by 5

$\therefore x = 7$

(v) $5x \equiv 4 \pmod 6$

$5x - 4 = 6n$

(Try $x = 1, 2, 3$ etc.)

$5 \times 2 - 4 = 6$ which is div. by 6.

$\therefore x = 2$.

② $x \equiv 13 \pmod{17}$

$x - 13 = 17n$

Putting $n = 1$, $x = 17 + 13 = 30$.

$7x - 3 \equiv y \pmod{17}$

$7 \times 30 - 3 \equiv y \pmod{17}$

$207 - y = 17n$.

If $y = 3$ $\quad 207 - 3 = 204$ is div. by 17.

$\therefore$ The required value $= 3$

③ $5x \equiv 4 \pmod 6$

$5x - 4 = 6n$

Putting $n = 1$, $5x = 6 + 4$

$x = 10/5 = 2$

$\therefore x = 2, (2+6), (2+12), \ldots$

$= 2, 8, 14, \ldots$

④ $3x - 2 \equiv 0 \pmod{11}$

$3x - 2 = 11n$.

Putting $n = 1$, $3x = 11 + 2$

$x = \frac{13}{3}$ (Fractional value)

If $n = 2$, $3x = 11 \times 2 + 2$

$x = \frac{24}{3} = 8$.

$x = 8, (8+11), (8+22), \ldots$

$= 8, 19, 30, \ldots$

Hints for Days Problem

1. Note what day is given.
2. Find the remainder for the days given with Mod 7.
3. If the remainder is Zero, the required day is the same day of given.
4. If remainder is 1, the required day is the next day of given day
5. Similarly and so on.

⑦ The given day is Tuesday.

The day to be found is after 45 days.

$45 = 7 \times 6 + 3$.

The remainder is 3.

The 3rd day after Tuesday is

Friday.

(8) $9 \equiv 2 \pmod 7$

As per modulo operations

$9^n \equiv 2^n \pmod 7$

$6\times 9^n \equiv 6\times 2^n \pmod 7$

$2^n + 6\times 9^n \equiv [2^n + 6\times 2^n] \pmod 7$

$2^n+6\times 9^n \equiv 2^n[1+6] \pmod 7$

$2^n+6\times 9^n \equiv 2^n[7] \pmod 7$

$2^n+6\times 9^n \equiv [2^n\times 0] \pmod 7$

$2^n+6\times 9^n \equiv 0 \pmod 7$

$\therefore$ $2^n+6\times 9^n$ is always div. by 7

Check: put $n=2$.

$2^2+6\times 9^2 = 490$ which is div. by 7

---

(9)

$16 \equiv -1 \pmod{17}$

$2^4 \equiv -1 \pmod{17}$

$(2^4)^n \equiv (-1)^n \pmod{17}$

$(2^4)^{20} \equiv (-1)^{20} \pmod{17}$

$2^{80} \equiv 1 \pmod{17}$

$2^{80}\times 2 \equiv 1\times 2 \pmod{17}$

$2^{81} \equiv 2 \pmod{17}$

$\therefore$ The remainder $= 2$

Hints For Timing Problems

1. convert given time into Railway time
2. Add or subtract the hours after or before accordingly.
3. Find the result in the mode of 24.
4. According to the remainder write it in 12 hrs Format.

---

(5) Given Time 7 am = 7.00 hr.

7.00 + 100.00 = 107 hr

107 = 24 × 4 + 11

The remainder is 11.00 hr.

$\therefore$ Time After 100 hr it is 11.00 am.

---

(6) Given time = 11. pm = 23.00 hr.

23.00 − 15.00 = 8.00.

8 = 24 × 0 + 8

The remainder 8.00 hr.

$\therefore$ Time Before 15 hr = 8.00 Am

---

(10) Starting Time = 23.30 hr Sunday.

Journey Time = 11.00 hr

34.30

34.30 = 24 × 1 + 10.30

The remainder is 10.30 hr. Monday

Time ahead (−) 4.30

6.00 hr.

The reaching time at London is 6.00 AM, Monday

10th Maths (N.S)

Example : 2.28 (i)

Let 'a' and 'd' be the first term and common difference.

We know that, $t_n = a+(n-1)d$.

$\therefore\ x = a+(l-1)d$ ——— ①

$y = a+(m-1)d$ ——— ②

$z = a+(n-1)d$ ——— ③

① − ② ⇒ $x-y = (l-m)d$

$\therefore (l-m) = \frac{x-y}{d}$

② − ③ ⇒ $y-z = (m-n)d$

$\therefore (m-n) = \frac{y-z}{d}$

③ − ① ⇒ $z-x = (n-l)d$

$\therefore (n-l) = \frac{z-x}{d}$

Now

$x(m-n)+y(n-l)+z(l-m)$

$= \frac{1}{d}\left[x(y-z)+y(z-x)+z(x-y)\right]$

$= \frac{1}{d}\left[xy-xz+yz-xy+xz-yz\right]$

$= \frac{1}{d}[0] = 0$

10th Maths (N.S)

Example: 2.34

Given: $t_{13} = 3$, $S_{13} = 234$.

$S_n = \frac{n}{2}[a+l]$

$S_{13} = \frac{13}{2}[a+3] = 234$

$[a+3] = \frac{\cancel{234}^{18} \times 2}{\cancel{13}}$

$a+3 = 36$

$a = 36-3$

$= 33$

$t_{13} \Rightarrow a+12d = 3$

$33+12d = 3$

$d = \frac{3-33}{12} = \frac{-30}{12}$

$= -\frac{5}{2}$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{21} = \frac{21}{2}\left[2\times33+(21-1)\left(-\frac{5}{2}\right)\right]$

$= \frac{21}{2}[66-50]$

$= \frac{21\times16}{2} = 168$

10th Maths (N.S)

Exercise : 2.5 (9)

Given: $x, 10, y, 24, z$ are in A.P.

Let it be: t_1, t_2, t_3, t_4, t_5.

W.K.T: $t_2 - t_1 = d$

$t_3 - t_1 = 2d$.

$\therefore\ t_4 - t_2 = 24 - 10$

$2d = 14$

$d = 7$

$\therefore\ x = 10-7 = 3$.

$y = 10+7 = 17$

$z = 24+7 = 31$.

10th Maths (N.S)

Example : 2.35

Given: $S_n = \frac{5n^2}{2} + \frac{3n}{2}$

$S_1 = \frac{5\times1^2}{2} + \frac{3\times1}{2} = 4$

$S_2 = \frac{5\times2^2}{2} + \frac{3\times2}{2} = 13$

$t_1 = S_1 = 4$

$t_2 = S_2 - S_1 = 13-4 = 9$

$\therefore\ a = 4,\ d = 5$

$t_n = a+(n-1)d$

$t_{17} = 4+16\times5$

$= 84$

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10th Maths N.S

Exercise : 2.6 (4)

Given : $S_n = 2n^2 - 3n$
$= n[2n-3]$

$S_1 = 1[2\times1-3] = -1$

$S_2 = 2[2\times2-3] = 2$

$S_3 = 3[2\times3-3] = 9$

$S_4 = 4[2\times4-3] = 20$

$t_1 = S_1 = -1$

$t_2 = S_2 - S_1 = 2-(-1) = 3$

$t_3 = S_3 - S_2 = 9-2 = 7$

$t_4 = 20-6 = 20-9 = 11$

Here the common differences are equal and it is 4, with first term 'a' as (-1)

So the given series an A.P.

10th Maths (N.S)

Exercise : 2.6 (11)

Given:

Series	First Term	Common diff.	No. of Terms.	Sum of Terms.
AP_1	1	1	n	S_1
AP_2	2	3	n	S_2
AP_3	3	5	n	S_3
…	…	…	…	…
AP_m	m	$(2m-1)$	n	S_m.

$S_1 = \frac{n}{2}[2\times1+(n-1)1] = \frac{n(n+1)}{2}$

$S_2 = \frac{n}{2}[2\times2+(n-1)3] = \frac{n}{2}[3n+1]$.

$S_3 = \frac{n}{2}[2\times3+(n-1)5] = \frac{n}{2}[5n+1]$

$S_2 - S_1 = \frac{n}{2}[3n+1-n-1] = n^2$

$S_3 - S_2 = \frac{n}{2}[5n+1-3n-1] = n^2$.

Here the $d = n^2$

So, $S_1+S_2+S_3+\dots+S_m$ is again an AP series with S_1 ie $a = \frac{n}{2}(n+1)$ and $d = n^2$. for m terms.

$\therefore S_1+S_2+S_3+\dots+S_m$ (m terms)

$= \frac{m}{2}\left[2\times\frac{n(n+1)}{2} + (m-1)n^2\right]$

$= \frac{m}{2}\left[\cancel{n^2}+n+mn^2-\cancel{n^2}\right]$

$= \frac{m}{2}[mn^2+n]$

$= \frac{mn}{2}[mn+1]$

10th Maths (N.S)

Exercise : 2.7 (12)

Given : a, b, c are A.P.
x, y, z are in G.P.

Let 'd' be the common diff. of A.P and 'r' be the common ratio of G.P.

$\therefore a-b = -d$
$b-c = -d$
$c-a = 2d$

And if $y = y$

Then $x = \frac{y}{r}$, $z = yr$

LHS $x^{b-c} \times y^{c-a} \times z^{a-b}$

$= \left(\frac{y}{r}\right)^{-d} \times y^{2d} \times (yr)^{-d}$

$= \frac{y^{-d}}{r^{-d}} \times y^{2d} \times y^{-d} r^{-d}$

$= \frac{r^d}{y^d} \times y^{2d} \times \frac{1}{y^d r^d}$

$= \frac{r^d y^{2d}}{r^d y^{2d}}$

$= 1$ (RHS)

10th Maths (N.S)

Exercise : 2.7 (8)

Given: a, b, c are in A.P.

Let 'd' be the Common diff.

$\therefore a = b - d \rightarrow (1)$

$c = b + d. \rightarrow (2)$

The given sequence: $3^a, 3^b, 3^c$

From (1) and (2)

The above sequence may be rewriten as,

$3^{b-d}, 3^b, c^{b+d}$

The common ratios are

$\frac{3^b}{3^{b-d}} = \frac{3^{b+d}}{3^b}$

$3^d = 3^d$

which are same

$\therefore 3^a, 3^b, 3^c$ are in G.P.

10^{th} Maths (N.S)

Exercise: 2.9 (7)

Given Series: $(2^3-1)+(4^3-3^3)+(6^3-5^3)+\cdots$

(i) $(2^3-1)+(4^3-3^3)+(6^3-5^3)+\ldots\ldots n$ terms.

$= (2^3+4^3+6^3+\ldots n \text{ term})$
$\quad - (1^3+3^3+5^3+\ldots n \text{ terms})$

$= 2^3(1^3+2^3+3^3+\ldots n \text{ terms})$
$\quad - (1^3+3^3+5^3+\ldots n \text{ terms})$

$= 2^3\left[\dfrac{n(n+1)}{2}\right]^2 - n^2(2n^2-1)$

$\left[\because \text{ Sum of cubes of odd natural Numbers } = n^2(2n^2-1)\right]$

$= 2^3\dfrac{n^2(n+1)^2}{2^2} - 2n^4+n^2$

$= 2n^2(n^2+2n+1) - 2n^4+n^2$

$= 2n^4+4n^3+2n^2-2n^4+n^2$

$= 4n^3+3n^2$

(ii) $(2^3-1)+(4^3-3^3)+(6^3-5^3)+\ldots 8$ terms

$= 4\times 8^3+3\times 8^2$

$= 2048+192$

$= 2240$

10^{th} Maths (N.S)

Exercise : 2.8 (9)

Given : $0.\overline{123}$

$0.\overline{123} = 0.123123123\ldots\ldots$

$= 0.123 + 0.000123 +$
$\quad 0.000000123 + \ldots\ldots$

Now it is sum of infinitive terms

$a = 0.123$, $r = \dfrac{0.000123}{0.123} = \dfrac{1}{1000}$

$S_\infty = \dfrac{a}{1-r}$

$= \dfrac{0.123}{1-\frac{1}{1000}}$

$= \dfrac{0.123\times 1000}{999}$

$= \dfrac{123}{999} = \dfrac{41}{333}$

$\therefore 0.\overline{123} = \dfrac{41}{333}$

10^{th} Maths (N.S)

Exercise : 2.8 (10)

Given : $S_n = (x+y)+(x^2+xy+y^2)+$
$\quad (x^3+x^2y+xy^2+y^3)+\ldots\ldots$

Multiply $(x-y)$ on both sides.

$(x-y)S_n = (x-y)(x+y)+(x-y)(x^2+xy+y^2)$
$\quad +(x-y)(x^3+x^2y+xy^2+y^3)+\ldots$

$= (x^2-y^2)+(x^3-y^3)+(x^4-y^4)+\ldots$

$= (x^2+x^3+x^4+\ldots\ldots n \text{ terms})$
$\quad - (y^2+y^3+y^4+\ldots\ldots n \text{ term})$

Here $a = x^2, r = x$; $a = y^2, r = y$.

$(x-y)S_n = \left[\dfrac{x^2(x^n-1)}{x-1} - \dfrac{y^2(y^n-1)}{y-1}\right]$

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10th Maths – Chapter 4 to 6 (Book in One Marks)

Green indicates Thinking Corner, Blue indicates Progress Check

Dear students,

1. Read and kept in mind the Points to Remember in all chapters.

2. Don't muck up the book back one marks answers.

3. Try to know how the answer has come. This method of practicing will help you in many ways.

4. If you have any doubts in this, clarify it with your teachers.

5. If you know the Basic and Logic very well, then Maths will become a Magic.

புரியாமற் படிப்பது எதற்கும் உதவாது
புரிந்து படிப்பது என்றும் மறவாது.

Chapter – 4 GEOMETRY

1. Are square and a rhombus similar or congruent. Discuss. Never
   Since in rhombus, the side angles are not equal to 90^0 and the two diagonals are also not equal.
2. Are a rectangle and a parallelogram similar. Discuss. Never
   Since in parallelogram, the side angles are not equal to 90^0 and the two diagonals are also not equal.
3. Are any two right angled triangles similar? If so why?
   Yes. If the corresponding sides are proportional.

4. A pair of equiangular triangles are similar.

5. If two triangles are similar, then they are equiangular.

6. If we change exactly one of the four given lengths, then we can make these triangles similar.

7. All circles are similar (congruent/ similar).

8. All squares are similar (similar/ congruent).

9. Two triangles are similar, if their corresponding angles are equal and their corresponding sides are proportional.

10. (a) All similar triangles are congruent – True/False. False
    (b) All congruent triangles are similar – True/False. True

11. Give two different examples of pair of non-similar figures.
Squares and Circles, Right triangles and Acute Triangles.

12. A straight line drawn parallel to a side of a triangle divides the other two sides proportionally

13. Basic Proportionality Theorem is also known as Thales Theorem.

14. Let ΔABC be equilateral. If D is a point on BC and AD is the internal bisector of ∠A. Using Angle Bisector Theorem, BD/DE is 1

15. The bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.

16. If the median AD to the side BC of a ΔABC is also an angle bisector of ∠A then AB/AC is 1.

17. In a right angled triangle, the side opposite to 90° (the right angle) is called the hypotenuse.

18. The other two sides are called legs of the right angled triangle.

19. The hypotenuse will be the longest side of the triangle.

20. In India, Pythagoras Theorem is also referred as "Baudhyana Theorem".

21. Write down any five Pythagorean triplets?
3, 4, 5   6, 8, 10   9, 12, 15   12, 16, 20   5, 12, 13

22. In a right angle triangle the sum of other two angles is 90^0.

23. Can all the three sides of a right angled triangle be odd numbers? No.
Why? Because sum of squares of any two odd numbers becomes an even number. Then the square root of such even number will never be an odd number.

24. Hypotenuse is the longest side of the right angled triangle.

25. The first theorem in mathematics is Thales Theorem.

26. If the square of the longest side of a triangle is equal to sums of squares of other two sides, then the triangle is Right triangle.

27. State True or False. Justify them.
(i) Pythagoras Theorem is applicable to all triangles. False
$AB^2 + AC^2 = BC^2$ will not be applicable for all triangles except right triangle.
(ii) One side of a right angled triangle must always be a multiple of 4. False.
$1^2 + (\sqrt{5})^2 = (\sqrt{6})^2$

28. A straight line cuts the circle is called as a secant.

29. The word "tangent" comes from the latin word "tangere" which means "to touch".

30. The longest chord in a circle is the diameter.

31. We can draw two tangents from a point outside the circle.

32. We can draw only one tangent from a point on the circle.

33. A straight line that touches a circle at a common point is called a tangent.

34. A chord is a sub-section of a secant.

35. The lengths of the two tangents drawn from an exterior point to a circle are equal.

36. No tangent can be drawn from an interior point of the circle.

37. Angle bisector is a cevian that divides the angle, into two equal halves.

38. Can we draw two tangents parallel to each other on a circle? - Yes.
From the end points of the diameter, we can draw two tangents parallel to each other.

39. Can we draw two tangents perpendicular to each other on a circle? - Yes.

40. The term cevian comes from the name of Italian engineer Giovanni Ceva,

41. A cevian is a line segment that extends from one vertex of a triangle to the opposite side.

42. A cevian that divides the opposite side into two congruent(equal) lengths is known as median.

43. A cevian that is perpendicular to the opposite side is known as altitude.

44. A cevian that bisects the corresponding angle is known as angle bisector.

45. The cevians do not necessarily lie within the triangle, although they do in the diagram.

Activity 1

Let us try to construct a line segment of length $\sqrt{2}$.

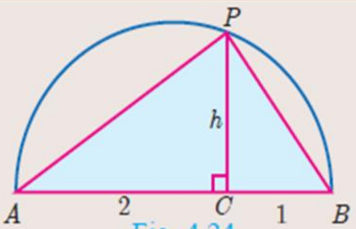


Fig. 4.24

For this, we consider the following steps.

Step1: Take a line segment of length 3 units. Call it as AB.

Step2: Take a point C on AB such that $AC=2$, $CB=1$.

Step3: Draw a semi-circle with AB as diameter as shown in the diagram

Step4: Take a point 'P' on the semi-circle such that CP is perpendicular to AB.

Step5: Join P to A and B. We will get two right triangles ACP and $B\cancel{C}P$. PCB

Step6: Verify that the triangles ACP and PCB $B\cancel{C}P$ are similar. ∠ACP = ∠PCB = 90^0; ∠PAC = ∠BPC; ∠CPA = ∠CPB

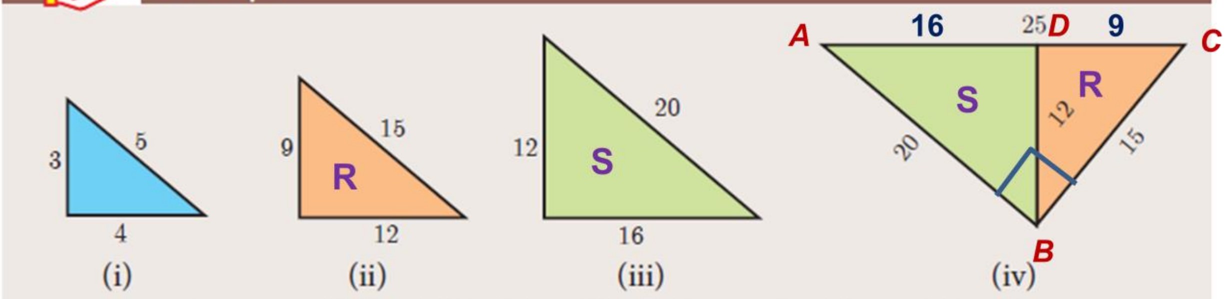
Step7: Let $CP = h$ be the common altitude. Using similarity, find h. $h/2 = 1/h$; $h^2 = 2$; $h = \sqrt{2}$

Step8: What do you get upon finding h? $h = \sqrt{(AC \times CB)}$

Repeating the same process, can you construct a line segment of lengths $\sqrt{3}$, $\sqrt{5}$, $\sqrt{8}$.

Yes, we can construct a line segment of lengths √3, √5, √8 by taking a line segment of lengths of 3+1, 5+1, 8+1 units respectively.

Activity 4



From fig. (iv) $25^2 = 20^2 + 15^2$; $20^2 = 16^2 + 12^2$; $15^2 = 12^2 + 9^2$

$\therefore (16 + 9)^2 = (16^2 + 12^2) + (12^2 + 9^2)$

$16^2 + 9^2 + 2 \times 16 \times 9 = 16^2 + 12^2 + 12^2 + 9^2$

$2 \times 16 \times 9 = 2 \times 12^2$

$16 \times 9 = 12^2$; ie $BD^2 = AD \times DC$

Activity 5

(i) Take two consecutive odd numbers.

(ii) Write the reciprocals of the above numbers and add them. You will get a number of the form $\frac{p}{q}$.

(iii) Add 2 to the denominator of $\frac{p}{q}$ to get $q+2$.

(iv) Now consider the numbers $p, q, q+2$. What relation you get between these three numbers?

Try for three pairs of consecutive odd numbers and conclude your answer.

Taking 5 and 7, their

Reciprocals are 1/5, 1/7

1/5 + 1/7 = 12/35

Now, p = 12, q = 35, q+2 = 37

The relation is $12^2 + 35^2 = 37^2$

144 + 1225 = 1369

$\therefore$ p, q, q+2 are the Pythagorean Triplet

Chapter – 5 COORDINATE GEOMETRY

1. Apollonius is hailed as "The Great Geometer". His greatest work was called "conics".

2. Coordinate geometry, also called Analytical geometry.

3. The first degree equation in two variables ax +by +c = 0 represents a straight line in a plane.

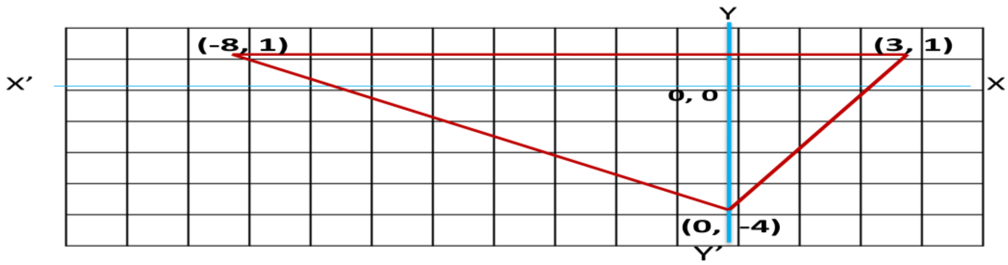
The vertices of DPQR are P(0,-4) , Q(3,1) and R(-8,1)

4. Draw ΔPQR on a graph paper. Graph drawn.

5. Check if ΔPQR is equilateral. It is not an equilateral triangle.

6. Find the area of ΔPQR . Area = 27.5 sq.unit

7. Find the coordinates of M, the mid-point of QP. (3/2, -3/2)



8. Find the coordinates of N, the mid-point of QR.   **(-5/2, 1)**

9. Find the area of ∆MPN.   **6.875sq.unit**.

10. What is the ratio between the areas of ∆MPN and ∆DPQR ?   **1 : 4**.

Progress Check

1. Complete the following table.

S.No.	Points	Distance	Mid Point	Internal Point	Internal Ratio	External Point	External Ratio
(i)	(3, 4), (5, 5)	√5	4, 4.5	19/5, 22/5	2:3	- 1, 2	2:3
(ii)	(–7,13),(–3,1)	4√10	- 5, 7	$\left(-\frac{13}{3}, 5\right)$	2 : 1	$(-13, \overset{31}{\cancel{15}})$	3 : 5

2. $A(0,5)$, $B(5,0)$ and $C(-4,-7)$ are vertices of a triangle then its centroid will be at **1/3, -2/3**.

11. How many triangles exist, whose area is zero? **None**

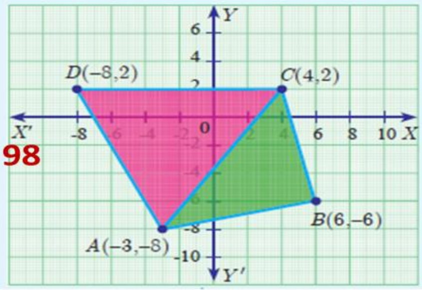
12. If the area of a quadrilateral formed by the points (a, a), (–a, a), (a, –a) and (–a, –a), where a ≠ 0 is 64 square units, then identify the type of the quadrilateral.
Square.

13. Find all possible values of a. **(4, 4), (–4, 4), (4, –4) and (–4, –4)**.

Progress Check

Given a quadrilateral $ABCD$ with vertices $A(-3,-8)$, $B(6,-6)$, $C(4,2)$, $D(-8,2)$

1. Find the area of ΔABC.   **38**
2. Find the area of ΔACD.   **60**
3. Calculate area of ΔABC + area of ΔACD. **98**
4. Find the area of quadrilateral $ABCD$. **98**
5. Compare the answers obtained in 3 and 4.
Both are equal.



14. The inclination of X axis and every line parallel to X axis is **0°** .

15. The inclination of Y axis and every line parallel to Y axis is **90°** .

16. The measure of steepness is called **slope** or **gradient**.

17. The slope of a vertical line is **undefined**.

18. Two non-vertical lines are **parallel** if and only if their **slopes are equal**.

19. When the line l_1 is parallel to l_2 if and only if **$m_1 = m_2$**.

20. When the line l_1 is perpendicular to line l_2 then **$m_1 m_2 = -1$** .

21. In any triangle, **exterior angle** is **equal to sum of the opposite interior angles**.

Progress check		
S.No.	Points	Slope
1	*A(-a, b) , B(3a, -b)*	-b/2a
2	*A(2, 3) , B(4, 7)*	2
3	*A(5, 8) , B(10, 8)*	0
4	*A(7, 3) , B(7, 10)*	Undefined

22. If the slopes of both the pairs of opposite sides are equal then the quadrilateral is a parallelogram.

23. Provide three examples of using the concept of slope in real-life situations.
1. Ghot road in the hilly area. 2. Ramps at the entrance of the house for vehicles.
3. Ramps at hospitals for handicapped persons.

24. For, the point (x, y) in a xy plane, the x coordinate x is called "Abscissae" and the y coordinate y is called "Ordinate".

25. Is it possible to express, the equation of a straight line in slope-intercept form, when it is parallel to Y axis?
Not possible. (Since slope is not defined for lines parallel to Y axis.)

Progress check				
S.No.	Equation	Slope	*x* intercpt	*y* intercept
1	$3x - 4y + 2 = 0$	3/4	-2/3	1/2
2	$y = 14x$	14	0	0
3	$3x - 2y - 6 = 0$	3/2	2	-3

Progress check					
S.No.	Equation	Parallel or Perpendicular	S.No.	Equation	Parallel or Perpendicular
1	$5x + 2y + 5 = 0$ $5x + 2y - 3 = 0$	Parallel	3	$8x - 10y + 11 = 0$ $4x - 5y + 16 = 0$	Parallel
2	$3x - 7y - 6 = 0$ $7x + 3y + 8 = 0$	Perpendicular	4	$2y - 9x - 7 = 0$ $27y + 6x - 21 = 0$	Perpendicular

26. How many straight lines do you have with slope 1? Infinite Lines.

27. Find the number of point of intersection of two straight lines.
One point.

28. Find the number of straight lines perpendicular to the line $2x - 3y + 6 = 0$.

Infinite Perpendicular lines can be drawn.

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இளமையில் விரும்பிக் கற்றிடு

Activity 1

(i) Take a graph sheet.

(ii) Consider a triangle whose base is the line joining the points (0,0) and (6,0)

(iii) Take the third vertex as (1,1), (2,2), (3,3), (4,4), (5,5) and find their areas. Fill in the details given:

Third vertex	Area of Triangle
(1,1)	$A_1 =$ **3 Sq.unit.**
(2,2)	$A_2 =$ **6 Sq.unit.**
(3,3)	$A_3 =$ **9 Sq.unit.**
(4,4)	$A_4 =$ **12 Sq.unit.**
(5,5)	$A_5 =$ **15 Sq.unit.**

(iv) Do you see any pattern with A_1, A_2, A_3, A_4, A_5 ? If so mention it. **It is an A.P. Sequence**

(v) Repeat the same process by taking third vertex in step (iii) as (1,2), (2,4), (3,8), (4,16), (5,32)

(vi) Fill the table with these new vertices

Third vertex	Area of Triangle
(1,2)	$A_1 =$ **6 Sq.unit.**
(2,4)	$A_2 =$ **12 Sq.unit.**
(3,8)	$A_3 =$ **24 Sq.unit.**
(4,16)	$A_4 =$ **48 Sq.unit.**
(5,32)	$A_5 =$ **96 Sq.unit.**

(vii) What pattern do you observe now with A_1, A_2, A_3, A_4, A_5 ? **It is a G.P. Sequence**

Activity 2

Find the area of the shaded region

= 1/2x7x(6 – 4)

= 7 sq.unit.

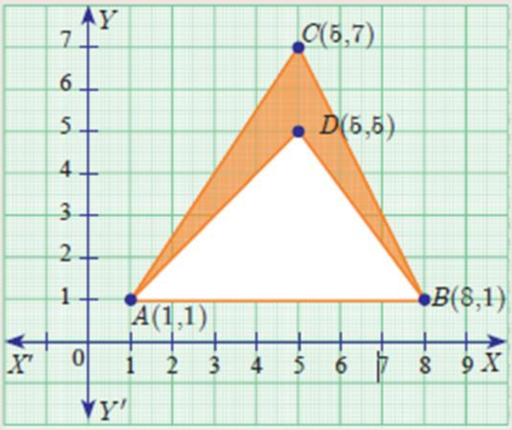


Fig. 5.15

Activity 3

The diagram contain four lines l_1, l_2, l_3 and l_4.

(i) Which lines have positive slope?

(ii) Which lines have negative slope?

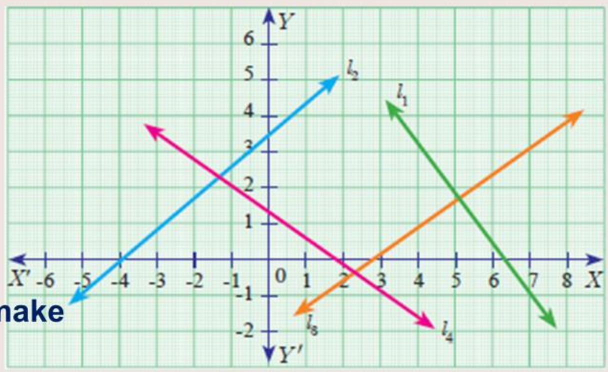


Fig. 5.19

(i). l_2, l_3 have positive slopes, because they make acute angles with X-axis

(ii). l_1, l_4 have negative slopes, because they make obtuse angles with X-axis

Activity 4

If line l_1 is perpendicular to line l_2 and line l_3 has slope 3 then

(i) find the equation of line l_1

(ii) find the equation of line l_2

(iii) find the equation of line l_3

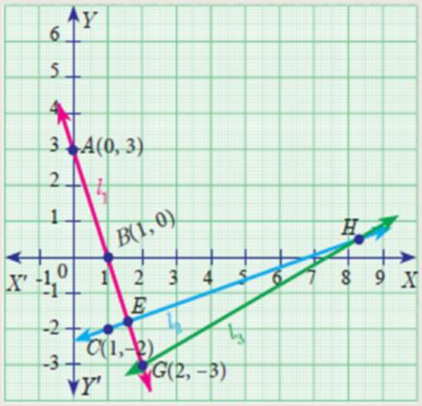


Fig. 5.38

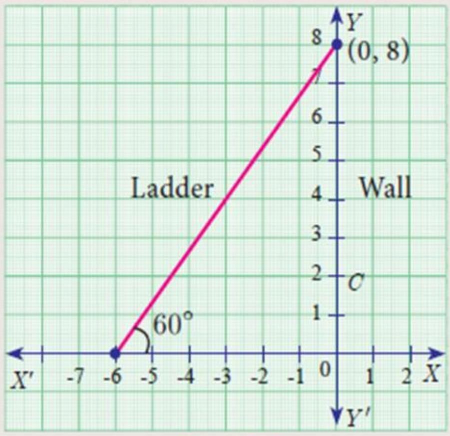
(i) Line l_1 equation : Here x-intercept = 1, y-intercept = 3, Using two intercept form
x/1 + y/3 = 1 from this $3x + y – 3 = 0$

(ii) Line l_2 equation : Here l_2 is perpendicular to l_1; Slope l_1 = -3 ; ∴ Slope l_2 = 1/3 and it passes through C(1, -2) ; Using Slope point form y – y1 = m(x –x1)
y + 2 = 1/3 (x –1) from this $x – 3y – 7 = 0$

(iii) Line l_3 equation : Here Slope l_3 = 3; and it passes through C(2, -3) ;
Using Slope point form y – y1 = m(x –x1) ; y + 3 = 3 (x –2) from this
$3x – y – 9 = 0$

Activity 5

A ladder is placed against a vertical wall with its foot touching the horizontal floor. Find the equation of the ladder under the following conditions.

No.	Condition	Picture	Equation of the ladder
(i)	The ladder is inclined at 60° to the floor and it touches the wall at (0,8) **Slope of ladder = 8/6 = 4/3** **It passes through (0, 8)** **y – 8 = 4/3 (x – 0)**	 Fig. 5.39	**4x – 3y + 24 = 0**
(ii)	The foot and top of the ladder are at the points (2,4) and (5,1)	**Using two point form** **(y – 4)/(1 – 4) = (x – 2)/(5 – 2)**	**x + y – 6 = 0**

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பாணமாய் விரைந்து கற்றிடு

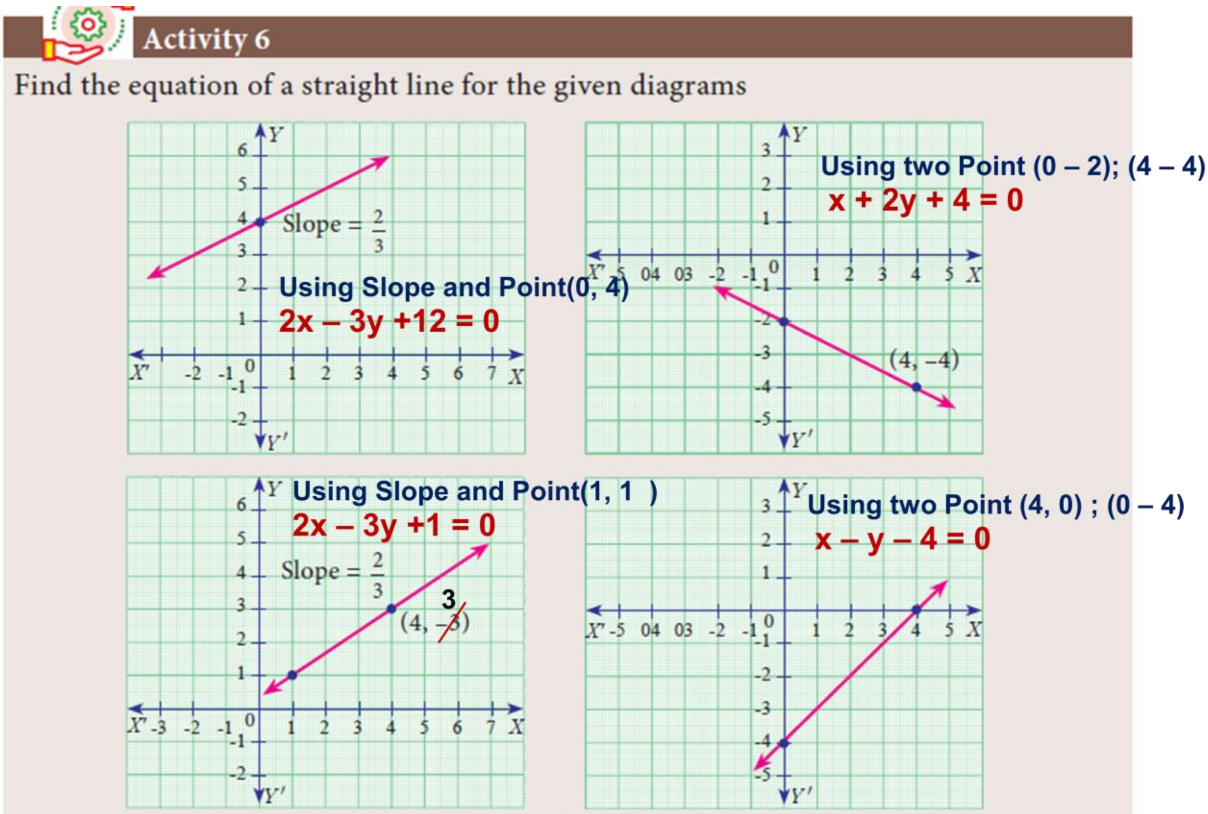

Fig. 5.41

Chapter – 6 TRIGONOMETRY

1. Hipparchus of Rhodes around 200 BC is considered as "The Father of Trigonometry"

2. When will the values of sin Ø and cos Ø be equal? Ø = 45°

3. For what values of Ø, sinØ = 2 ? No. (Since sinØ varies from 0 to 1 only.)

4. Among the six trigonometric quantities, as the value of angle increase from 0° to 90° , which of the six trigonometric quantities has undefined values?
   $\tan 90^0$, $\mathrm{cosec}\, 0^0$, $\sec 90^0$, $\cot 0^0$

5. Is it possible to have eight trigonometric ratios? No.
   (Since triangle has 3 sides only.From this we can make only 6 ratios)

6. Let 0°≤ Ø ≤ 90° . For what values of Ø does
   (i) sin Ø > cos Ø (ii) cos Ø > sin Ø (iii) sec Ø = 2 tan Ø (iv) cosec Ø = 2 cot Ø
   (i) 45°< Ø ≤ 90° (ii) 0°≤ Ø < 45° (iii) Ø = 30° iv) Ø = 60°

7. The number of trigonometric ratios is 6.

8. $1 - \cos^2 Ø$ is $\sin^2 Ø$.

9. (sec Ø + tan Ø)(sec Ø − tan Ø) is 1. ($= \sec^2 Ø - \tan^2 Ø$ $\because (a+b)(a-b) = a^2 - b^2$)

10. (cot Ø + cosec Ø)(cot Ø −cosec Ø) is –1 . ($= \cot^2 Ø - \mathrm{cosec}^2 Ø$)

11. $\cos 60° \sin 30° + \cos 30° \sin 60°$ is 1.
(cos 60° = sin 30°; sin 60° = cos 30°; ∴ It is = $\sin^2 30 + \cos^2 30 = 1$)

12. $\tan 60° \cos 60° + \cot 60° \sin 60°$ is $(\sqrt{3} + 1)/2$.

13. (tan 45°+ cot 45°)+(sec 45° cosec45°) is 4.

14. (i) sec Ø = cosec Ø if Ø is 45°. (ii) cot Ø = tan Ø if Ø is 45°.

15. What type of triangle is used to calculate heights and distances? Right Triangle.

16. When the height of the building and distances from the foot of the building is given, which trigonometric ratio is used to find the angle of elevation?
tan Ø = Height/Distance.

17. If the line of sight and angle of elevation is given, then which trigonometric ratio is used
(i) to find the height of the building. Height = sin Ø x Line of sight.

(ii) to find the distance from the foot of the building. Distance = cos Ø x Line of sight.

18. What is the minimum number of measurements required to determine the height or distance or angle of elevation? Two.

19. The line drawn from the eye of an observer to the point of object is Line of sight.

20. Which instrument is used in measuring the angle between an object and the eye of the observer? Theodolite.

21. When the line of sight is above the horizontal level, the angle formed is Angle of elevation.

22. The angle of elevation increases as we move towards the foot of the vertical object (tower). (Note : The angle of elevation decreases as we move away from the Tower).

23. When the line of sight is below the horizontal level, the angle formed is Angle of depression.

24. Angle of Depression and Angle of Elevation are equal since they are alternative angles.

Identity	Equal forms
$\sin^2Ø + \cos^2Ø = 1$	$\sin^2Ø = 1 - \cos^2Ø$ (or) $\cos^2Ø = 1 - \sin^2Ø$
$1+ \tan^2Ø = \sec^2Ø$	$\tan^2Ø = \sec^2Ø - 1$ (or) $\sec^2Ø - \tan^2Ø = 1$
$1+ \cot^2Ø = \mathrm{cosec}^2Ø$	$\cot^2Ø = \mathrm{cosec}^2Ø - 1$ (or) $\mathrm{cosec}^2Ø - \cot^2Ø = 1$

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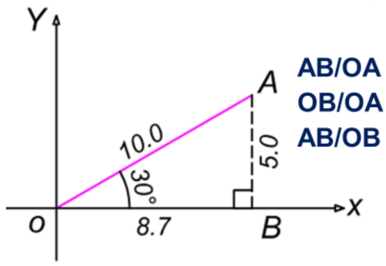
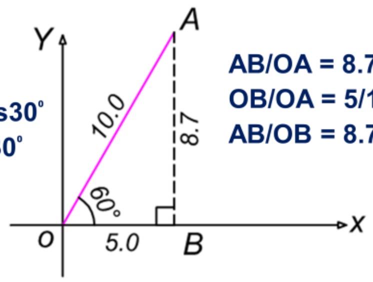
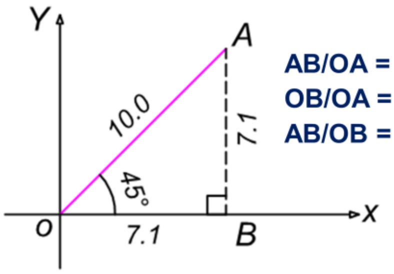
Activity 1



AB/OA = 5/10 = ½ = sin30°
OB/OA = 8.7/10 = 0.87 = cos30°
AB/OB = 5/8.7 = 0.57 = tan30°



AB/OA = 8.7/10 = 0.87 = sin60°
OB/OA = 5/10 = ½ = cos60°
AB/OB = 8.7/5 = 1.74 = tan60°

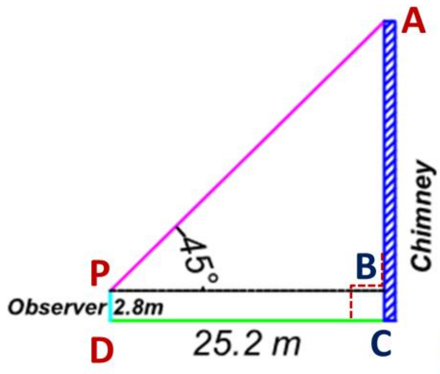


AB/OA = 7.1/10 = 0.71 = sin45°
OB/OA = 7.1/10 = 0.71 = cos45°
AB/OB = 7.1/7.1 = 1 = tan45°

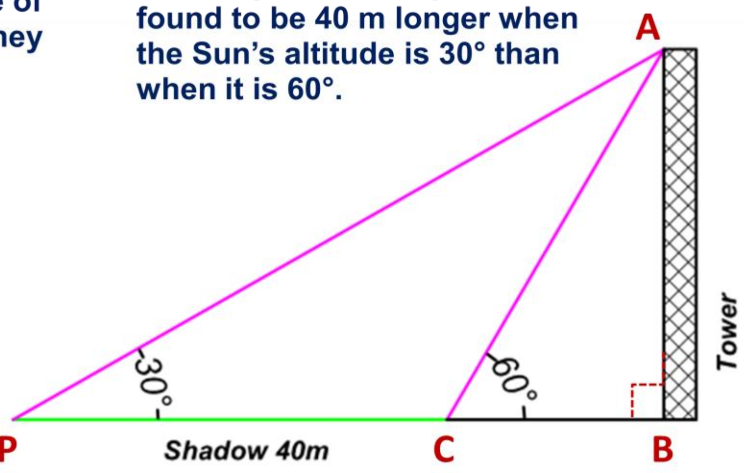
From these, we conclude that
sin30° = cos60° ; sin60° = cos30°
Sin 45^0 = cos45^0
tan30° = 1/tan60° = cot60^0

Activity 2

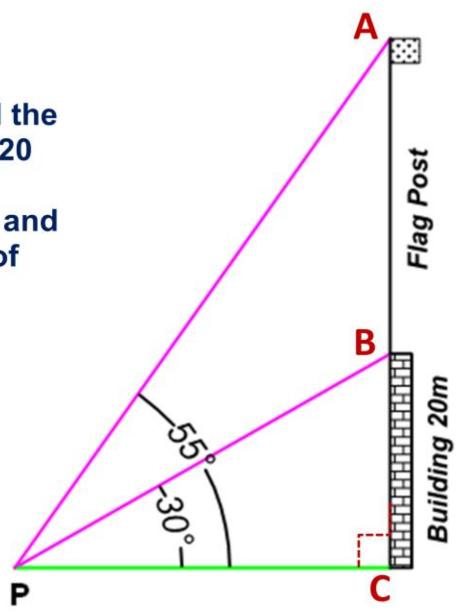
(ii) An observer 2.8 m tall is 25.2 m away from a chimney. The angle of elevation of the top of the chimney from her eyes is 45°.



(iv) The shadow of a tower standing on a level ground is found to be 40 m longer when the Sun's altitude is 30° than when it is 60°.



(iii) From a point P on the ground the angle of elevation of the top of a 20 m tall building is 30°. A flag is hoisted at the top of the building and the angle of elevation of the top of the flagstaff from P is 55° .



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10th Maths – Chapter 7 MENSURATION
(Book in One Marks) (With Solution to certain Problems)

Green indicates Thinking Corner, Blue indicates Progress Check

புரியாமற் படிப்பது எதற்கும் உதவாது
புரிந்து படிப்பது என்றும் மறவாது.

1. When ‘h’ coins each of radius ‘r’ units and thickness 1 unit is stacked one upon the other, what would be the solid object you get? Also find its C.S.A.
We get a solid circular cylinderof radius ‘r’ and height ‘h’; It’s CSA = $2\pi rh$ Sq.units.

2. When the radius of a cylinder is double its height, find the relation between its C.S.A. and base area.
Let the Height = r; ∴It’s Radius = 2r;
CSA = $2\pi rh = 2(2r)r = 4\pi r^2$
Base Area = $(2r)^2 = 4\pi r^2$ (CSA and Base Area are Equal)

3. Two circular cylinders are formed by rolling two rectangular aluminum sheets each of dimensions 12 m length and 5 m breadth, one by rolling along its length and the other along its width. Find the ratio of their curved surface areas.
Area of rectangular sheets = 12x5 = 60 m^2
On both way the sheets are covered as CSA = 60 m^2
The CSA ratio = 1 : 1

4. Right circular cylinder is a solid obtained by revolving a rectangle about one of its sides as axis.

5. In a right circular cylinder the axis is perpendicular to the diameter.

6. The difference between the C.S.A. and T.S.A. of a right circular cylinder is twice the base area.

7. The C.S.A. of a right circular cylinder of equal radius and height is twice the area of its base.

8. Give practical example of solid cone. Cone Ice Cream

9. Find surface area of a cone in terms of its radius when height is equal to radius.
Radius of the Cone = r; It’s Heght = r; $l = \sqrt{(h^2 + r^2)} = \sqrt{(r^2 + r^2)} = r\sqrt{2}$
C.Surface Area = $\sqrt{2}\pi r^2$

10. Compare the above surface area with the area of the base of the cone.

C.S.A = $\sqrt{2}\pi r^2$; Base Area = πr^2

C.S.A : Base Area = $\sqrt{2}$: 1

11. Right circular cone is a solid obtained by revolving a right angled triangle about one of the sides containing the right angle as axis

12. In a right circular cone the axis is perpendicular to the diameter.

13. The difference between the C.S.A. and T.S.A. of a cone is the Base Area.

14. When a sector of a circle is transformed to form a cone, then match the conversions taking place between the sector and the cone.

Sector		Cone	
Radius	1	Circumference of the base	3
Area	2	Slant height	1
Arc length	3	Curved surface area	2

15. Find the value of the radius of a sphere whose surface area is 36π sq. units.

$4\pi r^2 = 36\pi$; $r^2 = 9$; r = 3 unit

16. How many great circles can a sphere have? Infinitive.

17. Find the surface area of the earth whose diameter is 12756 kms.

Radius of the Earth = 12756/2 = 6378 km

Surface Area of the Earth = $4\pi r^2$ =$(4\text{x}22\text{x}6378^2)/7$ = 51,13,91,684.57 km^2

18. Every section of a sphere by a plane is a Circle.

19. The centre of a great circle is at the centre of the sphere.

20. The difference between the T.S.A. and C.S.A. of hemisphere is Base Area (πr^2).

21. The ratio of surface area of a sphere and C.S.A. of hemisphere is 2 : 1.

22. A section of the sphere by a plane through any of its great circle is a hemisphere.

23. Shall we get a hemisphere when a sphere is cut along the small circle? No.

24. T.S.A of a hemisphere is equal to how many times the area of its base? Thrice.

25. How many hemispheres can be obtained from a given sphere? Two.

26. Give two real life examples for a frustum of a cone. Glass, Bucket

27. Can a hemisphere be considered as a frustum of a sphere. No.

28. The portion of a right circular cone intersected between two parallel planes is a frustum.

29. How many frustums can a right circular cone have? Infinitive.

30. If the height is inversely proportional to the square of its radius, the volume of the cylinder is π. (Radius = r; Height = $1/r^2$; Volume = $\pi r^2 h = \pi r^2(1/r^2) = \pi$)

31. What happens to the volume of the cylinder with radius r and height h, when its
(a) radius is halved. The volume is reduced to (1/4)th.
(b) height is halved. The volume is reduced to half.

32. Is it possible to find a right circular cone with equal
(a) height and slant height No. (b) radius and slant height No. (c) height and radius. Yes.

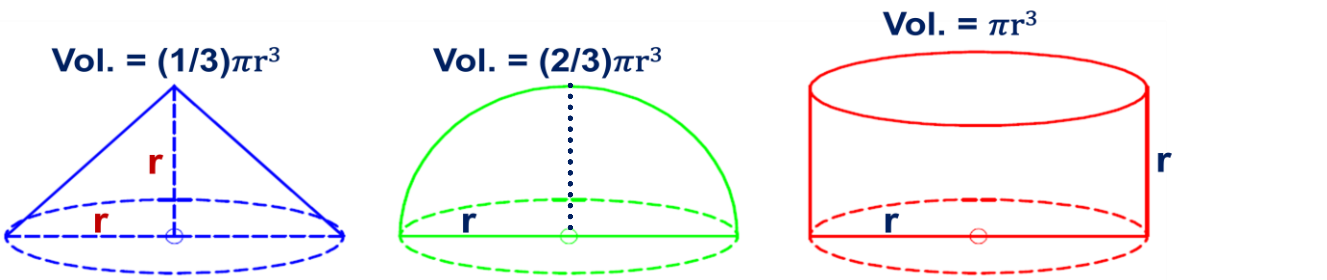
33. There are two cones with equal volumes. What will be the ratio of their radius and height? $r_1 : r_2 = \sqrt{h_2} : \sqrt{h_1}$.

Progress Check

1. Volume of a cone is the product of its base area and 1/3 height
2. If the radius of the cone is doubled, the new volume will be 4 times the original volume.
3. Consider the cones given in Fig.7.29
   (i) Without doing any calculation, find out whose volume is greater? Cone B
   (ii) Verify whether the cone with greater volume has greater surface area. Yes
   (iii) Volume of cone A : Volume of cone B =? 3 : 4

Fig. 7.29

34. A cone, a hemisphere and a cylinder have equal bases. The heights of the cone and cylinder are equal and are same as the common radius. Are they equal in volume? No.



35. Give any two real life examples of sphere and hemisphere.
Sphere : Earth glope in the schools, Balls, Lemon
Hemisphere : Cutting the above into two halves.

36. A plane along a great circle will split the sphere into two hemisphere parts.

37. If the volume and surface area of a sphere are numerically equal, then the radius of the sphere is 3 units.

38. What is the ratio of volume to surface area of a sphere? r : 3

39. The relationship between the height and radius of the hemisphere is equal.

40. The volume of a sphere is the product of its surface area and one-third of it's radius.

41. Is it possible to obtain the volume of the full cone when the volume of the frustum is known? No.

42. Frustum of a cylinder: CSA = $2\pi r(h_1 + h_2)/2$; Volume = $\pi r^2(h_1 + h_2)/2$.

Activity – 1

(i) Take a semi-circular paper with radius 7 cm and make it a cone. Find the C.S.A. of the cone.

Here, the CSA of the cone = The area of the semi-circular paper

$= (\pi r^2)/2 = (\pi 7^2)/2 = 77\text{ cm}^2$

(ii) Take a quarter circular paper with radius 3.5 cm and make it a cone. Find the C.S.A. of the cone.

Here, the CSA of the cone = The area of the quadrant paper

$= (\pi r^2)/4 = (\pi 3.5^2)/4 = 9.625\text{ cm}^2$

Activity - 2

(i). Sphere radius = r ;

(ii). Cylinder Radius = r; It's height = 2r

(iii). Rolling with a thread on both of them,

(iv). The length of the threads are equal .

(v). The SA of the sphere = The CSA of the Cylinder.

$= 2\pi rh = 2\pi r(2r) = 4\pi r^2$

Activity - 3

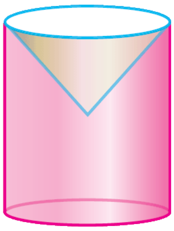
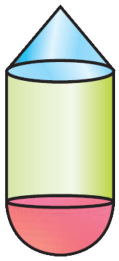
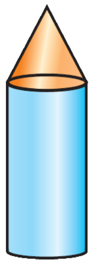
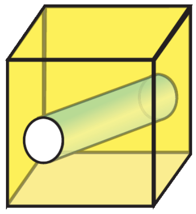
1. Using a globe list any two countries in the northern and southern hemispheres.

Northern hemisphere Countries : India, Japan.

Southern hemisphere Countries : Australia, New Zealand.

எளிதாய் விளங்கும் கல்வியை
இளமையில் விரும்பிக் கற்றிடு
வானமாய் விரிந்த கல்வியை
பாணமாய் விரைந்து கற்றிடு
தானமாய் பெற்ற கல்வியைத்
தரணியில் பலருக் களித்திடு.

Activity - 4

Combined solids				
List out the solids in each combined solid	Solid cylinder Circular cone.	Solid cylinder Hemisphere Circular cone	Solid cylinder Circular cone.	Cube Cylinder
Total Surface Area of the combined solid	$2\pi rh + \pi r^2 + \pi rl$ $\pi r(2h + r + l)$	$2\pi rh + 2\pi r^2 + \pi rl$ $\pi r(2h + 2r + l)$	$2\pi rh + \pi r^2 + \pi rl$ $\pi r(2h + r + l)$	$6a^2 + 2\pi rh - 2\pi r^2$ $6a^2 + 2\pi r(h - r)$

Activity - 5

Activity 5

The adjacent figure shows a cylindrical can with two balls. The can is just large enough so that two balls will fit inside with the lid on. The radius of each tennis ball is 3 cm. Calculate the following

(i) height of the cylinder. $4 \times 3 = 12$ cm

(ii) radius of the cylinder. 3 cm

(iii) volume of the cylinder. $\pi r^2h = \pi 3^2 \times 12 = 108\pi$ cm^3

(iv) volume of two balls. $2\times(4/3)\pi r^3 = 2\times(4/3)\pi 3^3 = 72\pi$ cm^3

(v) volume of the cylinder not occupied by the balls. 26π cm^3

(vi) percentage of the volume occupied by the balls. 66.67 %

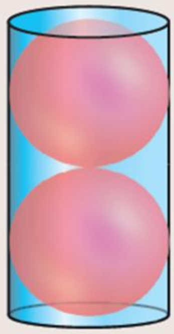


Fig. 7.43

Certain Solved Problems

Important Points :

1. In this chapter formulae take place a vital role. So, know them very well.

2. The main object of this chapter is
   - ❖ To determine the surface area and volume of cylinder, cone, sphere, hemisphere and frustum and sometimes to find the cost also.
   - ❖ To compute volume and surface area of combined solids.
   - ❖ To solve problems involving conversion of solids from one shape to another with no change in volume.

3. Quantities are two types.
   - Fundamental Quantities. ie Length, Breadth, Height, Radius, Diameter.

- Derived Quantities. ie Perimeter, Circumference, Area (CSA,TSA), Volume.

4. In this chapter, mainly we have to find out the derived quantities and cost for an individual shape or combined shape, secondly to find out the fundamental quantities from the given derived quantities and thirdly ratios problems

5. The main thing in this chapter is what dimension is given and what is to be found out.

6. The clear imaginations about the shapes and it's dimension are very very important.

TYPE - 1 : CSA and TSA

Exercise 7.1 (9). The internal and external diameters of a hollow hemispherical vessel are 20 cm and 28 cm respectively. Find the cost to paint the vessel all over at ` 0.14 per cm^2.

Given : Fundamental Qnty ie radii of hollow hemispherical vessel ;
To be found : Derived Qnty Plus Cost.

Solution : Hemispherical vessel :

Inner Radius (r) = 20/2 = 10 cm; Outer Radius (R) = 28/2 = 14 cm

Total Surface Area (TSA) = $3\pi R^2 + \pi r^2 = 3\pi 14^2 + \pi 10^2 = 2162.3$ cm^2

∴Cost of Painting = 2162.3 x 0.14 = Rs 302.72.

Exercise 7.1 (5). 4 persons live in a conical tent whose slant height is 19 cm. If each person require 22 cm2 of the floor area, then find the height of the tent. (Practically the units given in the problem is not sufficient to accomadate 4 persons. Anyhow let us do.)

Given : Derived qnty of Base Area
To be found : Fundamental qnty of height of conical tent.

Solution : Required Base area conical tent = 4x22 = 88cm^2; Slant height (l) = 19 cm.

$\pi r^2 = 88$; $r^2 = 88/\pi$; $r = \sqrt{28}$

Height of the Conical tent = $\sqrt{(l^2 - r^2)} = \sqrt{[19^2 - (\sqrt{28}^2)]} = \sqrt{(361 - 28)} = \sqrt{333} = 18.25$cm.

Exercise 7.1 (1). The radius and height of a cylinder are in the ratio 5:7 and its curved surface area is 5500 sq.cm. Find its radius and height.

Given : Derived qnty of CSA of cylinder; To be found : We have to find the fundamental qnty of radius and height of cylinder.

Solution : Solid Cylinder

r : h = 5 : 7; r/h = 5/7; r = 5h/7;   CSA = 5500 cm^2

$2\pi rh$ = CSA ; $2\pi(5h/7)h = 5500$;

2x22/7x5/7xh^2 = 5500; h^2 =(5500x7x7)/(2x22x5) = 25x7x7

h = $\sqrt{(25x7x7)}$ = 35 cm; ∴r = (5x35)/7 = 25 cm

TYPE - 2 : Volumes.

Exercise 7.2 (10). A container open at the top is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill a container at the rate of `40 per litre.

Given : Fundamental Qnty of a frustum of a cone ;
To be found : Derived Qnty of volume Plus Cost.

Solution : Frustum of a cone

Lower end radius (r) = 8 cm ; Upper end radius (R) = 20 cm

Volume of a frustum of a cone = $1/3x\,\pi\,[R^2 + Rr + r^2]h$

$= 1/3x\,\pi\,[20^2 + 20x8 + 8^2]16 = 10459.42\ cm^3$

$= 10.46$ lit [$\because$1000 cm^3 = 1 lit]

$\therefore$Cost of 10.46 lit milk = 10.46x40 = **Rs. 418.40**

Exercise 7.2 **(1). A 14 m deep well with inner diameter 10 m is dug and the earth taken out is evenly spread all around the well to form an embankment of width 5 m. Find the height of the embankment.**

Here the fundamental qnty of a Well is given; we have to find the fundamental qnty (Height) of the Embankment. The volume of earth excavated from the well is spread as an embankment around the well which is of hollow cylindrical shape. By equating these two volumes we can find height of the embankment.

Solution : Well : Radius (r) = 10/2 = 5 m; Height (H) = 14 m

Embankment : Inner radius (r) = 5 m; Outer radius (R) = 5 + 5 = 10 m.

Vol. embankment (Hollw Cylinder) = Vol. of well (Solid cylinder)

$(R^2 - r^2)\ h = \pi r^2 h$

$(10^2 - 5^2)\ h = \pi x 5^2 x 14$ [cancel the π on both sides]

h = 25x14/75 = **4.67 m**

TYPE - 3 : Combined Volumes.

Exercise 7.3 **:(2). Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made.**

Given : Fundamental Qnty of combined shape of cylinder with two cones at the ends

To be found : Volume of combined shape of cylinder with two cones at the ends.

Solution : Cylider radius (r) = 3/2 = 1.5 cm; Height (H) = 12-2-2 = 8 cm

Cone radius (r) = 1.5 cm; Height (h) = 2 cm

Total Volume = Vol. of cylinder + vol. of 2 cones = $\pi r^2 H + 2x(1/3)\,\pi r^2 h$

$= \pi r^2(H + 2x1/3xh) = \pi 1.5^2(8 + 2x\ 1/3\ x2) = 22/7x1.5^2x28/3 =$ **66 cm^3**

Exercise 7.3 **(3). From a solid cylinder whose height is 2.4 cm and the diameter 1.4 cm, a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest cm3.**

Given : Fundamental Qnty of Solid Cylinder and a Cone

To be found : Volume of remaining portion Cylinder after carving the cone from it.

Solution : Solid cylinder : r = 1.4/2 = 0.7 cm; h = 2.4 cm

Cone : r = 1.4/2 = 0.7 cm; h = 2.4 cm

Remaining volume = Vol. of cylinder – Vol. of cone

$= r^2h - 1/3x\,\pi r^2 h$

$= 2/3 \times \pi r^2 h$ [∵r and h are same for both]

$= 2/3 \times \pi \times 0.7^2 \times 2.4$

= **2.46 cm^3**

TYPE - 4 : Conversion of Volumes from one shape to another without changing the Volumes.

Exercise 7.4 (7). A solid sphere of radius 6 cm is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is 5 cm and its height is 32 cm, then find the thickness of the cylinder.

Given : Fundamental Qnty of Solid sphere

To be found : The thickness of hollow cylinder of same volume of the sphere

Solution : Solid Sphere : r = 6 cm;

Hollow cylinder : Outer radius = 5 cm ; Height = 32 cm

Let the thickness = t cm; Inner radius (r) = (5 – t) cm

Vol. of hollow cylinder = Vol. of Sphere

$$(R^2 - r^2)h = 4/3 \times \pi r^3$$

$$[5^2 - (5 - t)^2]32 = 4/3 \times 6^3$$

$$[25 - 25 + 10t - t^2] = (4 \times 6 \times 6 \times 6)/(3 \times 32)$$

$$10t - t^2 = 9$$

$$\text{(or) } t^2 - 10t + 9 = 0 ;$$

$$(t - 9)(t - 1) = 0 ; \text{ Solving } t = 9 \text{ cm or } 1\text{cm}$$

Here 9 cm can't be possible ∴ Thickness of the Cylinder = 1cm.

Exercise 7.4 (8). A hemispherical bowl is filled to the brim with juice. The juice is poured into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder then find the percentage of juice that can be transferred from the bowl into the cylindrical vessel.

Solution : Let the radius of the hemispherical bowl = r

∴ the radius of the cylinder = r [As per condition]

Also r = 1.5xh; ∴h = r/1.5 or h = (2/3)r

Vol. of the hemispherical bowl = $(2/3) \times \pi r^3$

Vol. of the cylindrical vessel = $\pi r^2 h = \pi r^2 (2/3) r = (2/3) \times \pi r^3$

Here both Volumes are equal

∴ 100 % of the juice can be transferred

TYPE - 5 : Problems with Ratios.

Exercise 7.1 (7). The ratio of the radii of two right circular cones of same height is 1:3. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone.

Solution : Ratio of radii = 1 : 3; So let us take the radii as r, 3r

Their height are 3r, 3r [As per condition]

Slanding length : $l = \sqrt{(h^2 + r^2)}$

l of Smaller cone = $\sqrt{[(3r)^2 + r^2]} = \sqrt{(9r^2 + r^2)} = \sqrt{(10r^2)} = r\sqrt{10} = r\sqrt{5}\sqrt{2}$

l of Larger cone = $\sqrt{[(3r)^2 + (3r)^2]} = \sqrt{(9r^2 + 9r^2)} = \sqrt{(18r^2)} = r\sqrt{18} = 3r\sqrt{2}$

CSA = πrl

CSA of Smaller cone/ CSA of Larger cone = (πxrx r√5√2)/(πx3rx 3r√2) = √5/9

Ratio of CSA = √5 : 9

Exercise 7.1 (8). Th e radius of a sphere increases by 25%. Find the percentage increase in its surface area.

Solution : Let the Radius of the sphere before increasing = r

Radius of the sphere after increasing = 1.25r

Surface before increasing = $4\pi r^2$

Surface after increasing = $4(1.25r)^2 = 4x\pi x1.5625r^2$

Increased surface area = $4x\pi x1.5625r^2 - 4\pi r^2 = 4x\pi x0.5625r^2$

Percentage increasing = $(4x\pi x0.5625r^2)/\ (4\pi r^2)$ x 100 = 56.25%

Exercise 7.2 (7). If the ratio of radii of two spheres is 4:7, find the ratio of their volumes.

Solution : Ratio of Radii of the spheres = 4 : 7

So Let the Radii of the spheres be 4r, 7r

Volume of the spheres = $4/3x(4r)^3$, $4/3x\pi(7r)^3$

= $4/3x\pi 64r^3$, $4/3x\pi 343r^3$

Ratio of volumes = $4/3x\pi 64r^3 : 4/3x\pi 343r^3$

= 64 : 343

Exercise 7.2 (8). A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is 3√3 : 4 .

Solution : Let the Radii of the Solid sphere and Hemisphere be R, r

As per condition, Surface areas are equal for both

$4\pi R^2 = 3\pi r^2$

R = (√3/2) r

Volume of the solid sphere = $4/3x\pi R^3 = 4/3x((\sqrt{3}/2)\ r)^3 = (\sqrt{3}/2)\pi r^3$

Volume of the Hemisphere = $4/3x\pi r^3/2 = 2/3x\pi r^3$

Vol. of the solid sphere/ Vol. of the Hemisphere = $((\sqrt{3}/2)r^3)/(2/3x\pi r^3)$

= 3√3/4

∴Ratio of their Vlumes = 3√3 : 4 (Proved)

Exercise 7.3 (7). A right circular cylinder just enclose a sphere of radius r units.

Calculate (i) the surface area of the sphere

(ii) the curved surface area of the cylinder

(iii) the ratio of the areas obtained in (i) and (ii).

Solution : Cylinder : Radius = r ; Height = 2r [∵It exactly fits the Sphere]

Sphere : Radius = r

(i) Surface area of the sphere = $4\pi r^2$

(ii) CSA of the Cylinder = $2\pi rh = 2\pi xrx2r = 4\pi r^2$

(iii) Their ratio = $4\pi r^2 : 4\pi r^2$ = 1: 1

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10th Maths – Chapter 8 STATISTICS AND PROBABILITY

(Book in One Marks & Solutions for Exercises)

Green indicates Thinking Corner, Blue indicates Progress Check

புதுமை படைக்க புரிந்து படி!

STATISTICS

1. Prasanta Chandra Mahalanobis introduced innovative techniques for conducting large-scale sample surveys and calculated acreages and crop yields by using the Method of random sampling.

2. He was awarded the Padma Vibhushan, one of India's highest honours, by the Indian government in 1968 and he is hailed as "Father of Indian Statistics".

3. The Government of India has designated 29th June every year, coinciding with his birth anniversary, as "National Statistics Day".

4. The most common Measures of Central Tendency are
   • Arithmetic Mean • Median • Mode

5. Does the mean, median and mode are same for a given data? Not always. But sometimes they may same like this data : 2, 4, 5, 5, 6, 8

6. What is the difference between the arithmetic mean and average? No difference

7. The mean of n observations is x , if first term is increased by 1 second term is increased by 2 and so on. What will be the new mean?

$$\textit{The mean of the increased value alone} = \frac{1+2+3+\ \dots\ +n}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

$$\textit{The new mean} = \bar{x} + \frac{n+1}{2}$$

8. The sum of all the observations divided by number of observations is Arithmetic Mean.

9. If the sum of 10 data values is 265 then their mean is 26.5. $\left(\frac{265}{10}\right)$

10. If the sum and mean of a data are 407 and 11 respectively, then the number of observations in the data are 37. $\left(\bar{x} = \frac{S}{n};\ \therefore n = \frac{S}{\bar{x}} = \frac{407}{11} = 37\right)$

11. Measures of Variation (or) Dispersion of a data provide an idea of how observations spread out (or) scattered throughout the data.

12. Different Measures of Dispersion are 1. Range 2. Mean deviation 3. Quartile deviation 4. Standard deviation 5. Variance 6. Coefficient of Variation.

13. The range of first 10 prime numbers is 27.

First 10 prime numbers are : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29

Range (R) = Largest value – Smallest value.= L – S = 29 – 2 = 27.

14. Can variance be negative? Never

Variance is always positive. Since it is the squares of the deviations from the mean.

15. Karl Pearson was the first person to use the word Standard deviation. German mathematician Gauss used the word Mean error.

16. Can the standard deviation be more than the variance? Yes. It can.

(If the Variance is less than one, Standard deviation is more than the Variance.)

(If the Variance is more than one, Standard deviation is less than the Variance.)

17. If the variance is 0.49 then the standard deviation is 0.7

18. For any collection of n values, can you find the value of

(i). $\sum(x_i - \bar{x}) = \sum d_i = 0$

(ii). $(\sum x_i) - \bar{x} = n\bar{x} - \bar{x} = \bar{x}(n-1)$

19. The standard deviation of a data is 2.8, if 5 is added to all the data values then the new standard deviation is 2.8.

⍟⍟ Note : The standard deviation of a given data will not change, if we add or subtract a constant to all the data values. But if we multiply the all data values with a constant, then the existing standard deviation also be multiplied with that constant to get the new standard deviation. ⍟⍟

20. If S is the standard deviation of values p, q, r then standard deviation of p–3, q–3, r–3 is S.

21. Coefficient of variation is a relative measure of Standard deviation.

22. When the standard deviation is divided by the mean we get Coefficient of variation .

23. The coefficient of variation depends upon Standard deviation and Mean.

24. If the mean and standard deviation of a data are 8 and 2 respectively then the coefficient of variation is 25 % . $\left(\frac{2}{8} \times 100\right)$

25. When comparing two data, the data with higher coefficient of variation is inconsistent.

Mere Muck up will put into the Dark Room.

குருட்டு மனப்பாடம் இருட்டறைக்குள் தள்ளிவிடும்.

PROBABILITY

1. An experiment in which a particular outcome cannot be predicted is called Random event.

2. The set of all possible outcomes is called a Sample space.

3. If an event E consists of only one outcome then it is called an elementary event.

4. Which of the following values cannot be a probability of an event?

$(a)\ -0.0001 \quad (b)\ 0.5 \quad (c)\ 1.001 \quad (d)\ 1 \quad (e)\ 20\% \quad (f)\ 0.253 \quad (g)\ \frac{1-\sqrt{5}}{2} \quad (h)\ \frac{\sqrt{3}+1}{4}$

a, c, g can not be the values of probability. (Since they are less than zero or more than one. Probability always 0 to 1 only.)

5. What will be the probability that a non leap year will have 53 Saturdays? $\frac{1}{7}$

6. What is the complement event of an impossible event? Certain events

7. 1. P(only A) = P(A) − P(A∩B).

8. P(Ā ∩ B) = P(only B) = P(B) − P(A∩B).

9. A ∩ B and Ā ∩ B are mutually exclusive events.

10. P(Ā ∩ B̄) = P($\overline{A \cup B}$) = 1 − P(A U B).

11. If A and B are mutually exclusive events then P(A ∩ B) = ∅.

12. If P(A ∩ B) = 0.3, P(Ā ∩ B) = 0.45 then P(B) = 0.75.

13. P(A U B)+P(A ∩ B) is P(A) + P(B).

14. A ∩ Ā = ∅ ; A U Ā = S

15. If A, B are mutually exclusive events, then P(A U B) = P(A) + P(B)

16. P (Union of mutually exclusive events) = ∑(Probability of events)

ACTIVITIES – 1

Find the standard deviation of the marks obtained by you in all five subjects in the quarterly examination and in the midterm test separately. What do you observe from your results.

S.No.	Test	Tamil	English	Maths	Science	S.S
1	Mid Term	80	81	100	92	97
2	Quarterly	92	88	90	90	90

Mid Term Test : Mean $\bar{x} = \frac{80+81+100+92+97}{5} = \frac{450}{5}$ = 90 (It is an integer)

x_i	$d_i = x_i - \bar{x}$ $= x_i - 90$	d_i^2
80	−10	100
81	−9	81
100	10	100
92	2	4
97	7	49
		334

$$Standard\ Deviation\ \sigma = \sqrt{\frac{\sum d_i^2}{n}}$$

$$= \sqrt{\frac{334}{5}}$$

$= \sqrt{66.8}$

= 8.17

Quarterly exam : Mean $\bar{x} = \frac{92+88+90+90+90}{5} = \frac{450}{5}$ = 90 (It is an integer)

x_i	$d_i = x_i - \bar{x}$ $= x_i - 90$	d_i^2
92	2	4
88	−2	4
90	0	0
90	0	0
90	0	0
		4

$$Standard\ Deviation\ \sigma = \sqrt{\frac{\sum d_i^2}{n}}$$

$$= \sqrt{\frac{4}{5}}$$

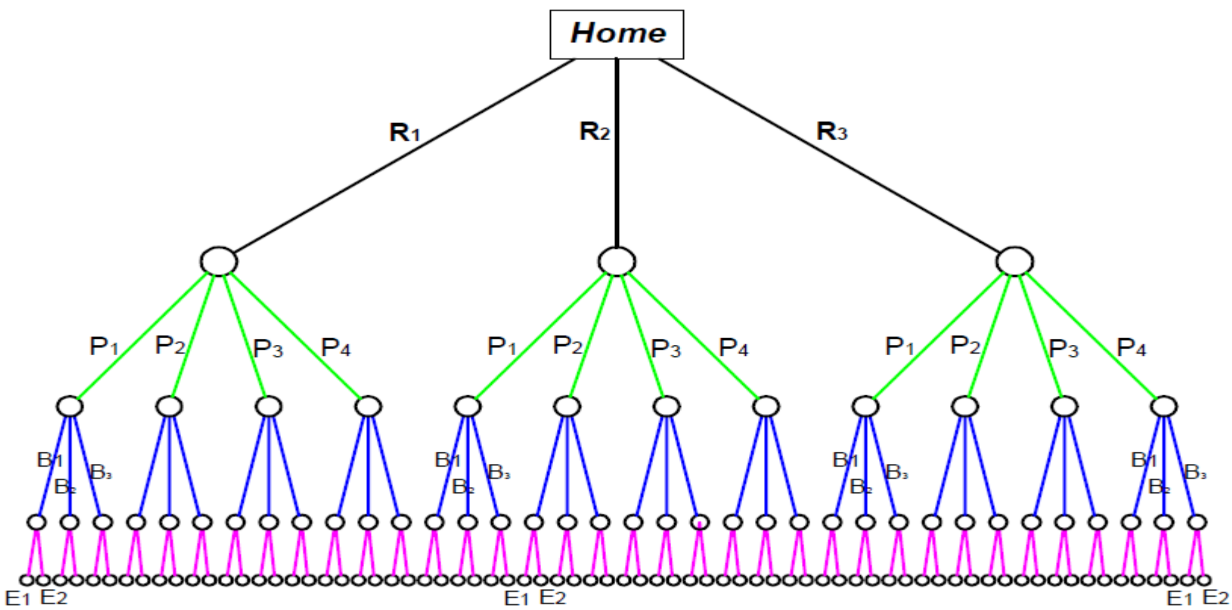
$= \sqrt{0.8}$

= 0.89

Observation : Eventhough the total and the means are same for both, there are much difference in the Standard deviations. It is because of the marks obtained in the Mid term are scatted towards the central value (Mean) than the Quarterly exam.

ACTIVITIES – 3

There are three routes R_1, R2 and R3 from Madhu's home to her place of work. There are four parking lots P1, P2, P3, P4 and three entrances B1, B2, B3 into the office building. There are two elevators E1 and E2 to her floor. Using the tree diagram explain how many ways can she reach her office?



Number of ways to reach the office = $3(R_1,R_2,R_3) \times 4(P_1,P_2,P_3,P_4) \times 3(B_1,B_2,B_3) \times 2(E_1,E_2)$

= **72 Ways**

ACTIVITIES – 4

Collect the details and find the probabilities of

(i) selecting a boy from your class. (ii) selecting a girl from your class.

(iii) selecting a student from tenth standard in your school.

(iv) selecting a boy from tenth standard in your school.

(v) selecting a girl from tenth standard in your school.

Solution : Let 10^{th} Std Boys = 32; Girls = 28 ; Total = 60; School Strength = 640

Sample space of 10^{th} Std = 60

Sample space of School = 640

(i) Probability of selecting a boy from 10^{th} Std = $\frac{32}{60}$ = 0.533

(ii) Probability of selecting a girl from 10^{th} Std = $\frac{28}{60}$ = 0.467

(iii) Probability of selecting a student from 10^{th} Std in the school = $\frac{60}{640}$ = 0.094

(iv) Probability of selecting a boy from 10^{th} Std in the school = $\frac{32}{640}$ = 0.05

(v) Probability of selecting a girl from 10^{th} Std in the school = $\frac{28}{640}$ = 0.044

ACTIVITIES – 5

The addition theorem of probability can be written easily using the following way.

$P(A \cup B) = S_1 - S_2$

$P(A \cup B \cup C) = S_1 - S_2 + S_3$

Where S1 $\rightarrow$ Sum of probability of events taken one at a time.

S2 $\rightarrow$ Sum of probability of events taken two at a time.

S3 $\rightarrow$ Sum of probability of events taken three at a time.

$$P(A \cup B) = \underbrace{P(A)+P(B)}_{S_1} - \underbrace{P(A \cap B)}_{S_2}$$

$$P(A \cup B \cup C) = \underbrace{P(A)+P(B)+P(C)}_{S_1} - \underbrace{(P(A\cap B)+P(B\cap C)+P(A\cap C))}_{S_2} + \underbrace{P(A\cap B\cap C)}_{S_3}$$

Find the probability of $P(A \cup B \cup C \cup D)$ using the above way. Can you find a pattern for the number of terms in the formula

Solition :

Let S1 $\rightarrow$ Sum of probability of events taken one at a time.

S2 → Sum of probability of events taken two at a time.

S3 → Sum of probability of events taken three at a time.

S4 → Sum of probability of events taken four at a time.

S5 → Sum of probability of events taken five at a time and so on…

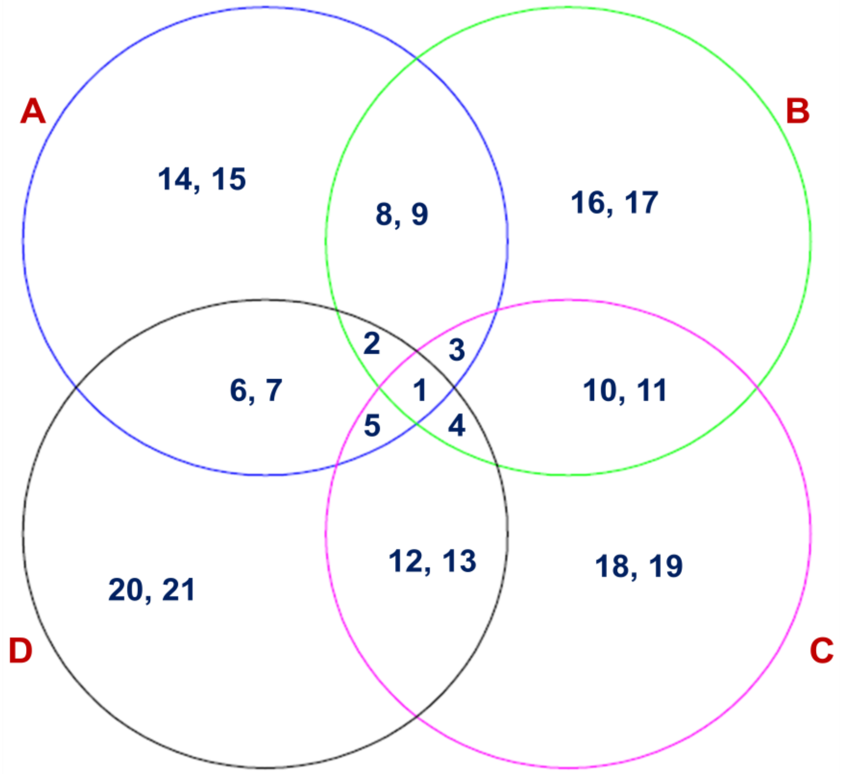
Example for four terms :

Let us take numbers from 1 to 21 and distribute it as shown in the fig. : n(S) = 21

∴ (A U B U C U D) = {1, 2, 3, …,21};   n(A U B U C U D) = 21

P(A U B U C U D) = $\frac{n(A \cup B \cup C \cup D)}{n(S)}$ = $\frac{21}{21}$ = 1 ------------- ①

And let the elements be distributed in A, B, C and D as shown in the Venn Diagram.



From the Venn diagram,

Taking One at a time,

A = {1, 2, 3, 5, 6, 7, 8, 9, 14, 15} ;   n(A) = 10;   P(A) = $10/21$

B = {1, 2, 3, 4, 8, 9, 10, 11, 16, 17} ;   n(B) = 10;   P(B) = $10/21$

C = {1, 3, 4, 5, 10, 11, 12, 13, 18, 19} ;   n(C) = 10;   P(C) = $10/21$

D = {1, 2, 4, 5, 6, 7, 12, 13, 20, 21} ;   n(D) = 10;   P(D) = $10/21$

∴P(A)+P(B)+P(C)+P(D) = $S_1 = \frac{40}{21}$

Taking Two at a time,

(A ∩ B) = {1, 2, 3, 8, 9} ;   n(A ∩ B) = 5;   P(A ∩ B) = $5/21$

(B ∩ C) = {1, 3, 4, 10, 11} ;   n(A ∩ B) = 5;   P(B ∩ C) = $5/21$

(C ∩ D) = {1, 4, 5, 12, 13} ;   n(A ∩ B) = 5;   P(C ∩ D) = $5/21$

(D ∩ A) = {1, 2, 5, 6, 7} ;   n(A ∩ B) = 5;   P(D ∩ A) = $5/21$

$(A \cap C) = \{1, 3, 5\}$; $n(A \cap C) = 3$; $P(A \cap C) = 3/21$

$(B \cap D) = \{1, 2, 4\}$; $n(A \cap C) = 3$; $P(B \cap D) = 3/21$

$\therefore P(A\cap B)+P(B\cap C)+P(C\cap D)+P(D\cap A)+P(A\cap C)+P(B\cap D) = S_2 = \frac{26}{21}$

<u>Taking Three at a time</u>,

$(A \cap B \cap C) = \{1, 3\}$; $n(A \cap B \cap C) = 2$; $P(A \cap B \cap C) = 2/21$

$(B \cap C \cap D) = \{1, 4\}$; $n(B \cap C \cap D) = 2$; $P(B \cap C \cap D) = 2/21$

$(C \cap D \cap A) = \{1, 5\}$; $n(B \cap C \cap D) = 2$; $P(C \cap D \cap A) = 2/21$

$(D \cap A \cap B) = \{1, 2\}$; $n(D \cap A \cap B) = 2$; $P(D \cap A \cap B) = 2/21$

$\therefore P(A\cap B\cap C)+ P(B\cap C\cap D)+ P(C\cap D\cap A)+ P(D\cap A\cap B) = S_3 = \frac{8}{21}$

<u>Taking Four at a time</u>,

$(A \cap B \cap C \cap D) = \{1\}$; $n(A \cap B \cap C \cap D) = 1$; $P(A \cap B \cap C \cap D) = 1/21$

$\therefore P(A \cap B \cap C \cap D) = S_4 = \frac{1}{21}$

$P(A \cup B \cup C \cup D) =$

$$\underbrace{P(A)+P(B)+P(C)+P(D)}_{S_1} - \underbrace{(P(A\cap B)+P(B\cap C)+P(C\cap D)+P(D\cap A)+P(A\cap C)+P(B\cap D))}_{S_2} +$$

$$\underbrace{(P(A\cap B\cap C)+ P(B\cap C\cap D)+ P(C\cap D\cap A)+ P(D\cap A\cap B))}_{S_3} - \underbrace{P(A\cap B\cap C\cap D)}_{S_4}$$

$P(A \cup B \cup C \cup D) = \frac{40}{21} - \frac{26}{21} + \frac{8}{21} - \frac{1}{21} = \frac{21}{21} = 1$ ------------- 2

Here 1 = 2, $\therefore P(A \cup B \cup C \cup D) = S_1 - S_2 + S_3 - S_4$

∴The Probability pattern for the number of terms is as follows.

$P(A \cup B)$ (2 terms) $= S_1 - S_2$

$P(A \cup B \cup C)$ (3 terms) $= S_1 - S_2 + S_3$

$P(A \cup B \cup C \cup D)$ (4 terms) $= S_1 - S_2 + S_3 - S_4$

$P(A \cup B \cup C \cup D \cup E)$ (5 terms) $= S_1 - S_2 + S_3 - S_4 + S_5$

And so on like this…

∴<u>The Probability pattern for the number of terms = Sum of odd terms – Sum of even terms</u>.

எளிதாய் விளங்கும் கல்வியை
இளமையில் விரும்பிக் கற்றிடு
வானமாய் விரிந்த கல்வியை
பாணமாய் விரைந்து கற்றிடு
தானமாய் பெற்ற கல்வியைத்
தரணியில் பலருக் களித்திடு.

Solution to Exercises

Exercise 8.1

1(i). Range = L – S = 125 – 63 = 62; L + S = 125 + 63 = 188

Coeff. of Range = $\frac{L-S}{L+S} = \frac{62}{188}$ = 0.33

(ii). Range = L – S = 61.4 – 13.6 = 47.8; L + S = 61.4 + 13.6 = 75

Coeff. of Range = $\frac{L-S}{L+S} = \frac{47.8}{75}$ = 0.64

2. Range = 36.8; S = 13.4; L = R + S = 36.8 + 13.4 = 50.2

3. Initial range of Income = 400 - 450; Final range of Income = 600 - 650;

Range = 650 – 400 = 250.

4. Total pages to be completed = 60 ; Total number of Students = 8

Pages completed by each = 32, 35, 37, 30, 33, 36, 35, 37

Pages to be completed by each = 28, 25, 23, 30, 27, 24, 25, 23

Mean $\bar{x} = \frac{28+25+23+30+27+24+25+37}{8} = \frac{205}{8}$ = 25.625

Since the mean is not an integer, let us adopt assumed mean method to find SD

Let the assumed mean be 25;

x_i	$d_i = x_i - A$ $= x_i - 25$	d_i^2
28	3	9
25	0	0
23	−2	4
30	5	25
27	2	4
24	−1	1
25	0	0
23	−2	4
	$\sum d_i$ = 5	47

Standard Deviation $\sigma = \sqrt{\frac{\sum di^2}{n} - \left(\frac{\sum di}{n}\right)^2}$

$= \sqrt{\frac{47}{8} - \left(\frac{5}{8}\right)^2}$

$= \sqrt{5.875 - (0.625)^2}$

$= \sqrt{5.875 - 0.39}$

$= \sqrt{5.485}$

= 2.34

5. Wages of 9 workers :₹310, ₹290, ₹320, ₹280, ₹300, ₹290, ₹320, ₹310, ₹280.

Mean $\bar{x} = \frac{310+290+320+280+300+290+320+310+280}{9} = \frac{2700}{9}$ = 300. (It is an integer.)

So let us use actual mean with step deviation method with C = 10

x_i	$d_i = (x_i - \bar{x})/10$ $= (x_i - 300)/10$	d_i^2
310	1	1
290	−1	1
320	2	4
280	−2	4
300	0	0

Standard Deviation $\sigma = C \times \sqrt{\frac{\sum di^2}{n}}$

$= 10 \times \sqrt{\frac{20}{9}}$

$= 10 \times \left(\frac{2}{3}\right) \times \sqrt{5}$

290	−1	1
320	2	4
310	1	1
280	−2	4
		20

σ = 14.91

Variance σ^2 = 14.91 x 14.91 = 222.30

6. Number of strikes in 12 hr = 1+2+3+…+12 = $\frac{n(n+1)}{2} = \frac{12x13}{2}$ = 78 times.

Number of strikes in 24 hr =2(1+2+3+…+12) = 2 x 78 = 156 times.

Standard Deviation of first 'n' natural number = $\sqrt{\frac{n^2-1}{12}}$

Standard Deviation of 12 hr = $\sqrt{\frac{12^2-1}{12}} = \sqrt{\frac{143}{4\times3}} = \frac{1}{2}\times\sqrt{\frac{143}{3}} = \frac{1}{2}\times\sqrt{47.67}$

Standard Deviation of 24 hr = $2\times\frac{1}{2}\times\sqrt{47.67} = \sqrt{47.67}$ = = 6.9

7. Standard Deviation of first 'n' natural number = $\sqrt{\frac{n^2-1}{12}}$

Standard Deviation of first 21 natural number = $\sqrt{\frac{21^2-1}{12}} = \sqrt{\frac{440}{12}} = \sqrt{\frac{110}{3}}$

$= \sqrt{36.67}$ = 6.05

8. The standard deviation of a data = 4.5

Each value of the data is decreased (subtracted) by 5

The standard deviation of a data will not change for addition or subtraction of a constant in each data.

∴The standard deviation of the new data after decrease is also = 4.5

9. The standard deviation of a data = 3.6

Each value of the data is divided by 3

For mulplication or divition of a constant in each data, the existing standard deviation of the data will also be multiplied or divided by that constant to get the new SD.

∴The standard deviation of the new data = 3.6/3 = 1.2

Problems 10 to 13 are grouped datas Standard Deviation. (Let us solve 12)

10. Hints : For any grouped use assumed mean method. For easy calculation use the middle midvalue as assumed mean.

Do it with assumed mean as 55 or 60 and C as 5 ; (Do as in Example : 8.13)

11. Hints : Here the mid values are 5,15,25,35,45,55,65.

Do it with assumed mean as 35 and C as 10; (Do as in Example : 8.13)

13. Hints : Here the mid values are 9,10,11,12,13.

Do it with assumed mean as 11; (Do as in Example : 8.12)

12. The given frequencies are 21-24, 25-28, 29-32, 33-36, 37-40, 41-44

It's not a continuous frequency one. So let us change it as a continuous.

The continuous frequency = 20.5-24.5, 24.5- 28.5, 28.5-32.5,32.5-36.5,36.5-40.5, 40.5-44.5

The Midvalue = 22.5, 26.5, 30.5, 34.5, 38.5, 42.5;

Being decimal and for easy calculation let us deduct 2.5 in each midvalue

The New Midvalue = 20, 24, 28, 32, 36, 40; Let the assumed mean A = 28 and C = 4

Diameters	Midvalue – 2.5	fi	$d_i = x_i - A = x_i - 28$	$d_i = (x_i - A)/4$	$f_i d_i$	$f_i d_i^2$
20.5-24.5	20	15	−8	−2	−30	60
24.5-28.5	24	18	−4	−1	−18	18
28.5-32.5	28	20	0	0	0	0
32.5-36.5	32	16	4	1	4	16
36.5-40.5	36	8	8	2	16	32
40.5-44.5	40	7	12	3	21	63
		N=84	$\sum d_i = 5$		$\sum f_i d_i = 5$	$\sum f_i d_i^2 = 189$

Standard Deviation $\sigma = C \times \sqrt{\frac{\sum f_i d_i^2}{N} - \left(\frac{\sum f_i d_i}{N}\right)^2}$

$$= 4 \times \sqrt{\frac{189}{84} - \left(\frac{5}{84}\right)^2}$$

$= 4 \times \sqrt{2.25 - 0.00}$ $\quad [\because \left(\frac{5}{84}\right)^2 = 0.06^2 = 0$ for two decimal places]

= 4 x 1.5 = 6.0

14. Candidates = 100; Mean = 60; SD = 15

Incorrect datas = 40, 27 ; Correct data = 45, 72

Correct total value = 100x60 – (40 + 27) + (45 + 72) = 6050;

Correct mean = 6050/100 = 60.5

Let us first find incorrect $\sum x_i^2$ Using direct method of SD

$$\sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} = \sigma$$

$$\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2 = \sigma^2$$

$$\frac{\sum x_i^2}{100} - (60)^2 = 15^2$$

$$\frac{\sum x_i^2}{100} = 225 + 3600 = 3825$$

$$\sum x_i^2 = 3825 \times 100 = 382500$$

Corrected $\sum x_i^2$ = 382500 – ($40^2 + 27^2$) + ($45^2 + 72^2$) = 387380

Corrected SD = $\sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} = \sqrt{\frac{387380}{100} - (60.5)^2}$

$= \sqrt{3873.80 - 3660.25}$

$= \sqrt{213.55} = 14.61$

15. Mean ($\bar{x}$) = 8; Variance (σ^2) = 16; Number of datas = 7; Five datas are : 2, 4, 10, 12, 14

Let the remaining two datas be x, y

$$\frac{Sum\ of\ the\ datas}{Number\ of\ datas} = \bar{x}$$

$$\frac{2+4+10+12+14+x+y}{7} = 8$$

$42 + x + y = 56$; $x + y = 14$; $y = 14 - x$ ------------- 1

$\sigma^2 = \left(\frac{1}{n}\right)\sum(x_i - \bar{x})^2 = 16$

$\sum(x_i - 8)^2 = 16x7 = 112$

$(2-8)^2 + (4-8)^2 + (10-8)^2 + (12-8)^2 + (14-8)^2 + (x-8)^2 + (y-8)^2 = 112$

$(-6)^2 + (-4)^2 + (2)^2 + (4)^2 + (6)^2 + (x-8)^2 + (6-x)^2 = 112$ [From 1 $y = 14 - x$]

$36 + 16 + 4 + 16 + 36 + x^2 - 16x + 64 + x^2 - 12x + 36 = 112$

$2x^2 - 28x + 208 - 112 = 0$; $x^2 - 14x + 48 = 0$; $(x-10)(x-4) = 0$

Solving x = 6 or 8; ∴ y = 8 or 6

The two datas are : 6, 8

Exercise 8.2

1. Standard Deviation σ = 6.5; Mean $\bar{x}$ = 12.5 ; C.V = ?

C.V = $\frac{\sigma}{\bar{x}} \times 100 = \frac{6.5}{12.5} \times 100 = 52\ \%$

2. Standard Deviation σ = 1.2; C.V = 25.6; Mean $\bar{x}$ = ?

C.V = $\frac{\sigma}{\bar{x}} \times 100$

Mean $\bar{x} = \frac{\sigma}{C.V} \times 100 = \frac{6.5}{25.6} \times 100 = 4.6875$ or 4.69

3. Mean $\bar{x}$ = 15; C.V = 48; Standard Deviation σ = ?

C.V = $\frac{\sigma}{\bar{x}} \times 100$

Standard Deviation $\sigma = \frac{C.V \times \bar{x}}{100} = \frac{48 \times 15}{100} = 7.2$

4. n = 5; $\bar{x}$ = 6 , $\sum x^2$ = 765; C.V = ?

Standard Deviation $\sigma = \sqrt{\frac{\sum {x_i}^2}{n} - \left(\frac{\sum x_i}{n}\right)^2} = \sqrt{\frac{765}{5} - (6)^2}$

$= \sqrt{153 - 36} = \sqrt{117} = 10.82$

C.V = $\frac{\sigma}{\bar{x}} \times 100 = \frac{10.82}{6} \times 100 = 180.33\ \%$

5. Given datas : 24, 26, 33, 37, 29, 31; Number of datas = 6

$$Mean\ (\bar{x}) = \frac{24+26+33+37+29+31}{6} = 30$$

x_i	$d_i = x_i - \bar{x}$ $= x_i - 30$	d_i^2
24	−6	36
26	−4	16
33	3	9
37	7	49
29	−1	1
31	1	1
		112

$$Standard\ Deviation\ \sigma = \sqrt{\frac{\sum d_i^2}{n}}$$

$$= \sqrt{\frac{112}{6}} = \sqrt{18.67} = 4.32$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{4.32}{30} \times 100$$

= 14.4 %

6. Time taken (in min.) by 8 students : 38, 40, 47, 44, 46, 43, 49, 53.

$$Mean\ (\bar{x}) = \frac{38+40+47+44+46+43+49+53}{8} = 45$$

x_i	$d_i = x_i - \bar{x}$ $= x_i - 45$	d_i^2
38	−7	49
40	−5	25
47	2	4
44	−1	1
46	1	1
43	−2	4
49	4	16
53	8	64
		165

$$Standard\ Deviation\ \sigma = \sqrt{\frac{\sum d_i^2}{n}}$$

$$= \sqrt{\frac{165}{8}} = \sqrt{20.625} = 4.54$$

$$C.V = \frac{\sigma}{\bar{x}} \times 100 = \frac{4.54}{45} \times 100$$

= 10.09 %

7. Sathya's total in 5 subjects = 460; Her SD = 4.6; Her mean($\bar{x}$) = $\frac{460}{5}$ = 92

Sathya's C.V = C.V = $\frac{\sigma}{\bar{x}} \times 100$ = $\frac{4.6}{92} \times 100$ = 5 ------------- 1

Vidhya's total in 5 subjects = 480; Her SD = 2.4; Her mean($\bar{x}$) = $\frac{480}{5}$ = 96

Vidhya's C.V = $\frac{2.4}{96} \times 100$ = 2.5 ------------- 2

Comparing 1 and 2 Vidhya is more consistent than Sathya.

8. Mathematic's mean = 56; It's SD = 12

Science's mean = 65; It's SD = 14

Social Science's mean = 60; It's SD = 10

Mathematic's C.V = C.V = $\frac{\sigma}{\bar{x}} \times 100$ = $\frac{12}{56} \times 100$ = 21.43 ------------- 1

Science's C.V = $\frac{14}{65} \times 100$ = 21.54 ------------- 2

Social Science's C.V = $\frac{10}{60} \times 100$ = 16.67 ------------- 3

Comparing 1, 2 and 3

Science shows the highest variation.

Social Science shows the lowest variation.

9. Temperature of city A (in degree Celsius) : 18, 20, 22, 24, 26
Temperature of city B (in degree Celsius) : 11, 14, 15, 17, 18

$$Mean\ of\ A = \frac{18+20+22+24+26}{5} = 22\ ;\ Mean\ of\ B = \frac{11+14+15+17+18}{5} = 15$$

To find SD for city A

x_i	$d_i = x_i - \bar{x} = x_i - 22$	d_i^2
18	−4	16
20	−2	4
22	0	0
24	2	4
26	4	16
		40

To find SD for city B

x_i	$d_i = x_i - \bar{x} = x_i - 15$	d_i^2
11	−4	16
14	−1	1
15	0	0
17	2	4
18	3	9
		30

SD of A (σ) = $\sqrt{\frac{\sum d_i^2}{n}} = \sqrt{\frac{40}{5}} = \sqrt{8} = 2\sqrt{2}$ = 2.828

SD of B (σ) = $\sqrt{\frac{\sum d_i^2}{n}} = \sqrt{\frac{30}{5}} = \sqrt{6}$ = 2.45

C.V of A = $\frac{\sigma}{\bar{x}} \times 100 = \frac{2.828}{22} \times 100$ = 12.85 ------------- (1)

C.V of B = $\frac{2.45}{15} \times 100$ = 16.33 ------------- (2)

Comparing (1) and (2) City A is more consistent.

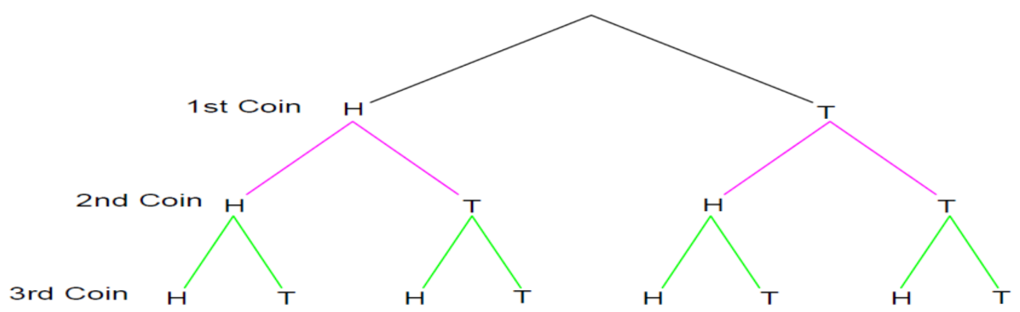
<u>Hints to find SD</u>

1. For ungrouped data first find the mean. If the mean is an integer ,use actual mean method. If the mean is in decimal, use assumed mean method.

2. For grouped data, use assumed mean method to find SD.

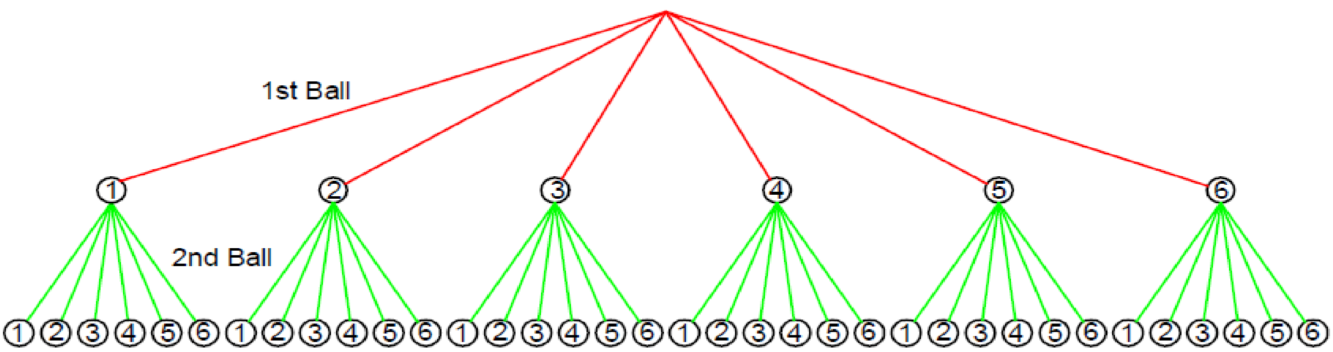
<u>Exercise 8.3</u>

1. <u>Tossing 3 Coins</u>



Sample space : {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

2. Selecting two balls from a bag containing 6 balls numbered 1 to 6



Sample space = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

3. $P(A) : P(\bar{A}) = 17 : 15$; $n(S) = 640$;

$15P(A) = 17P(\bar{A})$

$15[1 - P(\bar{A})] = 17P(\bar{A})$ $[\because P(A) + P(\bar{A}) = 1]$

$15 - 15P(\bar{A}) = 17P(\bar{A})$

$32P(\bar{A}) = 15$;

(i) $P(\bar{A}) = \frac{15}{32}$;

$\therefore P(A) = 1 - P(\bar{A}) = 1 - \frac{15}{32} = \frac{17}{32}$

(ii) $P(A) = \frac{n(A)}{n(S)} = \frac{17}{32}$

$\therefore n(A) = \frac{17}{32} \times n(S) = \frac{17}{32} x\, 640 = 340$

4. A coin is tossed thrice, S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT} ; n(S) = 8

Let A be the event of getting two consecutive tails

A = {HTT, TTH, TTT}; n(A) = 3

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{8}$

5. Number of cards = 1000; n(S) = 1000

Let A be the event of getting perfect square number above 500

A = {529, 576, 625, 676, 729, 784, 841, 900, 961};

(i). Chance of getting 1st prize winner n(A) = 9

$P(A) = \frac{n(A)}{n(S)} = \frac{9}{1000}$

(ii). After getting 1st prize, now n(S) = 1000 – 1 = 999

Let B be the event of getting perfect square number above 500

For getting 2nd prize, n(B) = 9 – 1 = 8

P(B) = $\frac{n(B)}{n(S)} = \frac{8}{999}$

6. (i) Blue balls = 12; Red falls = x; n(S) = x+12

Let A be the event of getting red balls

n(A) = x;   P(A) = $\frac{n(A)}{n(S)} = \frac{x}{x+12}$ ------------- (1)

(ii) After adding 8 Red falls, now n(S) = 12+x+8 = x+20

Now B be the event of getting red balls

n(B) = x+8; P(B) = $\frac{n(B)}{n(S)} = \frac{x+8}{x+20}$ ------------- (2)

As per condition, (2) = 2 x (1)

$\frac{x+8}{x+20} = 2 \times \frac{x}{x+12}$

$(x+8) \times (x+12) = 2x \times (x+20)$

$x^2 + 20x + 96 = 2x^2 + 40x$

$x^2 + 20x - 96 = 0$

$(x + 24)(x - 4) = 0$

x = – 24 or 4; Since negative value is impossible, x = 4; P(A) = $\frac{4}{16} = \frac{1}{4}$

7. Two unbiased dice are rolled once.

Then Sample space = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),
(4,1), (4,2), (4,3), (4,4), (4,5), (4,6)
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

n(S) = 36

(i). Let A be the event of getting a doublet

A = {(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)} ; n(A) = 6 ;

P(A) = $\frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(ii). Let B be the event of getting a product as a prime number

B = {(1,2), (1,3), (1,5), (2,1), (3,1), (5,1)} ; n(A) = 6

P(B) = $\frac{n(B)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(iii). Let C be the event of getting a Sum as a prime number

C = {(1,1), (1,2), (1,4), (1,6), (2,1), (2,3), (2,5), (3,2), (3,4), (4,1), (4,3), (5,2), (5,6), (6,1), (6,5)} ; n(C) = 15

P(C) = $\frac{n(C)}{n(S)} = \frac{15}{36} = \frac{5}{12}$

(iv). Let D be the event of getting the sum as 1

D = { } ; n(A) = 0

P(D) = $\frac{n(D)}{n(S)} = \frac{0}{36} = 0$

8. If threee coins are tossed together,

Then it's sample space : {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}; n(S) = 8

(i) All heads ; A = {HHH}; n(A) = 1; P(A) = $\frac{n(A)}{n(S)} = \frac{1}{8}$

(ii) Atleast one tail ; B = {HHT, HTH, HTT, THH, THT, TTH, TTT}; n(B) = 7

P(B) = $\frac{n(B)}{n(S)} = \frac{7}{8}$

(iii) Atmost one head; C = { HTT, THT, TTH, TTT }; n(C) 4; P(C) = $\frac{n(C)}{n(S)} = \frac{4}{8} = \frac{1}{2}$

(iv) Atmost two tails; D = { HHH, HHT, HTH, HTT, THH, THT, TTH}; n(D) = 7

P(D) = $\frac{n(D)}{n(S)} = \frac{7}{8}$

9. Two dice are numbered as : 1,2,3,4,5,6 and 1,1,2,2,3,3

Then it's Sample space = {(1,1), (1,1), (1,2), (1,2), (1,3), (1,3),
(2,1), (2,1), (2,2), (2,2), (2,3), (2,3),
(3,1), (3,1), (3,2), (3,2), (3,3), (3,3),
(4,1), (4,1), (4,2), (4,2), (4,3), (4,3),
(5,1), (5,1), (5,2), (5,2), (5,3), (5,3),
(6,1), (6,1), (6,2), (6,2), (6,3), (6,3)}

n(S) = 36

(i) Let A_1 be the event of getting a sum of 2.

A_1 = {(1,1), (1,1)}; n(A_1) = 2; P(A_1) = $\frac{n(A1)}{n(S)} = \frac{2}{36} = \frac{1}{18}$

(ii) Let A_2 be the event of getting a sum of 3.

A_2 = {(1,2), (1,2), (2,1), (2,1)}; n(A_2) = 4; P(A_2) = $\frac{n(A2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

(iii) Let A_3 be the event of getting a sum of 4.

A_3 = {(1,3), (1,3), (2,2), (2,2), (3,1), 3,1)}; n(A_3) = 6; P(A_3) = $\frac{n(A3)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(iv) Let A_4 be the event of getting a sum of 5.

A_4 = {(2,3), (2,3), (3,2), (3,2), (4,1), (4,1) }; n(A_4) = 6; P(A_4) = $\frac{n(A4)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(v) Let A_5 be the event of getting a sum of 6.

A_5 = {(3,3), (3,3), (4,2), (4,2), (5,1), (5,1)}; n(A_5) = 6; P(A_5) = $\frac{n(A5)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(vi) Let A_6 be the event of getting a sum of 7.

A_6 = {(4,3), (4,3), (5,2), (5,2), (6,1), (6,1)}; n(A_6) = 6; P(A_2) = $\frac{n(A6)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(vii) Let A_7 be the event of getting a sum of 8.

A_7 = {(5,3), (5,3), (6,2), (6,2)}; n(A_7) = 4; P(A_2) = $\frac{n(A7)}{n(S)} = \frac{4}{36} = \frac{1}{9}$

(viii) Let A_8 be the event of getting a sum of 9.

A_8 = {(6,3), (6,3)}; $n(A_2) = 2$; $P(A_8) = \frac{n(A8)}{n(S)} = \frac{2}{36} = \frac{1}{18}$

10. Bag contains : 5 red balls, 6 white balls, 7 green balls, 8 black balls.

It's sample space : n(S) = 5+6+7+8 = 26

(i) Probability white

Let A be the event of getting a white ball.

$n(A) = 6$; $P(A) = \frac{n(A)}{n(S)} = \frac{6}{26} = \frac{3}{13}$

(ii) Probability black or red

Let B be the event of getting a black or red ball.

$n(B) = 8+5 = 13$; $P(B) = \frac{n(B)}{n(S)} = \frac{13}{26} = \frac{1}{2}$

(iii) Probability of not white

From (i), Probability white $P(A) = \frac{3}{13}$

Probability not white is $P(\bar{A}) = 1 - P(A) = 1 - \frac{3}{13} = \frac{10}{13}$

(iv) Probability of neither white nor black

Let C be the event of getting either white nor black

$n(C) = 8+6 = 14$; $P(C) = \frac{n(C)}{n(S)} = \frac{14}{26} = \frac{7}{13}$

Probability neither white nor black is $P(\bar{C}) = 1 - P(C) = 1 - \frac{7}{13} = \frac{6}{13}$

11. Non-defective bulbs = 20 ; Let the defective bulbs (A) = x ; n(S) = 20 + x

Probability of getting a defective bulb = $\frac{n(A)}{n(S)} = \frac{x}{20+x} = \frac{3}{8}$

$8x = 3x + 60$;

$5x = 60$; $\therefore x = \frac{60}{5} = 12$; Defective bulbs = 12

12.

Cards removed in Diamonds (Red in colour)	King	Queen	
Cards removed in Hearts (Red in colour)		Queen	Jack
Cards removed in Spades (Black in colour)	King		Jack
Total cards removed	2 King	2 Queen	2 Jack

The remaining cards : n(S) = 52 – 6 = 46

(i) Probability of a clavor : (No cards removed in clavor)

Let A be the event of getting a clavor card.

$n(A) = 13$; $P(A) = \frac{n(A)}{n(S)} = \frac{13}{46}$

(ii) Probability of a queen of red card :

Let B be the event of getting a queen of red card.

$\because$Both red cards of queen have been removed from deck of cards

$n(B) = 0$; $P(B) = \frac{n(B)}{n(S)} = 0$

(iii). Probability of a king of black card

Let C be the event of getting a king of black card.

Out of two black kings, one has been removed spades.

n(C) = 1 ; P(C) = $\frac{n(C)}{n(S)} = \frac{1}{46}$

13. Total area of rectangular region = 4 x 3 = 12 ; n(S) = 12

Prize winning area of circular region = 3.14 x 1^2 = 3.14 ,

n(A) = 3.14; P(A) = $\frac{n(A)}{n(S)} = \frac{3.14}{12} = \frac{314}{1200} = \frac{157}{600}$

14. Both Priya and Amudhan are visiting the shop from Monday to Saturday.

It is as like as rolling of two dice of 1 to 6

∴ Their's Sample space = {(Mo,Mo), (Mo,Tu), (Mo,We), (Mo,Th), (Mo,Fr), (Mo,Sa),
(Tu,Mo), (Tu,Tu), (Tu,We), (Tu,Th), (Tu,Fr), (Tu,Sa),
(We,Mo), (We,Tu), (We,We), (We,Th), (We,Fr), (We,Sa),
(Th,Mo), (Th,Tu), (Th,We), (Th,Th), (Th,Fr), (Th,Sa),
(Fr,Mo), (Fr,Tu), (Fr,We), (Fr,Th), (Fr,Fr), (Fr,Sa),
(Sa,Mo), (Sa,Tu), (Sa,We), (Sa,Th), (Sa,Fr), (Sa,Sa)}

n(S) = 36

(i) Probability of visiting on the same day

Let A be the event of visiting on the same day.

A = {(Mo,Mo), (Tu,Tu), (We,We), (Th,Th), (Fr,Fr), (Sa,Sa)}

n(A) = 6 ; P(A) = $\frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(ii) Probability of visiting on different days

Let B be the event of visiting on different days.

Except the days as per (i), the all other days are different days

n(B) = 36 – 6 = 30 ; P(B) = $\frac{n(B)}{n(S)} = \frac{30}{36} = \frac{5}{6}$

(iii) Probability of visiting on consecutive days

Let C be the event of visiting on consecutive days.

C = {(Mo,Tu), (Tu,We), (We,Th), (Th,Fr), (Fr,Sa)}

n(C) = 5 ; P(C) = $\frac{n(C)}{n(S)} = \frac{5}{36}$

15. The game consists of tossing a coin 3 times.

∴It's sample space = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}; n(S) = 8

(i) To get double entry fee : She should throw 3 heads

Let A be the event of getting 3 heads.

A = {HHH}

n(A) = 1 ; P(A) = $\frac{n(A)}{n(S)} = \frac{1}{8}$

(ii) To get just her entry fee : She should throw one or two heads

Let B be the event of getting one or two heads.

B = {HHT, HTH, HTT,THH, THT, TTH}

n(B) = 6 ; P(B) = $\frac{n(B)}{n(S)} = \frac{6}{8} = \frac{3}{4}$

(iii) To loose the entry fee : Excluding both (i) and (ii)

Let C be the event of getting except both (i) and (ii).

P(C) = 1 – 〔P(A) + P(B)〕 = 1 – $\left(\frac{1}{8} + \frac{6}{8}\right) = \frac{1}{8}$

Exercise 8.4

1. P(A) = 2/3; P(B) = 2/5; P(A ∪ B) = 1/3; P(A ∩ B) = ?

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

P(A ∩ B) = (P(A) + P(B)) – P(A ∪ B)

= $\frac{2}{3} + \frac{2}{5} - \frac{1}{3}$ = $\frac{11}{15}$

2. P(A) = 0.42; P(B) = 0.48; P(A ∩ B) = 0.16

(i). P(not A) = P($\bar{A}$) = 1 – P(A) = 1 – 0.42 = 0.58

(ii). P(not B) = P($\bar{B}$) = 1 – P(B) = 1 – 0.48 = 0.52

(iii). P(A or B) = P(A ∪ B)

= P(A) + P(B) – P(A ∩ B)

= 0.42 + 0.48 – 0.16 = 0.74

3. P(not A) = 0.45; P(A ∪ B) = 0.65; P(B) = ?

P(not A) = P($\bar{A}$) = 0.45

P(A) = 1 – P($\bar{A}$) = 1 – 0.45 = 0.55

Being mutually exclusive events : P(A) + P(B) = P(A ∪ B)

P(B) = P(A ∪ B) – P(A) = 0.65 – 0.55 = 0.1

4. P(A ∪ B) = 0.6; P(A ∩ B) = 0.2; P($\bar{A}$) + P($\bar{B}$) = ?

P(A) + P(B) – P(A ∩ B) = P(A ∪ B)

P(A) + P(B) = P(A ∪ B) + P(A ∩ B) = 0.6 + 0.2 = 0.8

P($\bar{A}$) + P($\bar{B}$) = 1 – P(A) + 1 – P(B)

= 2 – (P(A) + P(B))

= 2 – 0.8

= 1.2

5. P(A) = 0.5; P(B) = 0.3; A and B are mutually exclusive events

P(A ∪ B) = P(A) + P(B) (For mutually exclusive events)

P(A ∪ B) = 0.5 + 0.3 = 0.8

Probability that neither A nor B = P($\overline{A \cup B}$)

P($\overline{A \cup B}$) = 1 – P(A ∪ B) = 1 – 0.8 = 0.2

6. Two dice are rolled once. It's Sample space = {(1,1), (1,2), (1,3), (1,4), (1,5), (1,6),
(2,1), (2,2), (2,3), (2,4), (2,5), (2,6),
(3,1), (3,2), (3,3), (3,4), (3,5), (3,6),

(4,1), (4,2), (4,3), (4,4), (4,5), (4,6),
(5,1), (5,2), (5,3), (5,4), (5,5), (5,6),
(6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

n(S) = 36

Let A be the event of getting an even number first

A = {(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)}

n(A) = 18; P(A) = $\frac{n(A)}{n(S)} = \frac{18}{36}$

Let B be the event of getting a face sum of 8

B = {(2,6), (3,5), (4,4), (5,3), (6,2)}

n(B) = 5; P(B) = $\frac{n(B)}{n(S)} = \frac{5}{36}$

A ∩ B = {(2,6), (4,4), (6,2)}

n(A ∩ B) = 5; P(A ∩ B) = $\frac{n(A \cap B)}{n(S)} = \frac{3}{36}$

Probability of getting an either A or B = P(A ∪ B)

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

= $\frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$

7. For a deck of cards : n(S) = 52

Let A be the event of getting a red king

A = {King(diamods), King(hearts)}

n(A) = 2; P(A) = $\frac{n(A)}{n(S)} = \frac{2}{52}$

Let B be the event of getting a black queen

A = {Queen(clavor), Queen(spade)}

n(B) = 2; P(B) = $\frac{n(B)}{n(S)} = \frac{2}{52}$

The above two events are mutually exclusive events

P(A ∪ B) = P(A) + P(B)

= $\frac{2}{52} + \frac{2}{52} = \frac{4}{52} = \frac{1}{13}$

8. The cards in the box = {3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37}

n(S) = 18

Let A be the event of getting a card which is multiples of 7

A = {7, 21, 35}

n(A) = 3; P(A) = $\frac{n(A)}{n(S)} = \frac{3}{18}$

Let B be the event of getting a card which is a prime number

B = {3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37}

n(B) = 11; P(B) = $\frac{n(B)}{n(S)} = \frac{11}{18}$

A ∩ B = {7}

n(A ∩ B) = 1; P(A ∩ B) = $\frac{n(A\cap B)}{n(S)} = \frac{1}{18}$

<u>Probability that the drawn card have either A or B</u> = P(A ∪ B)

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

$= \frac{3}{18} + \frac{11}{18} - \frac{1}{18} = \frac{13}{18}$

9. Sample space tossing 3 coins = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

<u>Let A be the event of getting atmost 2 tails</u>

A = {HHH, HHT, HTH, HTT, THH, THT, TTH}

n(A) = 7; P(A) = $\frac{n(A)}{n(S)} = \frac{7}{8}$

<u>Let B be the event of getting atleast 2 heads</u>

B = {HHH, HHT, HTH, THH }

n(B) = 4; P(B) = $\frac{n(B)}{n(S)} = \frac{4}{8}$

A ∩ B = {HHH, HHT, HTH, THH }

n(A ∩ B) = 4; P(A ∩ B) = $\frac{n(A\cap B)}{n(S)} = \frac{4}{8}$

Probability of getting either A or B = P(A ∪ B)

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

$= \frac{7}{8} + \frac{4}{8} - \frac{4}{8} = \frac{7}{8}$

10. Probability getting an electrification contract P(A) = $\frac{3}{5}$

robability not getting a plumbing contract P($\bar{B}$) = $\frac{5}{8}$

∴Probability getting a plumbing contract P(B) = 1 – P($\bar{B}$)

$= 1 - \frac{5}{8} = \frac{3}{8}$

The probability of getting atleast one contract = P(A ∪ B) = $\frac{5}{7}$

The probability that he will get both A and B = P(A ∩ B)

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

P(A ∩ B) = P(A) + P(B) – P(A ∪ B)

P(A ∩ B) = $\frac{3}{5} + \frac{5}{8} - \frac{5}{7} = \frac{168+105-200}{280} = \frac{73}{280}$ (LCM of 5,8,7 is 280)

11. Population of town = 8000 ; n(S) = 8000

People above 50 yrs = 1300 ; n(A) = 1300; P(A) = $\frac{1300}{8000} = \frac{13}{80}$

Female population = 3000; n(B) = 3000; P(B) = $\frac{3000}{8000} = \frac{30}{80}$

Female above 50 yrs = 30 % of 3000 ; n(A ∩ B) = 900; P(A ∩ B) = $\frac{900}{8000} = \frac{9}{80}$

Probability of selecting a female or over 50 years = P(A ∪ B)

P(A ∪ B) = P(A) + P(B) – P(A ∩ B)

P(A ∪ B) = $\frac{13}{80} + \frac{30}{80} - \frac{9}{80} = \frac{34}{80} = \frac{17}{40}$

12. Sample space tossing 3 coins = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

<u>Let A be the event of getting exactly 2 heads</u>

A = {HHT, HTH, THH}

n(A) = 3; P(A) = $\frac{n(A)}{n(S)} = \frac{3}{8}$

<u>Let B be the event of getting atleast 1 tail</u>

B = { HHT, HTH, HTT, THH, THT, TTH, TTT}

n(B) = 7; P(B) = $\frac{n(B)}{n(S)} = \frac{7}{8}$

<u>Let C be the event of getting cosecutive 2 heads</u>

C = {HHH, HHT, THH}

n(C) = 3; P(C) = $\frac{n(C)}{n(S)} = \frac{3}{8}$

A ∩ B = {HHT, HTH, THH}; n(A ∩ B) = 3; P(A ∩ B) = $\frac{3}{8}$

B ∩ C = {HHT, THH}; n(B ∩ C) = 2; P(B ∩ C) = $\frac{2}{8}$

C ∩ A = {HHT, THH}; n(C ∩ A) = 2; P(C ∩ A) = $\frac{2}{8}$

A ∩ B ∩ C = {HHT, THH}; n(A ∩ B ∩ C) = 2; P(A ∩ B ∩ C) = $\frac{2}{8}$

(i). Probability of selecting one at a time S_1 = P(A) + P(B) + P(C)

$= \frac{3}{8} + \frac{7}{8} + \frac{3}{8} = \frac{13}{8}$

(ii). Probability of selecting two at a time S_2 = P(A ∩ B) + P(B ∩ C) + P(C ∩ A)

$= \frac{3}{8} + \frac{2}{8} + \frac{2}{8} = \frac{7}{8}$

(iii). Probability of selecting three at a time S_3 = P(A ∩ B ∩ C) = $\frac{2}{8}$

Probability of above three ie P(A U B U C) = S

S = $S_1 - S_2 + S_3$

$$= \frac{13}{8} - \frac{7}{8} + \frac{2}{8} = \frac{8}{8} = \mathbf{1}$$

13. A, B, C are any three events ; P(B) = 2P(A); P(C) = 3P(A)

P(A) + P(B) + P(C) = P(A) + 2P(A) + 3P(A) = 6P(A)

P(A ∩ B) = $\frac{1}{6}$; P(B ∩ C) = $\frac{1}{4}$; P(C ∩ A) = $\frac{1}{8}$; P(A ∩ B ∩ C) = $\frac{1}{15}$

P(A U B U C) = $\frac{9}{10}$

P(A U B U C) = P(A) + P(B) + P(C) – P(A ∩ B) – P(B ∩ C) – P(C ∩ A) + P(A ∩ B ∩ C)

$\frac{9}{10}$= 6P(A) – P(A ∩ B) – P(B ∩ C) – P(C ∩ A) + P(A ∩ B ∩ C)

$$\frac{9}{10} = 6P(A) - \frac{1}{6} - \frac{1}{4} - \frac{1}{8} + \frac{1}{15}$$

$$6P(A) = \frac{9}{10} + \frac{1}{6} + \frac{1}{4} + \frac{1}{8} - \frac{1}{15} = \frac{108+20+30+15-8}{120}$$

$$6P(A) = \frac{165}{120}$$

$$P(A) = \frac{165}{120 \times 6} = \frac{11}{48}$$

P(B) = 2 P(A) = $2 \times \frac{11}{48} = \frac{11}{24}$

P(C) = 3 P(A) = $3 \times \frac{11}{48} = \frac{11}{16}$

14. Class strength = 35; Boys : Girls = 4 : 3

Number of boys = $35 \times \left(\frac{4}{7}\right)$ = 20

Number of girls = 15

Let A be the event of selecting a boy with prime roll number

A = {2, 3, 5, 7, 11, 13, 17, 19}

n(A) = 8; P(A) = $\frac{n(A)}{n(S)} = \frac{8}{35}$

Let B be the event of selecting a girl with composite roll number

B = {21, 22, 24, 25, 26, 27, 28, 30, 32, 33, 34, 35}

n(B) = 12; P(B) = $\frac{n(B)}{n(S)} = \frac{12}{35}$

Let C be the event of selecting an even roll number

C = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34}

n(C) = 12; P(C) = $\frac{n(C)}{n(S)} = \frac{17}{35}$

A ∩ B = ∅; n(A ∩ B) = 0; P(A ∩ B) = 0

B ∩ C = {22, 24, 26, 28, 30, 32, 34}; n(B ∩ C) = 7; P(B ∩ C) = $\frac{7}{35}$

C ∩ A = {2}; n(C ∩ A) = 1; P(C ∩ A) = $\frac{1}{35}$

A ∩ B ∩ C = ∅; n(A ∩ B ∩ C) = 0; P(A ∩ B ∩ C) = 0

(i). Probability of selecting one at a time S_1 = P(A) + P(B) + P(C)

$= \frac{8}{35} + \frac{12}{35} + \frac{17}{35} = \frac{37}{35}$

(ii). Probability of selecting two at a time S_2 = P(A ∩ B) + P(B ∩ C) + P(C ∩ A)

$= 0 + \frac{7}{35} + \frac{1}{35} = \frac{8}{35}$

(iii). Probability of selecting three at a time S_3 = P(A ∩ B ∩ C) = 0

Probability of above three ie P(A U B U C) = S

S = $S_1 - S_2 + S_3$

$= \frac{37}{35} - \frac{8}{35} + 0 = \frac{29}{35}$

Important note:

1. For any chapter don't muck up the book back one mark answer.
2. Try to know how the answer has come. It will help you in many ways.
3. Students can raise your doubts through mail or whatsapp and I try to give solution as much as I can.

Wish you all the Best.

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