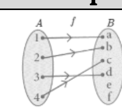
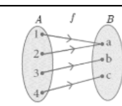
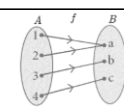
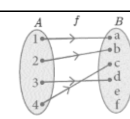
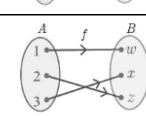
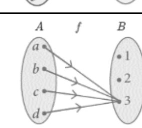
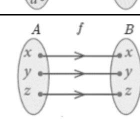
Definitions and Theorems

1. Relations and Functions

Definition
Cartesian Product: If A and B are two non-empty sets, then the set of all ordered pairs (a, b) such that $a \in A,\ b \in B$ is called the Cartesian Product of A and B. Thus $A \times B = \{(a, b) \mid a \in A, b \in B\}$
Relation (R): Let A and B be any two non-empty sets. A relation (R) from A to B is a subset of $A \times B$ satisfying some specified conditions. If $x \in A$ is related to $y \in B$ through R, then we write it as xRy. xRy if and only if $(x, y) \in$ R
Vertical line test: A curve drawn in a graph represents a function, if every vertical line intersects the curve at only one point.
Horizontal Line Test: A function represented in a graph in one – one, if every horizontal line intersects the curve at only one point.
Composition of function: Let $f: A \to B$ and $g: B \to C$ be two functions. Then the composition of f and g denoted by $g \circ f$ is defined as the function $g \circ f(x) = g(f(x))$ for all $x \in A$.
Linear function: A function $f: R \to R$ defined by $f(x) = mx + c, m \neq 0$ is called a linear function.
Quadratic function: A function $f: R \to R$ defined by $f(x) = ax^2 + bx + c\ \ (a \neq 0)$ is called a quadratic function.
Cubic function: A function $f: R \to R$ defined by $f(x) = ax^3 + bx^2 + cx + d\ \ (a \neq 0)$ is called a cubic function.
Reciprocal function: A function $f: R - \{0\} \to R$ defined by $f(x) = \frac{1}{x}$ is called a reciprocal function.
Constant function: A function $f: R \to R$ defined by $f(x) = c$ for all $x \in R$ is called a constant function.

Types of Functions

Definition	Example
One-One function (Injection): A function $f: A \to B$ is called one-one function if distinct elements of A have distinct images in B.	
Many-one function: A function $f: A \to B$ is called many-one function if two or more elements of A have same image in B	
Onto function (Surjection): A function $f: A \to B$ is said to be onto function if the range of f is equal to the co-domain of f.	
Into function: A function $f: A \to B$ is called an into function if there exists at least one element in B which is not the image of any element of A	
One to one and onto function (Bijection): If a function $f: A \to B$ is both one-one and onto, then f is called a bijection from A to B	
Constant function: A function $f: A \to B$ is called a constant function if the range of f contains only one element. That is, $f(x) = c$ for all $x \in A$ and for some fixed $c \in B$.	
Identity function: Let A be a non-empty set. Then the function $f: A \to A$ defined by $f(x) = x$ for all $x \in A$ is called an identity function on A and is denoted by I_A.	

2. Numbers and Sequences

Definition

Sequences: A real valued sequence is a function defined on the set of natural numbers and taking real values.
Finite sequence: If the number of elements in a sequence is finite [countable]
Infinite sequence : If the number of elements in a sequence is infinite. [uncountable]
Arithmetic progression: Let a and d be real numbers. Then the numbers of the form $a, a+d, a+2d, \ a+3d, a+4d, \ldots$ is said to Arithmetic progression. $a \rightarrow$ first term $d \rightarrow$ common difference.
Series : The sum of the terms of a sequence is called series. Let $a_1, a_2, a_3, \ldots a_n \ldots$ be the sequence of real numbers. Then the real number $a_1 + a_2 + a_3 + \cdots$ is defined as the series of real numbers.
Finite series: If a series has finite number of terms. **Infinite series:** If a series has infinite number of terms.
Arithmetic series : A series whose terms are in Arithmetic progression is called Arithmetic series.
Geometric progression: It is a sequence in which each term is obtained by multiplying a fixed non-zero number to the preceding term except the first term. Let a and $r \neq 0$ be real numbers. $a, ar, ar^2, \ldots ar^{n-1}$ is called General form of $G.P.$ $a \rightarrow$ first term, $r \rightarrow$ common ratio.
Geometric Series: A series whose terms are in Geometric progression is called Geometric series.
Congruence Modulo: Two integers a and b are congruence modulo n if they differ by an integer multiple of n. That $b - a = kn$ for some integer k. This can also be written as $a \equiv b \pmod{n}$
Here, the number n is called modulus. In other words, $a \equiv b \ (mod\ n)$ means $a - b$ is divisible by n.

Theorems

Theorem 1: Euclid's division Lemma : Let a and b be any two positive integers. Then, there exist unique integers q and r such that $a = bq + r, 0 \leq r < b$.
Generalised form of Euclid's division lemma: If a and b are $(b \neq 0)$ any two integers then there exist unique integers q and r such that $a = bq + r$, where $0 \leq r < |b|$
Theorem 2: If a and b are positive integers such that $a = bq + r$, then every common divisor of a and b is a common divisor of b and r and vice–versa.
Theorem 3: If a and b are two positive integers with $a > b$ then G.C.D of (a, b) =GCD of $(a - b, b)$
Theorem 4 : Fundamental Theorem of Arithmetic: Every positive integer (except the number 1) can be represented in exactly one way apart from rearrangement as a product of one or more primes.
Theorem 5 : a, b, c and d are integers and m is a positive integer such that if $a \equiv b \ (mod\ m)$ and $c \equiv d \ (mod\ m)$ then (i) $(a + c) \equiv (b + d)(mod\ m)$ (ii) $(a - c) \equiv (b - d)(mod\ m)$ (iii) $(a \times c) \equiv (b \times d)(mod\ m)$
Theorem 6: If $a \equiv b \ (mod\ m)$then (i) $ac = bc \ (mod\ m)$ (ii) $a \pm c = b \pm c \ (mod\ m)$ for any integer in c.

3. Algebra

Definition

Rational Expression: An expression can be written in the form $\frac{p(x)}{q(x)}$ then it is called a rational expression where $p(x)$ and $q(x)$ are polynomials and $q(x) \neq 0$. A rational expression is the ratio of two polynomials.

Excluded value: A value that makes a rational expression $\frac{p(x)}{q(x)}$ (in its lowest form) undefined is called exclude value.

Quadratic Expression: An expression of degree 2 is called a Quadratic Expression. It is expressed as $p(x) = ax^2 + bx + c, a \neq 0, a, b, c$ are real numbers.

Zeros of a quadratic expression: For a polynomial $p(x)$, if $p(a) = 0$ then $x = a$ is called zero of $p(x)$.

Roots of the quadratic equation: The values of x such that the expression $ax^2 + bx + c$ becomes zero are called roots of the quadratic equation $ax^2 + bx + c = 0$. The roots are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Matrix: A Matrix is a rectangular array of elements. The horizontal arrangements are called rows and vertical arrangements are called columns.

Order of a matrix: If a matrix A has m rows and n columns, then the order of A is $m \times n$

Transpose of a matrix: It is obtained by interchanging rows and columns of a matrix of the given A is called transpose of A and is denoted by A^T

Equal Matrices: Two marices A and B are said to be equal if and only if they have the same order and each element of matrix A is equal to the corresponding element of matrix B.

The negative of a matrix: The negative of a matrix $A_{m\times n}$ denoted by $(-A)_{m\times n}$ is the matrix formed by replacing each element in the matrix $A_{m\times n}$ with its additive inverse.

Additive identity : Null matrix (or) zero matrix is the identity of matrix addition. Let A be any matrix, $A + O = O + A = A$ Where, O is the null matrix or zero matrix of same order as that of A.

Additive inverse: If A be any given matrix then $-A$ is the additive inverse of A. $A + (-A) = (-A) + A = O$

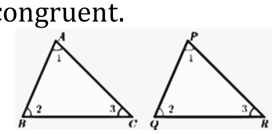
Types of matrices

Definition	Example
Row matrix: A matrix that has only one row. Order $= 1 \times n$	$A = [5 \quad 3 \quad 4 \quad 1]$ Order $= 1 \times 4$
Column matrix: A matrix that has only one column Order $= m \times 1$	$A = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$; Order $= 3 \times 1$
Square matrix: A matrix in which have equal number of rows and columns. Order $= m \times m$	$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$; Order $= 2 \times 2$
Diagonal matrix: A square matrix which have 0 value for elements above and below the leading diagonal. Order $= m \times m$	$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$; Order$= 3 \times 3$
Scalar matrix: A diagonal matrix which have equal, non-zero constant value for all elements along the leading diagonal. Order $= m \times m$	$A = \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$; Order $= 3 \times 3$
Identity (or) Unit matrix ($\boldsymbol{I}$): A diagonal matrix which have value 1 for all elements along the leading diagonal. Order $= m \times m$	$I_2 = A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Zero matrix (or) Null matrix ($\boldsymbol{O}$): A matrix which have all element value as 0. Order $= m \times n$	$O = A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$
Lower triangular matrix: A square matrix in which all the entries above the leading diagonal are zero is called a lower triangular matrix.	$A = \begin{bmatrix} 7 & 0 & 0 \\ 4 & 8 & 0 \\ 6 & 1 & 9 \end{bmatrix}$
Upper triangular matrix: If all the entries below the leading diagonal are zero, then it is called an upper triangular matrix.	$B = \begin{bmatrix} 7 & 5 & 2 \\ 0 & 6 & 1 \\ 0 & 0 & 9 \end{bmatrix}$

4. Geometry

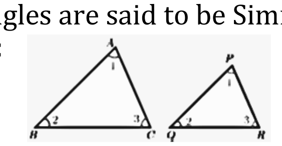
Two figures are said to be similar if every aspect of one figure is proportional to other figure.

Congruent triangle:
If three angles and their corresponding sides of two triangles are same, then the two triangles are said to be congruent.
Example:



$AB = PQ$
$BC = QR$
$AC = PR$

Similar triangle:
If three angles of two triangles are same and their corresponding sides are proportional, then the two triangles are said to be Similar.
Example:



$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

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5. Coordinate Geometry

Slope: If θ is the angle of inclination of a non-vertical straight line, then $tan\theta$ is called the slope or gradient of the line and is denoted by m.
Therefore, the slope of the straight line is $m = tan\theta,\ 0 \le \theta \le 180^{\circ}, \theta \ne 90^{\circ}$

(i) $\theta = 0^{\circ}$, The line is parallel to the positive direction of X axis.
(ii) $0 < \theta < 90^{\circ}$, The line has positive slope (A line with positive slope rises from left to right).
(iii) $90^{\circ} < \theta < 180^{\circ}$, The line has negative slope (A line with negative slope falls from left to right).
(iv) $\theta = 180^{\circ}$, The line is parallel to the negative direction of X axis.
(v) $\theta = 90^{\circ}$, The slope is undefined.

6. Trigonometry

Definition
Angle of elevation: It is an angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level. That is, the case when we raise our head to look at the object.
Angle of Depression : It is an angle formed by the line of sight with the horizontal when the point is below the horizontal level. That is, the case when we lower our head to look at the point being viewed.

8. Statistics and Probability

Definition
Variable: The quantities which are being considered in a survey are called variables. Variables are generally denoted by x_i, where $i = 1,2,3, \dots, n$.
Frequencies: The number of times, a variable occurs in a given data is called the frequency of that variable. Frequencies are generally denoted as f_i, where $i = 1,2,3, \dots, n$.
Arithmetic Mean: The Arithmetic Mean or Mean of the given values is sum of all the observations divided by the total number of observations. It is denoted by $\bar{x}$.
Range: The difference between the largest value and the smallest value is called Range.
Variance: The mean of the squares of the deviations from the mean is called Variance. It is denoted by σ^2
Standard Deviation: The positive square root of Variance is called Standard deviation. That is, standard deviation is the positive square root of the mean of the squares of deviations of the given values from their mean. It is denoted by σ.
Coefficient of variation: For comparing two or more data for corresponding changes the relative measure of standard deviation called Coefficient of variation.
Random experiment: A random experiment is an experiment in which (i) The set of all possible outcomes are known (ii) Exact outcome is not known
Sample space: The set of all possible outcomes in a random experiment is called a sample space.
Sample point: Each element of a sample space is called a sample point.
Tree diagram: Tree diagram allow us to see visually all possible outcomes of an random experiment. Each branch in a tree diagram represent a possible outcome.
Probability of an event: In a random experiment, let S be the sample space and $E \subseteq S$. Then if E is an event, the probability of occurrence of E is defined as $P(E)$.

Types of Events

Events	Example
Equally likely events: Two or more events are said to be equally likely if each one of them has an equal chance of occurring.	Head and tail are equally likely events in tossing a coin.
Certain events / Sure Event: In an experiment, the event which surely occur is called certain event.	When we roll a die, the event of getting any natural number from 1 to 6 is a certain event.
Impossible events / Null Event: In an experiment if an event has no scope to occur then it is called an impossible event.	When we toss two coins, the event of getting three heads is an impossible event.
Mutually exclusive events: Two or more events are said to be mutually exclusive if they don't have common sample points. i.e., events A, B are said to be mutually exclusive if, $A \cap B = \emptyset$.	When we roll a die the events of getting odd numbers and even numbers are mutually exclusive events.
Exhaustive events: The collection of events whose union is the whole sample space are called exhaustive events.	When we toss a coin twice, the collection of events of getting two heads, exactly one head, no head are exhaustive events.
Complementary events: The complement of an event A is the event representing collection of sample points not in A. It is denoted A' or A^c or $\bar{A}$. The event A and its complement A' are mutually exclusive and exhaustive.	When we roll a die, the event 'rolling 5 or 6' and the event of rolling 1, 2, 3 or 4 are complementary events.

Addition theorem on Probability:

(i) If A and B are any two events then, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(ii) If A, B and C are any three events then

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$