Important Formulae

1. Relations and Functions

✓ If $n(A) = p$ and $n(B) = q$ then $n(A \times B) = pq$

✓ Distributive property of Cartesian product: (i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

2. Numbers and Sequences

Arithmetic progression

(i) The numbers of the form	$a, a+d, a+2d, a+3d, \dots a+(n-1)d$
(ii) n^{th} term	$t_n = a + (n-1)d$
(iii) Common difference	$d = t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \cdots$ $d = t_n - t_{n-1}, n = 2,3,4, \dots$
(iv) Total number of terms	$n = \left(\frac{l-a}{d}\right) + 1$
(v) The sum of first n terms	$S_n = \frac{n}{2}[2a + (n-1)d]$ (OR) $S_n = \frac{n}{2}(a + l)$

Total amount for compound interest is $A = P\left(1 + \frac{r}{100}\right)^n$

Geometric progression

(i) The numbers of the form	$a, ar, ar^2, \dots . ar^{n-1}, \dots$	
(ii) n^{th} term	$t_n = ar^{n-1}$	
(iii) Common ratio	$r = \frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = \cdots$ $r = \frac{t_n}{t_{n-1}}, n = 2,3,4, \dots$	
(iv) The sum of first n terms	$r \neq 1, r > 1$	$S_n = \frac{a(r^n - 1)}{r-1}$
	$r = 1$	$S_n = a + a + a + \cdots + a = na$
	$r < 1$	$S_n = \frac{a(1-r^n)}{1-r}$
(v) The sum of infinite terms	$\frac{a}{1-r}, -1 < r < 1$	

Special Series

Sum of first n natural numbers

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

Sum of first n odd natural numbers

$$1 + 3 + 5 + \cdots + (2n-1) = \frac{n}{2} \times 2n = n^2$$

Sum of squares of first n natural numbers

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

Sum of cubes of first n natural numbers

$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

3. Algebra

Relationship between LCM and GCD $f(x) \times g(x) = LCM[f(x), g(x)] \times GCD[f(x), g(x)]$

Operations of rational expressions: $\frac{p(x)}{q(x)}, \frac{r(x)}{s(x)} \Rightarrow$ two rational expressions where $q(x) \neq 0, s(x) \neq 0$ then

Their product is: $\frac{p(x)}{q(x)} \times \frac{r(x)}{s(x)} = \frac{p(x) \times r(x)}{q(x) \times s(x)}$ **Their division:** $\frac{p(x)}{q(x)} \div \frac{r(x)}{s(x)} = \frac{p(x)}{q(x)} \times \frac{s(x)}{r(x)} = \frac{p(x) \times s(x)}{q(x) \times r(x)}$

The Relation between Roots and Co- efficient of a Quadratic Equation

α, β are the roots $\Rightarrow ax^2 + bx + c = 0$; $\alpha = \frac{-b+\sqrt{b^2-4ac}}{2a}$, $\beta = \frac{-b-\sqrt{b^2-4ac}}{2a}$

Sum of the roots $\alpha + \beta = -\frac{b}{a} = \frac{-\text{Co-efficient of } x}{\text{Co-efficient of } x^2}$; Product of the roots $\alpha\beta = \frac{c}{a} = \frac{\text{Constant term}}{\text{Co-efficient of } x^2}$

$\therefore$ Quadratic Equation $= x^2 - (\alpha + \beta)x + \alpha\beta = 0$

5. Coordinate Geometry

Distance between two points		$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Mid – point of line segment		$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
Section Formula	Internal Division	$P\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$
	External Division	$P\left(\frac{mx_2-nx_1}{m-n}, \frac{my_2-ny_1}{m-n}\right)$
Centroid of a triangle		$G\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$
Area of a Triangle		$= \frac{1}{2}\left\{\begin{matrix} x_1(y_2 - y_3) + x_2(y_3 - y_1) \\ +x_3(y_1 - y_2) \end{matrix}\right\}$ Sq. units Area of triangle $= 0 \Rightarrow$ Collinear
Area of the quadrilateral		$= \frac{1}{2}\left\{\begin{matrix} (x_1y_2 + x_2y_3 + x_3y_4 + x_4y_1) \\ -(x_2y_1 + x_3y_2 + x_4y_3 + x_1y_4) \end{matrix}\right\}$ Sq. units

Equation of Straight line in various forms:

	Name	Form
1	General form	$ax + by + c = 0$
2	Point – slope form	$y - y_1 = m(x - x_1)$
3	Slope - intercept	$y = mx + c$
4	Two point form	$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$
5	Intercept form	$\frac{x}{a} + \frac{y}{b} = 1$
6	Parallel to Y axis	$x = c$
7	Parallel to X axis	$y = b$

Slope of a straight line $ax + by + c = 0 \Rightarrow$ Slope $m = \frac{-\text{coefficient of } x}{\text{coefficient of } y} = -\frac{a}{b}$, y intercept $= \frac{-\text{constant term}}{\text{coefficient of } y} = -\frac{c}{b}$

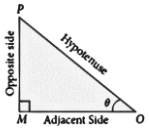
6. Trigonometry

Trigonometric Ratios:

$\sin\theta = \frac{Opp}{Hyp}$; $\cos\theta = \frac{Adj}{Hyp}$,

$tan\theta = \frac{sin\theta}{cos\theta}$ (or) $\frac{1}{\cot\theta}$;

$cot\theta = \frac{cos\theta}{sin\theta}$ (or) $\frac{1}{\tan\theta}$; $cosec\theta = \frac{1}{sin\theta}$; $sec\theta = \frac{1}{cos\theta}$



Identities : $\sin^2\theta + \cos^2\theta = 1$, $1 + \tan^2\theta = \sec^2\theta$,

$1 + \cot^2\theta = \text{cosec}^2\theta$

Complementary angle :

$sin(90^\circ - \theta) = cos\theta$ $cos(90^\circ - \theta) = sin\theta$

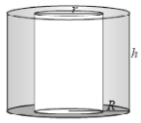
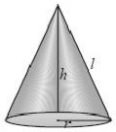
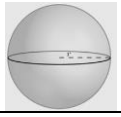
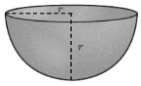
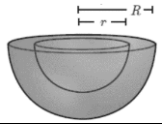
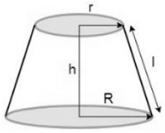
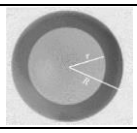
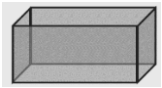
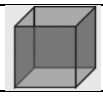
$tan(90^\circ - \theta) = cot\theta$ $cot(90^\circ - \theta) = tan\theta$

$cosec(90^\circ - \theta) = sec\theta$ $sec(90^\circ - \theta) = cosec\theta$

Table of Trigonometric Ratios for $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$

Trigonometric Ratios \ θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined
cosec θ	Undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Undefined
$\cot\theta$	Undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

7. Mensuration

Sl.No	Solid	Figure	CSA (sq.units)	TSA (sq.units)	Volume (cu.units)
1	Right circular cylinder		$2\pi rh$	$2\pi r(h+r)$	$\pi r^2 h$
2.	Hollow cylinder		$2\pi h(R+r)$	$2\pi(R+r)(R-r+h)$	$\pi(R^2-r^2)h$ (Or) $\pi h(R+r)(R-r)$
3	Right circular cone		πrl $\left(l=\sqrt{h^2+r^2}, r=\sqrt{l^2-h^2}\right)$	$\pi r(l+r)$	$\frac{1}{3}\pi r^2 h$
4	Sphere		$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$
5	Hemisphere		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$
6	Hollow hemisphere		$2\pi(R^2+r^2)$	$\pi(3R^2+r^2)$	$\frac{2}{3}\pi(R^3-r^3)$
7	Frustum		$\pi(R+r)l$ $\left(l=\sqrt{h^2+(R-r)^2}\right)$	$\pi l(R+r)+\pi R^2+\pi r^2$	$\frac{1}{3}\pi h(R^2+r^2+Rr)$
8	Hollow sphere		$4\pi R^2$ = Outer Surface area	$4\pi(R^2+r^2)$	$\frac{4}{3}\pi(R^3-r^3)$
9	Cuboid		$2h(l+b)$	$2(lb+bh+lh)$	$l\times b\times h$
10	Cube		$4a^2$	$6a^2$	a^3

8. Statistics and Probability

Arithmetic Mean		$\bar{x} = \frac{\text{Sum of all the observations}}{\text{Number of observations}}$	
Range	$R = L - S$	Coefficient of range	$\frac{L-S}{L+S}$
Variance	$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}$	Standard Deviation	$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n}}$

Calculation of Standard Deviation for ungrouped data

(i)	Direct Method	$\sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2}$
(ii)	Mean Method	$d_i = x_i - \bar{x}$, $\sigma = \sqrt{\frac{\Sigma d_i^2}{n}}$
(iii)	Assumed Mean Method	$\sigma = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$
(iv)	Step deviation Method	$\sigma = c \times \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2}$

Calculation of Standard Deviation for grouped data

(i)	Mean Method	$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N}}$, where $N = \sum_{i=1}^{n} f_i$
(ii)	Assumed Mean Method	$d_i = x - A$, $\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

Calculation of Standard deviation for continuous frequency distribution

(i)	Mean Method	$\sigma = \sqrt{\frac{\Sigma f_i (x_i-\bar{x})^2}{N}}$, x_i = Middle value of the ith class f_i = Frequency of the ith class
(ii)	Shortcut Method (or) Step deviation method	$d_i = \frac{x_i - A}{c}$, $\sigma = c \times \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$

To Find the Square root:

$$\sqrt{X} = \sqrt{S} + \frac{(X-S)}{2\sqrt{S}}$$

X – the number you want the square root

S – the closet square number you know to X

Coefficient of variation:

C.V$_1$= $\frac{\sigma_1}{\bar{x}_1} \times 100\%$,

C.V$_2$= $\frac{\sigma_2}{\bar{x}_2} \times 100\%$

i) C.V is more ⇒ Data is less consistent
ii) C.V is less ⇒ Data is more consistent
iii) C.V is equal ⇒ One data depend other

Verbal description	Set notation
Not A	$\overline{A}$
A or B (at least one of Aor B)	$A \cup B$
A and B	$A \cap B$
A but not B	$A \cap \overline{B}$
Neither A nor B	$\overline{A} \cap \overline{B}$
At least one of A, B or C	$A \cup B \cup C$
Exactly one of A and B	$(A \cap \overline{B}) \cup (\overline{A} \cap B)$
All three of A, B and C	$A \cap B \cap C$
Exactly two of A, B *and* C	$(A \cap B \cap \overline{C}) \cup (A \cap \overline{B} \cap C) \cup (\overline{A} \cap B \cap C)$

Probability of an event: $P(E) = \frac{n(E)}{n(S)}$

- The probability of **sure event** $= P(S) = 1$
- The probability of **impossible event** $= P(\emptyset) = 0$
- $P(E) + P(\overline{E}) = 1$
- $P(A \cap \overline{B}) = P$ (only A)$= P(A) - P(A \cap B)$
- $P(\overline{A} \cap B) = P$(only B)$= P(B) - P(A \cap B)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- If A and B are mutually exclusive events then $P(A \cup B) = P(A) + P(B)$
- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$