PART – II & III

1. Relations and Functions

Exercise 1.1

1. Find $A \times B, A \times A$ and $B \times A$ PTA-1 2M

(iii) $A = \{m, n\}; B = \emptyset$

$A \times B = \{\ \ \}$

$A \times A = \{m, n\} \times \{m, n\}$

$= \{(m, m), (m, n), (n, m), (n, n)\}$

$B \times A = \{\ \ \}$

Similar Problems (Solve Your Self) 2M

1. Find $A \times B, A \times A$ and $B \times A$

(i) $A = \{2, -2, 3\}$ and $B = \{1, -4\}$ (ii) $A = B = \{p, q\}$

Eg.1.1: If $A = \{1, 3, 5\}$ and $B = \{2, 3\}$ then (i) find $A \times B$ and $B \times A$. (ii) Is $A \times B = B \times A$? If not why? (iii) Show that $n(A \times B) = n(B \times A) = n(A) \times n(B)$ **(SEP-21)**

2. Let $A = \{1, 2, 3\}$ and $B = \{x | x$ is a prime number less than 10$\}$. Find $A \times B$ and $B \times A$. MAY-22 2M

$A = \{1,2,3\}$

$B = \{x | x$ is a prime number less than $10\} = \{2,3,5,7\}$

$A \times B = \{1,2,3\} \times \{2,3,5,7\}$

$= \{(1, 2), (1, 3), (1, 5), (1, 7), (2, 2), (2, 3), (2, 5), (2, 7), (3, 2), (3, 3), (3, 5), (3, 7)\}$

$B \times A = \{2,3,5,7\} \times \{1,2,3\}$

$= \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3), (7, 1), (7, 2), (7, 3)\}$

Similar Problems (Solve Your Self) 2M

3. If $B \times A = \{(-2,3), (-2,4), (0,3), (0,4), (3,3), (3,4)\}$ Find A and B. **(APR-23)**

Eg.1.2: If $A \times B = \{(3, 2), (3, 4), (5, 2), (5, 4)\}$ then find A and B. **(SEP-20, JUL-22)**

6. Let $A = \{x \in \mathbb{W} | x < 2\}, B = \{x \in \mathbb{N} | 1 < x \leq 4\}$ and $C = \{3, 5\}$. Verify that

(ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$ JUN-23, SEP-21, PTA-5 5M

LHS: $B \cap C = \{2,3,4\} \cap \{3,5\} = \{3\}$

$A \times (B \cap C) = \{0,1\} \times \{3\} = \{(0, 3), (1, 3)\}$ …………..(1)

RHS: $A \times B = \{0,1\} \times \{2,3,4\} = \{(0, 2), (0, 3), (0, 4), (1, 2), (1, 3), (1, 4)\}$

$A \times C = \{0,1\} \times \{3,5\} = \{(0,3), (0,5), (1,3), (1,5)\}$

$(A \times B) \cap (A \times C) = \{(0,2), (0, 3), (0,4), (1,2), (1, 3), (1,4)\} \cap \{(0, 3), (0,5), (1, 3), (1,5)\}$

$= \{(0, 3), (1, 3)\}$ ……………(2)

From (1) and (2), $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Similar Problems (Solve Your Self) 5M

4. If $A = \{5,6\}, B = \{4,5,6\}, C = \{5,6,7\}$, show that $A \times A = (B \times B) \cap (C \times C)$. **(JUL-22)**

5. Given $A = \{1,2,3\}, B = \{2,3,5\}, C = \{3,4\}$ and $D = \{1,3,5\}$, check if $(A \cap C) \times (B \cap D) = (A \times B) \cap (C \times D)$ is true ?

6. Let $A = \{x \in \mathbb{W} | x < 2\}, B = \{x \in \mathbb{N} | 1 < x \leq 4\}$ and $C = \{3,5\}$. Verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ **(PTA-2)** (iii) $(A \cup B) \times C = (A \times C) \cup (B \times C)$

7. Let $A =$ The set of all natural numbers less than 8, $B =$ The set of all prime numbers less than 8, $C =$ The set of even prime number, Verify that (i) $(A \cap B) \times C = (A \times C) \cap (B \times C)$ **(SEP-20)**

Eg.1.3: Let $A = \{x \in \mathbb{N} \mid 1 < x < 4\}$, $B = \{x \in \mathbb{W} \mid 0 \leq x < 2\}$ and $C = \{x \in \mathbb{N} \mid x < 3\}$. Then verify that

(i) $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

CQ: 1. Let $A = \{x \in W / x < 3\}$, $B = \{x \in N / 1 < x \leq 5\}$ and $C = \{3,5,7\}$ verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$ **(APR-23)**

2. Let $A = \{x \in W / 0 < x < 5\}$, $B = \{x \in W / 0 \leq x \leq 2\}$, $C = \{x \in W / x < 3\}$ then verify that $A \times (B \cap C) = (A \times B) \cap (A \times C)$ **(PTA-3)**

7. Let A = The set of all natural numbers less than 8, B = The set of all prime numbers less than 8, C = The set of even prime number, Verify that 5M

(ii) $A \times (B - C) = (A \times B) - (A \times C)$ MAY-22

LHS: $B - C = \{\cancel{2},3,5,7\} - \{\cancel{2}\} = \{3,5,7\}$

$A \times (B - C) = \{1,2,3,4,5,6,7\} \times \{3,5,7\}$

$= \{(\mathbf{1},\mathbf{3}),(\mathbf{1},\mathbf{5}),(\mathbf{1},\mathbf{7}),(\mathbf{2},\mathbf{3}),(\mathbf{2},\mathbf{5}),(\mathbf{2},\mathbf{7}),(\mathbf{3},\mathbf{3}),(\mathbf{3},\mathbf{5}),(\mathbf{3},\mathbf{7}),(\mathbf{4},\mathbf{3}),(\mathbf{4},\mathbf{5}),(\mathbf{4},\mathbf{7}),(\mathbf{5},\mathbf{3}),(\mathbf{5},\mathbf{5}),(\mathbf{5},\mathbf{7}),(\mathbf{6},\mathbf{3}),(\mathbf{6},\mathbf{5}),(\mathbf{6},\mathbf{7}),(\mathbf{7},\mathbf{3}),(\mathbf{7},\mathbf{5}),(\mathbf{7},\mathbf{7})\}$............(1)

RHS: $A \times B = \{1,2,3,4,5,6,7\} \times \{2,3,5,7\}$

$= \{(1,2),(1,3),(1,5),(1,7),(2,2),(2,3),(2,5),(2,7),(3,2),(3,3),(3,5),(3,7),(4,2),(4,3),(4,5),(4,7),(5,2),(5,3),(5,5),(5,7),(6,2),(6,3),(6,5),(6,7),(7,2),(7,3),(7,5),(7,7)\}$

$A \times C = \{1,2,3,4,5,6,7\} \times \{2\} = \{(1,2),(2,2),(3,2),(4,2),(5,2),(6,2),(7,2)\}$

$(A \times B) - (A \times C)$

$= \{\cancel{(1,2)},(1,3),(1,5),(1,7),\cancel{(2,2)},(2,3),(2,5),(2,7),\cancel{(3,2)},(3,3),(3,5),(3,7),\cancel{(4,2)},(4,3),(4,5),(4,7),\cancel{(5,2)},(5,3),(5,5),(5,7),\cancel{(6,2)},(6,3),(6,5),(6,7),\cancel{(7,2)},(7,3),(7,5),(7,7)\}$

$- \{\cancel{(1,2)},\cancel{(2,2)},\cancel{(3,2)},\cancel{(4,2)},\cancel{(5,2)},\cancel{(6,2)},\cancel{(7,2)}\}$

$= \{(\mathbf{1},\mathbf{3}),(\mathbf{1},\mathbf{5}),(\mathbf{1},\mathbf{7}),(\mathbf{2},\mathbf{3}),(\mathbf{2},\mathbf{5}),(\mathbf{2},\mathbf{7}),(\mathbf{3},\mathbf{3}),(\mathbf{3},\mathbf{5}),(\mathbf{3},\mathbf{7}),(\mathbf{4},\mathbf{3}),(\mathbf{4},\mathbf{5}),(\mathbf{4},\mathbf{7}),(\mathbf{5},\mathbf{3}),(\mathbf{5},\mathbf{5}),(\mathbf{5},\mathbf{7}),(\mathbf{6},\mathbf{3}),(\mathbf{6},\mathbf{5}),(\mathbf{6},\mathbf{7}),(\mathbf{7},\mathbf{3}),(\mathbf{7},\mathbf{5}),(\mathbf{7},\mathbf{7})\}$............(2)

From (1) and (2), $\mathbf{A \times (B - C) = (A \times B) - (A \times C)}$

Exercise 1.2

1. Let $A = \{1,2,3,7\}$ and $B = \{3,0,-1,7\}$, which of the following are relation from A to B? 2M

(i) $R_1 = \{(2,1),(7,1)\}$

$A \times B = \{1,2,3,7\} \times \{3,0,-1,7\}$

$= \{(1,3),(1,0),(1,-1),(1,7),(2,3),(2,0),(2,-1),(2,7),(3,3),(3,0),(3,-1),(3,7),(7,3),(7,0),(7,-1),(7,7)\}$

We know that, $(2,1)$ and $(7,1) \in R_1$

but $(2,1),(7,1) \notin A \times B$

So, R_1 is **not a relation** from A to B

(iii) $R_3 = \{(2,-1),(7,7),(1,3)\}$

Here $R_3 \subseteq A \times B$

Hence R_3 is **a relation** from A to B

Similar Problems (Solve Your Self) 2M

1. Let $A = \{1,2,3,7\}$ and $B = \{3,0,-1,7\}$, which of the following are relation from A to B?
   (ii) $R_2 = \{(-1,1)\}$ (iv) $R_4 = \{(7,-1),(0,3),(3,3),(0,7)\}$

Eg.1.4: Let $A = \{3,4,7,8\}$ and $B = \{1,7,10\}$. Which of the following sets are relations from A to B?
(i) $R_1 = \{(3,7),(4,7),(7,10),(8,1)\}$
(ii) $R_2 = \{(3,1),(4,12)\}$
(iii) $R_3 = \{(3,7),(4,10),(7,7),(7,8),(8,11),(8,7),(8,10)\}$

2. Let $A = \{1,2,3,4,\ldots,45\}$ and R be the relation defined as "is square of a number" on A. Write R as a subset of $A \times A$. Also, find the domain and range of R. 2M SEP-21

Given $A = \{1,2,3,4,\ldots,45\}$

$A \times A = \{(1,1),(1,2),(1,3),(1,4)\ldots\ldots(45,45)\}$

Then, R be the relation defined as is "square of a number" on A.

Hence, $R = \{(1,1),(2,4),(3,9),(4,16),(5,25),(6,36)\}$

So $R \subseteq A \times A$

The domain of $R = \{\mathbf{1},\mathbf{2},\mathbf{3},\mathbf{4},\mathbf{5},\mathbf{6}\}$

The range of $R = \{\mathbf{1},\mathbf{4},\mathbf{9},\mathbf{16},\mathbf{25},\mathbf{36}\}$

Similar Problems (Solve Your Self) 2M

3. A Relation R is given by the set $\{(x,y)/y = x + 3, x \in \{0,1,2,3,4,5\}\}$. Determine its domain and Range. **(JUN-23, PTA-5)**

CQ - A Relation R is given by the set $\{(x,y)/y = x^2 + 3, x \in \{0,1,2,3,4,5\}\}$ Determine its domain and range. **(PTA-2)**

4. **Represent each of the given relations by (a) an arrow diagram (b) a graph and (c) a set in roster form, wherever possible.**

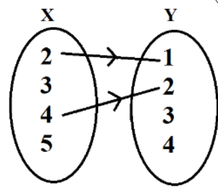
(i) $\{(x, y)|x = 2y, x \in \{2, 3, 4, 5\}, y \in \{1, 2, 3, 4\}\}$ 5M

(a) Arrow diagram

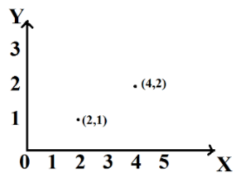
Given, $x = 2y$

If $y = 1 \Rightarrow x = 2$

If $y = 2 \Rightarrow x = 4$



(b) a graph



(c) a set in roster form

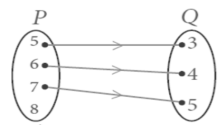
$R = \{(2,1), (4,2)\}$

Similar Problems (Solve Your Self) 5M

4. Represent each of the given relation by (a) an arrow diagram (b) a graph and (c) a set in roster form, wherever possible.

(ii) $\{(x, y)|y = x + 3, x, y$ are natural numbers $<10\}$ **(JUL-22)**

Eg.1.5: The arrow diagram shows a relationship between the sets P and Q. Write the relation in (i) Set builder form (ii) Roster form (iii) What is the domain and range of R. **(MAY-22)**



5. **A company has four categories of employees given by Assistants (A), Clerks (C), Managers (M) and an Executive Officer (E). The company provide ₹ 10,000, ₹ 25,000, ₹ 50,000 and ₹ 1,00,000 as salaries to the people who work in the categories A, C, M and E respectively. If A_1, A_2, A_3, A_4 and A_5 were Assistants; C_1, C_2, C_3, C_4 were Clerks; M_1, M_2, M_3 were managers and E_1, E_2 were Executive officers and if the relation R is defined by xRy, where x is the salary given to person y, express the relation R through an ordered pair and an arrow diagram.** 2M

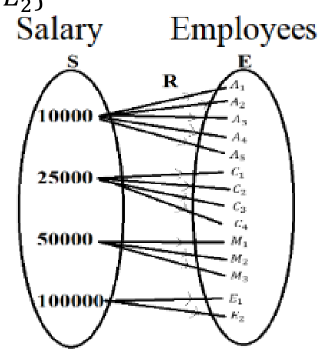
Salaries (S) = {10000, 25000, 50000, 100000}

Employees (E) = $\{A_1, A_2, A_3, A_4, A_5, C_1, C_2, C_3, C_4, M_1, M_2, M_3, E_1, E_2\}$

(a) Ordered Pairs:

$R = \{(10000, A_1), (10000, A_2), (10000, A_3), (10000, A_4), (10000, A_5), (25000, C_1), (25000, C_2), (25000, C_3), (25000, C_4), (50000, M_1), (50000, M_2), (50000, M_3), (100000, E_1), (100000, E_2)\}$

(b) An arrow diagram:



Exercise 1.3

1. **Let $f = \{(x, y)|x, y \in \mathbb{N}$ and $y = 2x\}$ be a relation on N. Find the domain, co-domain and range. Is this relation a function?** 2M

$y = f(x) = 2x$

$f(1) = 2(1) = 2$

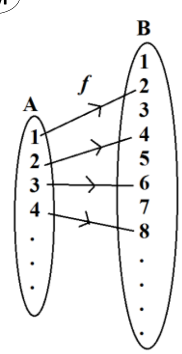
$f(2) = 2(2) = 4$

$f(3) = 2(3) = 6$

$f(4) = 2(4) = 8$

$\vdots$

f be a relation on N



Similar Problems (Solve Your Self) 2M

Eg.1.6: Let $X = \{1, 2, 3, 4\}$ and $Y = \{2, 4, 6, 8, 10\}$ and $R = \{(1, 2), (2, 4), (3, 6), (4, 8)\}$. Show that R is a function and find its domain, co-domain and range?

Domain of $f = \{1,2,3,4, \ldots\}$

Codomain of $f = \{1,2,3,4, \ldots\}$, Range of $f = \{2,4,6,8, \ldots\}$

From the arrow diagram of f, for each $x \in A$ and $y \in B$. **Yes,** f is a function.

2. Let $X = \{3, 4, 6, 8\}$. Determine whether the relation $\mathbb{R} = \{(x, f(x)) | x \in X, f(x) = x^2 + 1\}$ is a function from X to $\mathbb{N}$? (2M)

Given $X = \{3,4,6,8\}$

$Y = \{1,2,3,4, \dots.\}$

$R = \{(x, f(x)/x \in X, f(x) = x^2 + 1)\}$

Let $y = f(x) = x^2 + 1$

$f(3) = 3^2 + 1 = 10$,

$f(4) = 4^2 + 1 = 17$,

$f(6) = 6^2 + 1 = 37$, $f(8) = 8^2 + 1 = 65$

$R = \{(3,10), (4,17), (6,37), (8,65)\}$, **Yes,** it is a function from X to N

> **Similar Problems (Solve Your Self)** (2M)
>
> **Eg.1.7:** A relation $f: X \to Y$ is defined by $f(x) = x^2 - 2$ where, $x \in \{-2, -1, 0, 3\}$ and $Y = R$ (i) List the elements of f (ii) Is f a function?
>
> **Eg.1.8:** If $X = \{-5, 1, 3, 4\}$ and $Y = \{a, b, c\}$, then which of the following relations are functions from X to Y?
> (i) $R_1 = \{(-5, a), (1, a), (3, b)\}$
> (ii) $R_2 = \{(-5, b), (1, b), (3, a), (4, c)\}$

3. Given the function $f: x \to x^2 - 5x + 6$, evaluate (i) $f(-1)$ (ii) $f(2a)$ (iii) $f(2)$ (iv) $f(x - 1)$ (2M)

$f(x) = x^2 - 5x + 6$

(i) $f(-1)$

$f(-1) = (-1)^2 - 5(-1) + 6 = 1 + 5 + 6 = \mathbf{12}$

(ii) $f(2a)$

$f(2a) = (2a)^2 - 5(2a) + 6 = \boldsymbol{4a^2 - 10a + 6}$

(iii) $f(2)$

$f(2) = 2^2 - 5(2) + 6 = 4 - 10 + 6 = \mathbf{0}$

(iv) $f(x - 1)$

$f(x - 1) = (x - 1)^2 - 5(x - 1) + 6$

$= x^2 - 2x + 1 - 5x + 5 + 6 = \boldsymbol{x^2 - 7x + 12}$

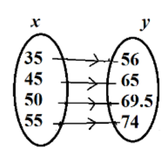
> **Similar Problems (Solve Your Self)** (2M)
>
> 5. Let $f(x) = 2x + 5$. If $x \neq 0$ then find $\frac{f(x+2)-f(2)}{x}$
> 6. A function f is defined by $f(x) = 2x - 3$
> (i) find $\frac{f(0)+f(1)}{2}$ (ii)find x such that $f(x) = 0$
> (iii) find x sucht that $f(x) = x$
> (iv) find x such that $f(x) = f(1 - x)$
>
> **Eg.1.9:** Given $f(x) = 2x - x^2$, find (i) $f(1)$ (ii) $f(x + 1)$ (iii) $f(x) + f(1)$

10. The data in the adjacent table depicts the length of a person forehand and their corresponding height. Based on this data, a student finds a relationship between the height (y) and the forehand length (x) as $y = ax + b$, where a, b are constants. (i) Check if this relation is a function. (ii) Find a and b (iii) Find the height of a person whose forehand length is 40cm (iv) Find the length of forehand of a person if her height is 53.3 inches. (5M) PTA-4

Length x of forehand (in cm)	Height 'y' (in inches)
35	56
45	65
50	69.5
55	74

Given $y = ax + b$

(i) Arrow diagram



Each element in x is associated with a unique element in y

Yes, this relation is a function

(ii) find a and b

From the table

$35a + \cancel{b} = 56$ ....(1)

$45a + \cancel{b} = 65$ ....(2)

$\underline{(-) \quad (-) \quad (-)}$

$-10a = -9$

$a = \frac{9}{10} = 0.9$

$a = 0.9$ substitute in (1)

$35(0.9) + b = 56$

$31.5 + b = 56$

$b = 56 - 31.5 = 24.5$

$\boldsymbol{a = 0.9}$ **and** $\boldsymbol{b = 24.5}$

(iii) Length $= 40$cm, $a = 0.9, b = 24.5$

$y = ax + b$

$= (0.9)(40) + 24.5 \quad = 60.5$

The height of a person whose forehand length is 40 cm **= 60.5 inches.**

(iv) Height $= 53.3$ inches

$y = ax + b$

$53.3 = (0.9)x + 24.5 \quad = 0.9x + 24.5$

$53.3 - 24.5 = 0.9x$

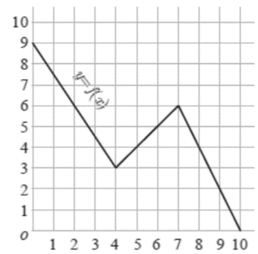
$28.8 = 0.9x$

$x = \frac{28.8}{0.9} = 32 \Rightarrow x = 32$ cm

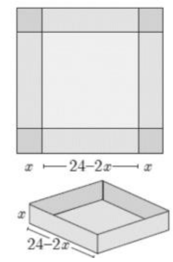
The length of forehand of a person = **32 cm**

Similar Problems (Solve Your Self) 2M

4. A graph representing the function $f(x)$ is given in adjacent figure. It is clear that $f(9) = 2$
   (i) Find the following values of the function
   (a) $f(0)$ (b) $f(7)$ (c) $f(2)$ (d) $f(10)$
   (ii) For what value of x is $f(x) = 1$
   (iii) Describe the following (i)Domain (ii) Range
   (iv) What is the image of 6 under f?



7. An open box is to be made from a square piece of material, 24 cm on a side, by cutting equal squares from the corners and turning up the sides as shown figure. Express the volume V of the box as a function of x.

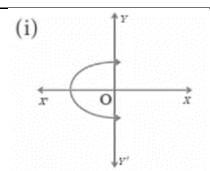


8. A function f is defined by $f(x) = 3 - 2x$. Find x such that $f(x^2) = (f(x))^2$
9. A plane is flying at a speed of 500 km per hour. Express the distance d travelled by the plane as function of time t in hours.

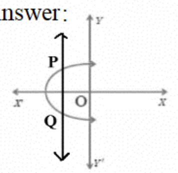
Exercise 1.4

1. Determine whether the graph given below represent functions. Give reason for your answers concerning each graph. 2M

(i)

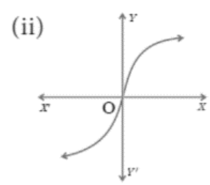


Answer:

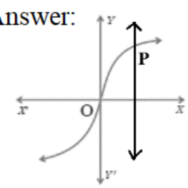


Using vertical line test, the curve **does not represent a function** as the vertical line meets the curve in two points P and Q.

(ii)



Answer:

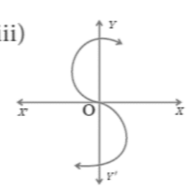


The curve represent **a function** as the vertical line meets the curve in at most one point.

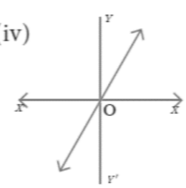
Similar Problems (Solve Your Self) 2M

1. Determine whether the graph given below represent functions. Give reason for your answers concerning each graph.

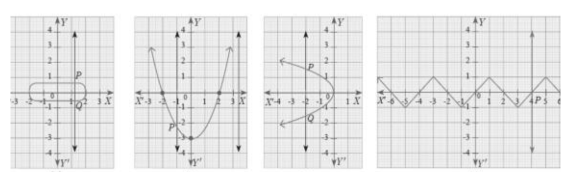
(iii)



(iv)

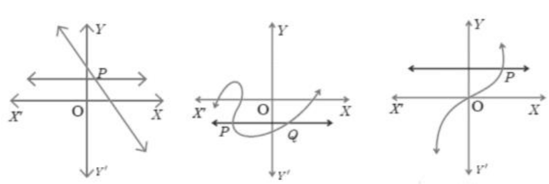


Eg.1.10. Using vertical line test, determine which of the following curves ((a), (b), (c), (d)) represent a function?



Similar Problems (Solve Your Self) 2M

Eg.1.12: Using horizontal line test ((a), (b), (c)), determine which of the following functions are one – one.



2. **Let $f: A \to B$ be a function defined by $f(x) = \frac{x}{2} - 1$, where $A = \{2, 4, 6, 10, 12\}, B = \{0, 1, 2, 4, 5, 9\}$. Represent f by (i) set of ordered pairs (ii) a table (iii) an arrow diagram (iv) a graph** APR-23 5M

Given, $A = \{2,4,6,10,12\}, B = \{0,1,2,4,5,9\}$

Now $f(x) = \frac{x}{2} - 1$

Thus, $f(2) = \frac{2}{2} - 1 = 1 - 1 = \mathbf{0}$

$f(4) = \frac{4}{2} - 1 = 2 - 1 = \mathbf{1}$

$f(6) = \frac{6}{2} - 1 = 3 - 1 = \mathbf{2}$

$f(10) = \frac{10}{2} - 1 = 5 - 1 = \mathbf{4}$

$f(12) = \frac{12}{2} - 1 = 6 - 1 = \mathbf{5}$

Similar Problems (Solve Your Self) 5M

3. Represent the function $f = \{(1,2), (2,2), (3,2), (4,3), (5,4)\}$ through (i) an arrow diagram (ii) a table form (iii) a graph

Eg.1.11: Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 5, 8, 11, 14\}$ be two sets. Let $f: A \to B$ be a function given by $f(x) = 3x - 1$. Represent this function **(PTA-3, SEP-20)**
(i) by arrow diagram (ii) in a table form
(iii) as a set of ordered pairs (iv) in a graphical form

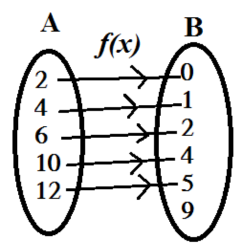
(i) set of ordered pairs,

$\boldsymbol{f} = \{(\mathbf{2}, \mathbf{0}), (\mathbf{4}, \mathbf{1}), (\mathbf{6}, \mathbf{2}), (\mathbf{10}, \mathbf{4}), (\mathbf{12}, \mathbf{5})\}$

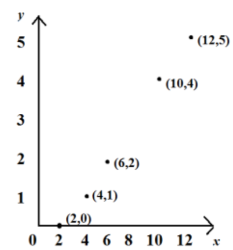
(ii) Table

x	2	4	6	10	12
$f(x)$	0	1	2	4	5

(iii) An arrow diagram



(iv) Graph



4. **Show that the function $f: \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 2x - 1$ is one-one but not onto.** 2M

The function $f: \mathbb{N} \to \mathbb{N}$ defined by $f(x) = 2x - 1$

If $x = 1, f(1) = 2(1) - 1 = 1$

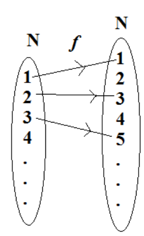
If $x = 2, f(2) = 2(2) - 1 = 3$

If $x = 3, f(3) = 2(3) - 1 = 5$

Then f is a function from N to N and for different elements in N, there are different images in N. Hence f one-one function.

But the even numbers in the co-domain do not have any pre-images of the domain. Hence f is not onto, So f is **one-one but not onto function.**

Arrow diagram:



Similar Problems (Solve Your Self) 2M

Eg.1.13: Let $A = \{1, 2, 3\}$, $B = \{4, 5, 6, 7\}$ and $f = \{(1, 4), (2, 5), (3, 6)\}$ be a function from A to B. Show that f is one – one but not onto function.

5. **Show that the function $f: \mathbb{N} \to \mathbb{N}$ defined by $f(m) = m^2 + m + 3$ is one – one function.** SEP-20 2M

The function $f: \mathbb{N} \to \mathbb{N}$ defined by

$f(m) = m^2 + m + 3$

$m = 1, f(1) = (1)^2 + 1 + 3 = 1 + 1 + 3 = 5$

$m = 2, f(2) = (2)^2 + 2 + 3 = 4 + 2 + 3 = 9$

$m = 3, f(3) = (3)^2 + 3 + 3 = 9 + 3 + 3 = 15$

$m = 4, f(4) = (4)^2 + 4 + 3 = 16 + 4 + 3 = 23$

Since different elements of N have different images in the co-domain the function of $\boldsymbol{f}$ **is one-one function.**

6. **Let $A=\{1,2,3,4\}$ and $B=\mathbb{N}$. Let $f:A\to B$ be defined by $f(x)=x^3$ then,** (2M)

(i) find the range of f (ii) identify the type of function

Now $=\{1,2,3,4\}$, $B=\{1,2,3,\ldots\}$

Given $f:A\to B$ and $f(x)=x^3$

$f(1)=1^3=1, f(3)=3^3=27$

$f(2)=2^3=8,\ f(4)=4^3=64$

(i) Range of $\boldsymbol{f}=\{\mathbf{1},\mathbf{8},\mathbf{27},\mathbf{64}\}$

(ii) Since distinct elements in A are mapped into distinct images in B, it is a **one-one function.**

$2\in B$ is not the image of any element of A. So, it is **Into function.**

Similar Problems (Solve Your Self) (2M)

Eg.1.15: Let f be a function $f:\mathbb{N}\to\mathbb{N}$ be defined by $f(x)=3x+2,\ x\in\mathbb{N}$

(i) Find the images of 1, 2, 3 (ii) Find the pre-images of 29, 53 **(PTA-3)**

(iii) Identify the type of function **(MDL)**

CQ: Let $=\{1,2,3,4\}$, $B=\mathbb{N}$. Let $f:A\to B$ be defined by $f(x)=x^2$.

Find (i) the range of f (ii) identify the type of function **(PTA-5)**

7. **In each of the following cases state whether the function is bijective or not. Justify your answer.**

(i) $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=2x+1$ (5M)

$f(x)=2x+1$

$f(0)=2(0)+1=1$

$f(1)=2(1)+1=3$

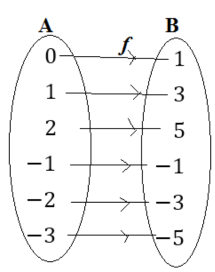
$f(2)=2(2)+1=5$

$f(-1)=2(-1)+1=-1$

$f(-2)=2(-2)+1=-3$

$f(-3)=2(-3)+1=-5$

Range of $f=\{1,3,5,-1,-3,-5\}$



As distinct elements of A have distinct images in B and every elements in B has a pre-image in A. The function is **bijective.**

(ii) $f:\mathbb{R}\to\mathbb{R}$ defined by $f(x)=3-4x^2$ (5M)

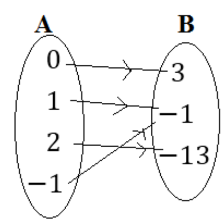
$f(x)=3-4x^2$

$f(0)=3-4(0)^2=3$

$f(1)=3-4(1)^2=-1$

$f(2)=3-4(2)^2=-13$

$f(-1)=3-4(-1)^2=-1$



Thus two distinct elements 1 and -1 in A have same image -1 in B. Hence f is not a one-one function. But every elements in B has a pre-image in A. Hence f is a onto function.

Therefore f is not one-one but onto.

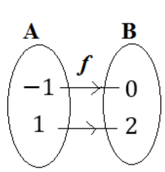
Hence f is **not bijective.**

8. **Let $A=\{-1,1\}$ and $B=\{0,2\}$. If the function $f:A\to B$ defined by $f(x)=ax+b$ is an onto function? Find a and b.** (2M)

Given $A=\{-1,1\}$ and $B=\{0,2\}$

Then $f:A\to B$ defined by

$f(x)=ax+b$ is an onto function.



[Range of f = co-domain]

$f(-1)=0\Rightarrow -a+b=0$

$f(1)=2\Rightarrow a+b=2$

Now $-\cancel{a}+b=0$ ....(1)

$\cancel{a}+b=2$ ....(2)

(1)+(2) $2b=2$

$b=1$

Substitute $b=1$ in (2)

$a+1=2$

$a=2-1=1$

Thus, $\boldsymbol{a}=\mathbf{1}$ **and** $\boldsymbol{b}=\mathbf{1}$

Similar Problems (Solve Your Self) (2M)

Eg.1.14: If $A=\{-2,-1,0,1,2\}$ and $f:A\to B$ is an onto function defined by $f(x)=x^2+x+1$ then find B.

Eg.1.17: Let f be a function from $\mathbb{R}$ to $\mathbb{R}$ defined by $f(x)=3x-5$. Find the value of a and b given that $(a,4)$ and $(1,b)$ belong to f. **(PTA-6)**

CQ: $R=\{(x,-2),(-5,y)\}$ represents the identity function, find the values of x and y. **(PTA-6)**

10. A function $f:[-5,9]\to\mathbb{R}$ is defined as follows: $f(x)=\begin{cases}6x+1; & -5\le x<2\\ 5x^2-1; & 2\le x<6\\ 3x-4; & 6\le x\le 9\end{cases}$ 5M

Find (i) $f(-3)+f(2)$ (ii) $f(7)-f(1)$ (iii) $2f(4)+f(8)$ (iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$

$$f(x)=\begin{cases}6x+1; & -5\le x<2 \text{ ; Where } x=-5,-4,-3,-2,-1,0,1\\ 5x^2-1; & 2\le x<6 \text{ ; Where } x=2,3,4,5\\ 3x-4; & 6\le x\le 9 \text{ ; Where } x=6,7,8,9\end{cases}$$

(i) $f(-3)+f(2)$ When $x=-3$ $f(x)=6x+1$ $f(-3)=6(-3)+1=-18+1=-17$ When $x=2$ $f(x)=5x^2-1$ $f(2)=5(2)^2-1=20-1=19$ $f(-3)+f(2)=-17+19=\mathbf{2}$	(ii) $f(7)-f(1)$ PTA-4 When $x=7$ $f(x)=3x-4$ $f(7)=3(7)-4=21-4=17$ When $x=1$ $f(x)=6x+1$ $f(1)=6(1)+1=6+1=7$ $\therefore f(7)-f(1)=17-7=\mathbf{10}$
(iii) $2f(4)+f(8)$ When $x=4$, $f(x)=5x^2-1$ $f(4)=5(4)^2-1=80-1=79$ When $x=8$, $f(x)=3x-4$ $f(8)=3(8)-4=24-4=20$ $2f(4)+f(8)=2(79)+20$ $=158+20=\mathbf{178}$	(iv) $\frac{2f(-2)-f(6)}{f(4)+f(-2)}$ PTA-4 When $x=-2$, $f(x)=6x+1$ $f(-2)=6(-2)+1=-12+1=-11$ When $x=6$, $f(x)=3x-4$ $f(6)=3(6)-4=18-4=14$ When $x=4$, $f(x)=5x^2-1$ $f(4)=5(4)^2-1=80-1=79$ $\frac{2f(-2)-f(6)}{f(4)+f(-2)}=\frac{2(-11)-14}{79+(-11)}=\frac{-22-14}{79-11}=\frac{-36}{68}=-\mathbf{\frac{9}{17}}$

Similar Problems (Solve Your Self) 5M

9. If the function f is defined by $f(x)=\begin{cases}x+2; & x>1\\ 2; & -1\le x\le 1\\ x-1; & -3<x<-1\end{cases}$ find the values of (i) $f(3)$ (ii) $f(0)$ (iii) $f(-1.5)$ (iv) $f(2)+f(-2)$

Eg.1.18: If the function $f:\mathbb{R}\to\mathbb{R}$ is defined by $f(x)=\begin{cases}2x+7, & x<-2\\ x^2-2, & -2\le x<3\\ 3x-2, & x\ge 3\end{cases}$, then find the values of

(i) $f(4)$ (ii) $f(-2)$ (iii) $f(4)+2f(1)$ (iv) $\frac{f(1)-3f(4)}{f(-3)}$

11. The distance S an object travels under the influence of gravity in the time t seconds is given by $S(t)=\frac{1}{2}gt^2+at+b$ where, (g is the acceleration due to gravity), a,b are constants. Verify whether the function $S(t)$ is one-one or not. 5M PTA-3

Given $S(t)=\frac{1}{2}gt^2+at+b$ (a,b constants)

Now take $t=1,2,3,\ldots$ seconds

$t=1$, $S(1)=\frac{1}{2}g(1)^2+a(1)+b$

$=\frac{1}{2}g+a+b=\mathbf{0.5g+a+b}$

$t=2$, $S(2)=\frac{1}{2}g(2)^2+a(2)+b$

$=\mathbf{2g+2a+b}$

$t=3$, $S(3)=\frac{1}{2}g(3)^2+a(3)+b$

$=\mathbf{4.5g+3a+b}$

Since distinct elements of A have distinct image in B. **Yes, it is an one-one function.**

Similar Problems (Solve Your Self) 5M

Eg.1.16: Forensic scientists can determine the height (in cm) of a person based on the length of their thigh bone. They usually do so using the function $h(b)=2.47b+54\cdot 10$ where b is the length of the thigh bone.

(i) Check if the function h is one – one or not

(ii) Also find the height of a person if the length of his thigh bone is 50 cm.

(iii) Find the length of the thigh bone if the height of a person is 147.96 cm.

12. The function $'t'$ which maps temperature in Celsius (C) into temperature in Fahrenheit (F) is defined by $t(C) = F$ where $F = \frac{9}{5}C + 32$. Find 5M PTA-1

(i) $t(0)$ (ii) $t(28)$ (iii) $t(-10)$ (iv) the value of C when $t(C) = 212$

(v) the temperature when the Celsius value is equal to the Fahrenheit value

The function t is defined by, $t(C) = F$, where $F = \frac{9}{5}C + 32$

(i) $t(0) = \frac{9}{5}(0) + 32 = \mathbf{32°F}$

(ii) $t(28) = \frac{9}{5}(28) + 32$

$= 9(5.6) + 32$

$= 50.4 + 32$

$= \mathbf{82.4°F}$

(iii) $t(-10) = \frac{9}{5}(-10) + 32$

$= -18 + 32$

$= \mathbf{14°F}$

(iv) When $t(C) = 212$

$\frac{9}{5}C + 32 = 212$

$\frac{9}{5}C = 212 - 32 = 180$

$C = \frac{180\times5}{9} = \mathbf{100°C}$

(v) we know that

$t(C) = F$ where $F = \frac{9}{5}C + 32$

$t(F) = C$ where $C = \frac{9}{5}F + 32$

If the temperatures are same then two $'t'$s in the formula should represent the same temperature. So then we multiply each side by $\left(-\frac{5}{4}\right)$

$t = \frac{9}{5}t + 32°$

$t - \frac{9}{5}t = 32°$

Multiply each side by $\left(-\frac{5}{4}\right)$

$-\frac{5}{4}\left(t - \frac{9}{5}t\right) = 32° \times \left(-\frac{5}{4}\right)$

$-\frac{5}{4}t + \frac{9}{4}t = -40°$

$\frac{-5t+9t}{4} = -40°$

$\frac{4t}{4} = -40°$

$\boldsymbol{t = -40°}$

Exercise 1.5

1. Using the functions f and g given below, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$ 2M

(i) $f(x) = x - 6, g(x) = x^2$ JUN-23

$f \circ g = f \circ g(x)$

$= f(g(x))$

$= f(x^2)$

$= x^2 - 6$..............(1)

$g \circ f(x) = g(f(x))$

$= g(x - 6)$

$= (x - 6)^2$...............(2)

From (1) and (2) $\boldsymbol{f \circ g \neq g \circ f}$

(iv) $f(x) = 3 + x, g(x) = x - 4$

$f \circ g(x) = f(g(x))$

$= f(x - 4)$

$= 3 + x - 4$

$= x - 1$ ....(1)

$g \circ f(x) = g(f(x))$

$= g(3 + x)$

$= 3 + x - 4$

$= x - 1$ ....(2)

From (1) and (2) we get that, $\boldsymbol{f \circ g = g \circ f}$

Similar Problems (Solve Your Self) 2M

1. Using the functions f and g given below, find $f \circ g$ and $g \circ f$. Check whether $f \circ g = g \circ f$

(ii) $f(x) = \frac{2}{x}, g(x) = 2x^2 - 1$ (iii) $f(x) = \frac{x+6}{3}, g(x) = 3 - x$ (v) $f(x) = 4x^2 - 1, g(x) = 1 + x$

Eg.1.19: Find $f \circ g$ and $g \circ f$ when $f(x) = 2x + 1$ and $g(x) = x^2 - 2$

Eg.1.20: Represent the function $f(x) = \sqrt{2x^2 - 5x + 3}$ as a composition of two functions.

2. Find the value of k, such that $f \circ g = g \circ f$ (5M)

(i) $f(x) = 3x + 2, g(x) = 6x - k$

$$f \circ g(x) = f(g(x)) = f(6x - k)$$
$$= 3(6x - k) + 2 = 18x - 3k + 2$$
$$g \circ f(x) = g(f(x)) = g(3x + 2)$$
$$= 6(3x + 2) - k = 18x + 12 - k$$

Given that, $f \circ g = g \circ f$

$$18x - 3k + 2 = 18x + 12 - k$$
$$18x - 18x - 3k + k = 12 - 2$$
$$-2k = 10$$
$$\boldsymbol{k = -5}$$

Similar Problems (Solve Your Self)

2. Find the value of k, such that $f \circ g = g \circ f$ (5M)

(ii) $f(x) = 2x - k, g(x) = 4x + 5$

Eg.1.21: If $f(x) = 3x - 2$, $g(x) = 2x + k$ and if $f \circ g = g \circ f$, then find the value of k. (5M)

Eg.1.22: Find k if $f \circ f(k) = 5$ where $f(k) = 2k - 1$. **(APR-23, PTA-4)** (2M)

3. If $f(x) = 2x - 1, g(x) = \frac{x+1}{2}$, show that $f \circ g = g \circ f = x$ (2M)

Given $f(x) = 2x - 1, g(x) = \frac{x+1}{2}$

$$f \circ g(x) = f(g(x)) = f\left(\frac{x+1}{2}\right) = 2\left(\frac{x+1}{2}\right) - 1$$
$$= x + 1 - 1$$
$$= x \text{ ....(1)}$$
$$g \circ f(x) = g(f(x)) = g(2x - 1)$$
$$= \frac{2x-1+1}{2} = \frac{2x}{2} = x \text{ .....(2)}$$

From (1) and (2), $\boldsymbol{f \circ g = g \circ f = x}$

4. If $f(x) = x^2 - 1, g(x) = x - 2$ find a, if $g \circ f(a) = 1$ PTA-2 (5M)

Given $f(x) = x^2 - 1, g(x) = x - 2$

$$g \circ f(x) = g(f(x)) = g(x^2 - 1)$$
$$= x^2 - 1 - 2$$
$$= x^2 - 3$$

Given $g \circ f(a) = 1$

Hence $a^2 - 3 = 1$

$$a^2 = 1 + 3$$
$$a^2 = 4$$
$$\boldsymbol{a = \pm 2}$$

5. Let $A, B, C \subseteq N$ and a function $f: A \to B$ be defined by $f(x) = 2x + 1$ and $g: B \to C$ be defined by $g(x) = x^2$. Find the range of $f \circ g$ and $g \circ f$ (2M)

$f: A \to B$ be defined by $f(x) = 2x + 1$, $g: B \to C$ be defined by $g(x) = x^2$ and $A, B, C \subseteq N$

$f \circ g(x) = f(g(x)) = f(x^2) = \boldsymbol{2x^2 + 1}$, Range of $f \circ g = \{y / y = 2x^2 + 1, x \in N\}$

$g \circ f(x) = g(f(x)) = g(2x + 1) = (\boldsymbol{2x + 1})^2$, Range of $g \circ f = \{y / y = (2x + 1)^2, x \in N\}$

6. Let $f(x) = x^2 - 1$. Find

(i) $f \circ f$ (2M)

$$f \circ f(x) = f(f(x))$$
$$= f(x^2 - 1)$$
$$= (x^2 - 1)^2 - 1$$
$$= x^4 - 2x^2 + 1 - 1$$
$$= \boldsymbol{x^4 - 2x^2}$$

(ii) $f \circ f \circ f$ (2M)

$$f \circ f \circ f(x) = f\left(f(f(x))\right)$$
$$= f(f(x^2 - 1))$$
$$= f((x^2 - 1)^2 - 1)$$
$$= f(x^4 - 2x^2 + 1 - 1)$$
$$= f(x^4 - 2x^2)$$
$$= \boldsymbol{[x^4 - 2x^2]^2 - 1}$$

Similar Problems (Solve Your Self) (2M)

Eg.1.24: Find x if $gff(x) = fgg(x)$, given $f(x) = 3x + 1$ and $g(x) = x + 3$.

Unit Exercise:

8. If $f(x) = \frac{x-1}{x+1}$, $x \neq 1$ show that $f(f(x)) = -\frac{1}{x}$, provided $x \neq 0$.

9. The functions f and g are defined by $f(x) = 6x + 8$; $g(x) = \frac{x-2}{3}$

(i) Calculate the value of $gg\left(\frac{1}{2}\right)$

(ii) Write an expression for $gf(x)$ in its simplest form.

7. **If $f: R \to R$ and $g: R \to R$ are defined by $f(x) = x^5$ and $g(x) = x^4$ then check if f, g are one-one and $f \circ g$ is one-one?** PTA-6 5M

$f: R \to R$ defined by $f(x) = x^5$

$f \circ f(x) = f(f(x))$

$= f(x^5)$

$= (x^5)^5 = x^{25}$

$f \circ f(1) = (1)^{25} = 1$

$f \circ f(2) = (2)^{25}$

$f \circ f(3) = (3)^{25}$

Since each elements in f have distinct images, f is one-one

$g: R \to R$ defined by $g(x) = x^4$

$g \circ g(x) = g(g(x)) = g(x^4)$

$= (x^4)^4$

$= x^{16}$

$g \circ g(-1) = (-1)^{16} = 1$

$g \circ g(1) = (1)^{16} = 1$

$g \circ g(2) = (2)^{16}$

Thus two distinct elements -1 and 1 have same images.

Hence g is not one-one

$f \circ g(x) = f(g(x))$

$= f(x^4)$

$= (x^4)^5 = x^{20}$

$f \circ g(1) = (1)^{20} = 1$

$f \circ g(-1) = (-1)^{20} = 1$

Thus two distinct elements -1 and 1 have same images. Hence **$f \circ g$ is not one-one.**

8. **Consider the functions $f(x), g(x), h(x)$ as given below, show that $(f \circ g) \circ h = f \circ (g \circ h)$ in each case.** 5M PTA-2

(iii) $f(x) = x - 4$, $g(x) = x^2$ & $h(x) = 3x - 5$

$f \circ g(x) = f(g(x))$

$= f(x^2) = x^2 - 4$

Then $(f \circ g) \circ h(x) = f \circ g(h(x))$

$= f \circ g(3x - 5)$

$= (3x - 5)^2 - 4$

$= 9x^2 - 30x + 25 - 4$

$= 9x^2 - 30x + 21$........(1)

$(g \circ h)x = g(h(x))$

$= g(3x - 5) = (3x - 5)^2$

$= 9x^2 - 30x + 25$

$f \circ (g \circ h)(x) = f(9x^2 - 30x + 25)$

$= 9x^2 - 30x + 25 - 4$

$= 9x^2 - 30x + 21$ ....(2)

From (1) and (2), $(\boldsymbol{f \circ g}) \circ \boldsymbol{h} = \boldsymbol{f} \circ (\boldsymbol{g \circ h})$

Similar Problems (Solve Your Self) 5M

8. Consider the functions $f(x), g(x), h(x)$ as given below, show that $(f \circ g) \circ h = f \circ (g \circ h)$ in each case.
   (i) $f(x) = x - 1, g(x) = 3x + 1$ and $h(x) = x^2$
   (ii) $f(x) = x^2, g(x) = 2x$ and $h(x) = x + 4$

Eg.1.23: If $f(x) = 2x + 3$, $g(x) = 1 - 2x$ and $h(x) = 3x$. Prove that $f \circ (g \circ h) = (f \circ g) \circ h$ **(PTA-5)**

Unit Exercise:

6. If $f(x) = x^2$, $g(x) = 3x$ and $h(x) = x - 2$, Prove that $(f \circ g) \circ h = f \circ (g \circ h)$.

9. **Let $f = \{(-1, 3), (0, -1), (2, -9)\}$ be a linear function from $\mathbb{Z}$ into $\mathbb{Z}$. Find $f(x)$.** 2M

Let $f = \{(-1,3), (0,-1), (2,-9)\}$ be a linear function from Z into Z

$f(x) = mx + c$ can be written

$f = \{(x, mx + c)/\ x \in Z\}$

$f(-1) = 3$

$-m + c = 3$ ....(1)

$f(0) = -1$

$c = -1$ ....(2)

Substitute $c = -1$ in (1)

$-m + c = 3$

$-m - 1 = 3$

$m = -1 - 3 = -4$

$\therefore f(x) = \boldsymbol{-4x - 1}$

10. **In electrical circuit theory, a circuit $C(t)$ is called a linear circuit if it satisfies the superposition principle given by $C(at_1 + bt_2) = aC(t_1) + bC(t_2)$, where a, b are constants. Show that the circuit $C(t) = 3t$ is linear.** 2M

$C(t_1) = t$

$C(t_2) = 2t$ where $t = t_1 + t_2$

$C(t) = C(t_1 + t_2)$

$= C(t_1) + C(t_2)$

$t + 2t = 3t$

$\therefore$ **$C(t) = 3t$ is linear.**

Unit Exercise - 1

4. Let $A=\{9,10,11,12,13,14,15,16,17\}$ and let $f: A \to N$ be defined by $f(n)=$ the highest prime factor of $n \in A$. Write f as a set of ordered pairs and find the range of f. (2M)

Let $A=\{9,10,11,12,13,14,15,16,17\}$

$f: A \to N$ defined by $f(n)=$ the highest prime factor of $n \in A$.

$f=\{(9,3),(10,5),(11,11),(12,3),$
$(13,13),(14,7),(15,5),(16,2),(17,17)\}$

Range of $f=\{2,3,5,7,11,13,17\}$

$9=3\times3$	$10=5\times2$	$11=1\times11$
$12=3\times4$	$13=1\times13$	$14=2\times7$
$15=5\times3$	$16=2\times8$	$17=1\times17$

Note for Unit Exercise – 1

Q.No: 1, 2, 3 – For Practice

Q.No: 6 – Similar to Exercise 1.5 -8th Question

Q.No: 8,9 – Similar to Exercise 1.5 -6th Question

5. Find the domain of the function (2M)

$$f(x)=\sqrt{1+\sqrt{1-\sqrt{1-x^2}}}$$

Given $f(x)=\sqrt{1+\sqrt{1-\sqrt{1-x^2}}}$

If $x\in(-\infty,-1)\cup(1,\infty)$, $f(x)$ is not real.

If $x\in[-1,1]$, $f(x)$ is real.

$\therefore$ Domain is $-1\le x\le 1$

7. Let $A=\{1,2\}$ and $B=\{1,2,3,4\}$, $C=\{5,6\}$ and $D=\{5,6,7,8\}$. Verify whether $A\times C$ is a subset of $B\times D$? (2M)

$A=\{1,2\}$, $B=\{1,2,3,4\}$, $C=\{5,6\}$ and $D=\{5,6,7,8\}$

Now, $A\times C=\{1,2\}\times\{5,6\}=\{(1,5),(1,6),(2,5),(2,6)\}$ .... (1)

$B\times D=\{1,2,3,4\}\times\{5,6,7,8\}$

$=\{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),(3,8),$
$(4,5),(4,6),(4,7),(4,8)\}$ .... (2)

we observe from (1) and (2), $A\times C\subseteq B\times D$.

10. Write the domain of the following real functions (2M)

(i) $f(x)=\frac{2x+1}{x-9}$ (ii) $p(x)=\frac{-5}{4x^2+1}$ (iii) $g(x)=\sqrt{x-2}$ (iv) $h(x)=x+6$

i) $f(x)=\frac{2x+1}{x-9}$ PTA-6

If $x=9$ then $f(-9)$ is not defined

Hence f is defined for all real numbers except at $x=9$.

So domain of $f=R-\{9\}$

ii) $p(x)=\frac{-5}{4x^2+1}$

Here p is defined for all real number.

Hence domain of $p=R$

iii) $g(x)=\sqrt{x-2}$ PTA-6

If $x\in(-\infty,2)$ $g(x)$ is not real

If $x\in[2,\ \infty)$ $g(x)$ is real

$\therefore$ the Domain is $[2,\infty)$

iv) $h(x)=x+6$

Here h is defined for all real numbers.

Hence domain of $h=R$

For Practice:

1. If the ordered pairs $(x^2-3x,\ y^2+4y)$ and $(-2,5)$ are equal, then find x and y. **(2M)**
2. The Cartesian product $A\times A$ has 9 elements among which $(-1,0)$ and $(0,1)$ are found. Find the set A and the remaining elements of $A\times A$. **(2M)**
3. Given that $f(x)=\begin{cases}\sqrt{x-1} & x\ge1\\ 4 & x<1\end{cases}$.

   Find (i) $f(0)$ (ii) $f(3)$ (iii) $f(a+1)$ in terms of a. (Given that $a\ge0$) **(2M)**

2. Numbers and Sequences

Exercise 2.1

1. Find all positive integers, when divided by 3 leaves remainder 2. (2M)

Let q is a positive integer given,

The divisor $= 3$ and remainder $= 2$

By Euclid's division lemma,

$$a = bq + r,\ 0 \le r < b$$

Let $q = 0,1,2,3,4 \ldots \quad (\because b = 3, r = 2)$

$3q + 2 = a$

$q = 0 \Rightarrow 3(0) + 2 = 0 + 2 = 2$

$q = 1 \Rightarrow 3(1) + 2 = 3 + 2 = 5$

$q = 2 \Rightarrow 3(2) + 2 = 6 + 2 = 8$

$q = 3 \Rightarrow 3(3) + 2 = 9 + 2 = 11$

$\therefore$ The positive integers are,

$\mathbf{2, 5, 8, 11} \ldots$

2. A man has 532 flower pots. He wants to arrange them in rows such that each row contains 21 flower pots. Find the number of completed rows and how many flower pots are left over. PTA-1 (2M)

Given: A man has 532 flower pots.

Each row contains 21 flower pots.

Thus, Dividend $= 532$ & Divisor $= 21$

By Euclid division lemma,

$a = bq + r\ , 0 \le r < b$

$532 = 21(25) + 7$

$$\begin{array}{r|l} & 25 \\ 21 & 532 \\ & 42 \\ & 112 \\ & 105 \\ & 7 \end{array}$$

The quotient $= 25$, remainder $= 7$

$\therefore$ **25 rows** are completed and **7 flower pots** are left over.

Similar Problems Solve Your Self

Eg. 2.1: We have 34 cakes. Each box can hold 5 cakes only. How many boxes we need to pack and how many cakes are unpacked? (2M)

UE-2: A milk man has 175 litres of cow's milk and 105 litres of buffalow's milk. He wishes to sell the milk by filling the two types of milk in cans of equal capacity. Calculate the following
(i) capacity of a can (ii) Number of cans of cow's milk
(iii) Number of cans buffalow's milk. (5M)

UE-4: Show that 107 is of the form $4q$+3 for any integer q. (2M)

3. Prove that the product of two consecutive positive integers is divisible by 2. (2M)

Let n & $n + 1$ be the two consecutive positive integers.

$n(n + 1) = n^2 + n$

Let $n = 2m + 1$ be odd number.

$n^2 + n = (2m + 1)^2 + (2m + 1)$

$= 4m^2 + 2(2m)(1) + 1^2 + 2m + 1$

$= 4m^2 + 4m + 1 + 2m + 1$

$= 4m^2 + 6m + 2$

$= 2(2m^2 + 3m + 1)$

$\therefore$ It is divisible by 2

Let $n = 2m$ be an even number.

$n^2 + n = (2m)^2 + 2m$

$= 4m^2 + 2m$

$= 2(2m^2 + m)$.

$\therefore$ It is divisible by 2

$\therefore$ Hence proved.

Similar Problems Solve Your Self

5. Prove that square of any integer leaves the remainder either 0 or 1 when divided by 4 (5M)

Eg. 2.3: Show that the square of an odd integer is of the form $4q + 1$, for some integer q. (2M)

UE-1. Prove that $n^2 - n$ divisible by 2 for every positive integer n. (5M)

4. When the positive integers a, b & c are divided by 13, the respective remainders are 9, 7 & 10. show that $a + b + c$ is divisible by 13. (5M)

Given: The positive integers a, b and c are divided by 13, leaves remainders are 9, 7 & 10 respectively.

To show that $a + b + c$ is divisible by 13.

Thus, divisor $= 13$

By Euclid's Division Lemma,

$a = bq + r\ ,0 \le r < b$

$a = 13q_1 + 9,\ b = 13q_2 + 7,\ c = 13q_3 + 10$

$a + b + c = 13q_1 + 9 + 13q_2 + 7 + 13q_3 + 10$

$= 13q_1 + 13q_2 + 13q_3 + 26$

$a + b + c = 13(q_1 + q_2 + q_3 + 2) + 0$

$(\because \text{Remainder} = 0)$

$\therefore a + b + c$ is multiple of 13.

$\therefore$ It is divisible by 13.

Similar Problems Solve Your Self

UE-3. When the positive integers a, b & c are divided by 13 the respective remainders are 9, 7 and 10. Find the remainder when $a + 2b + 3c$ is divided by 13. (5M)

6. Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of (2M)

(iv) 84, 90 and 120

Let $a = 120, b = 90, c = 84$

HCF of $a, b; a > b$.

By Euclid's division lemma,

$a = bq + r, 0 \leq r < b$

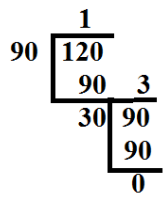
$120 = 90(1) + 30$

The remainder is $30 \neq 0$

$90 = 30(3) + 0$

The remainder is 0.

$\therefore$ The HCF of 120 and 90 is 30.



Next to find HCF of $d = 30$ and $c = 84$.

By using Euclid's division lemma, $c > d$

$84 = 30(2) + 24$

The remainder $24 \neq 0$

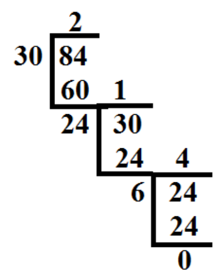
$30 = 24(1) + 6$

The remainder $6 \neq 0$

$24 = 6(4) + 0$

The remainder is 0

$\therefore$ The HCF of 84,90 and 120 is **6**



Similar Problems
Solve Your Self

6. Use Euclid's Division Algorithm to find the Highest Common Factor (HCF) of (2M)
   (i) 340 and 412
   (ii) 867 and 255
   (iii) 10224 and 9648

Eg. 2.2: Find the quotient and remainder when a is divided by b in the following cases (2M)
(i) $a = -12,\ b = 5$
(ii) $a = 17,\ b = -3$
(iii) $a = -19,\ b = -4$

Eg. 2.6: Find the HCF of 396, 504, 636. **(SEP-21)** (5M)

7. Find the largest number which divides 1230 and 1926 leaving remainder 12 in each case. (5M)

Given, 1230 and 1926 leaving remainder 12 in each case, when divided by largest number.

$1230 - 12 = 1218$ and $1926 - 12 = 1914$. JUL-22

Let $a = 1914$ and $b = 1218$ $\quad a > b$

By using Euclid's lemma, $a = bq + r, 0 \leq r < b$

$1914 = 1218(1) + 696$

The remainder $696 \neq 0$

$1218 = 696(1) + 522$

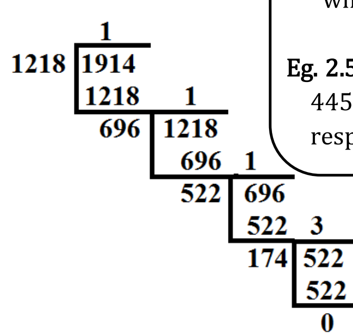
The remainder $522 \neq 0$

$696 = 522\ (1) + 174$

The remainder $174 \neq 0$

$522 = 174\ (3) + 0$

The remainder is 0



$\therefore$ The largest number is **174** which divides 1230 and 1926 and leaves remainder 12.

Similar Problems (Solve Your Self) (2M)

9. A positive integer when divided by 88 gives the remainder 61. What will be the remainder when the same number is divided by 11?

Eg. 2.5: Find the greatest number that will divide 445 and 572 leaving remainders 4 and 5 respectively.

8. If d is the Highest Common Factor of 32 and 60, find x and y satisfying $d = 32x + 60y$. 5M

Given: HCF of 32 and 60 is d

Using Euclid's lemma, $a = bq + r, 0 \le r < b$

Let $a = 60, b = 32, a > b$

$60 = 32\,(1) + 28$ ……………………(1)

$32 = 28\,(1) + 4$ ……………(2)

$28 = 4(7) + 0$ ………(3)

$\therefore\ d = 4$

$d = 32x + 60$ (given)

$4 = 32x + 60$

From (2) $\Rightarrow$ $32 = 28(1) + 4$

$32 - (28 \times 1) = 4$

From (1) $\Rightarrow$ $60 = 32(1) + 28$

$60 - (32 \times 1) = (28 \times 1)$

$4 = 32 - [60 - (32 \times 1)]$

$4 = (32 \times 1) - 60 + (32 \times 1)$

$4 = 32(1 + 1) - 60(1)$

$4 = 32(2) + 60(-1)$

$\therefore\ \boldsymbol{x = 2,\ \ y = -1}$

	1			
32	60			
	32	1		
	28	32		
		28	7	
		4	28	
			28	
			0	

Similar Problems (Solve Your Self) 5M

Eg. 2.4: If the Highest Common Factor of 210 and 55 is expressible in the form $55x - 325$, find x.

10. Prove that two consecutive positive integers are always coprime. 5M

Let $n, (n + 1)$ be the consecutive terms.

Using Euclid lemma, $a = bq + r,\ 0 \le r < b$

$n + 1 > n$. Then $(n + 1) = n(1) + 1$

$n = (1)(n) + 0$

Remainder $= 0$, Divisor $= 1$ HCF $=$ 1.

$\therefore$ It is always coprime.

	1		
n	$n+1$		
	n	n	
	1	n	
		n	
		0	

Exercise 2.2

1. For what values of natural number n, 4^n can end with the digit 6? 2M

Given: $n \in N$ and 4^n

$n = 1,2,3,4$ ….

$4^1 = 4$

$4^2 = 16$

$4^3 = 64$

$4^4 = 256$

$4^5 = 1024$

$4^6 = 4096$

⋮

4^n can end with the digit 6, when the value of n is **even number.**

Similar Problems Solve Your Self 2M

Eg. 2.8: Can the numbers are 6^n, n being a natural number end with the digit 5? Give reason for your answer.

3. Find the HCF of 252525 and 363636. 2M

Factorize of 252525

$= 5 \times 5 \times 10101$

Factorize of 363636

$= 2 \times 2 \times 3 \times 3 \times 10101$

$\therefore$ The HCF of 252525 and 363636 is **10101**

5	252525
5	50505
	10101

2	363636
2	181818
3	90909
3	30303
	10101

2. If m, n are natural numbers, for what values of m, does $2^n \times 5^m$ ends in 5? SEP-20 2M

Given: $m, n \in N$ and $2^n \times 5^m$

$n = 1, m = 1 \Rightarrow 2^1 \times 5^1 = 2 \times 5 = 10$

$n = 1, m = 2 \Rightarrow 2^1 \times 5^2 = 2 \times 25 = 50$

$n = 2, m = 3 \Rightarrow 2^2 \times 5^3 = 4 \times 125 = 500$

$\therefore 2^n$ is always even.

So that, the product of 5 is in always end digit is 0.

Hence, **No value** of $2^n \times 5^m$ end with the digit 5.

4. If $13824 = 2^a \times 3^b$ then a and b. MAY-22

Given: $13824 = 2^a \times 3^b$ 2M

The number 13824 can be factorized.

As, $13824 = 2^9 \times 3^3$

Hence, $2^a \times 3^b = 2^9 \times 3^3$

$\therefore a = 9$ and $b = 3$

3	13824
3	4608
3	1536
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
	2

Similar Problems (Solve Your Self) 2M

Eg. 2.10 : 'a' and 'b' are two positive integers such that $a^b \times b^a = 800$. Find 'a' and 'b'.

CQ - If $p^2 \times q^1 \times r^4 \times s^3 = 3{,}15{,}000$ then find p, q, r & s. **(APR-23)**

6. Find the LCM and HCF of 408 & 170 by applying the fundamental theorem of arithmetic. (2M)

$$\begin{array}{r|l} 2 & 170 \\ \hline 5 & 85 \\ \hline & 17 \end{array} \qquad \begin{array}{r|l} 2 & 408 \\ \hline 2 & 204 \\ \hline 2 & 102 \\ \hline 3 & 51 \\ \hline & 17 \end{array}$$

Given: The numbers 408 and 170 can be factorized as,

$408 = 2 \times 2 \times 2 \times 3 \times 17 = 2^3 \times 3 \times 17$

$170 = 2 \times 5 \times 17$

LCM of 408 and 170 $= 2^3 \times 3 \times 5 \times 17 = 8 \times 15 \times 17 = \mathbf{2040}$

HCF of 408 and 170 $= 2 \times 17 = \mathbf{34}$

7. Find the greatest number consisting of 6 digits which is exactly divisible by 24,15,36 ? (2M)

Given: The number is exactly divisible by 24,15,36.

Thus, LCM of 24,15,36 is 360.

The number is exactly divisible by 360.

Greatest number of 6 digits is 999999

$\therefore$ The greatest number $= 99999 - 279$

$= \mathbf{999720}$

$$\begin{array}{r|l} & 2777 \\ 360 & 999999 \\ & 720 \\ \hline & 2799 \\ & 2520 \\ \hline & 2799 \\ & 2520 \\ \hline & 2799 \\ & 2520 \\ \hline & 279 \end{array} \qquad \begin{array}{r|l} 3 & 24, 15, 36 \\ \hline 2 & 8, 5, 12 \\ \hline 2 & 4, 5, 6 \\ \hline & 2, 5, 3 \end{array}$$

$3 \times 2 \times 2 \times 2 \times 5 \times 3 = 360$

8. What is the smallest number that when divided by three numbers such as 35,56 and 91 leaves remainder 7 in each case? (2M)

The number divided by 35, 56 and 91 leaves remainder 7 in each case.

$$\begin{array}{r|l} 7 & 35, 56, 91 \\ \hline & 5, 8, 13 \end{array}$$

LCM of 35, 56 and 91= 3640

By Euclid's lemma, $a = bq + r, 0 \le r < b$

$n = 35q_1 + 7$ $\quad (\because\ 7 \times 5 \times 8 \times 13 = 3640)$

$n = 56q_2 + 7$

$n = 91q_3 + 7$

$n = 3640 + 7 = 3647$

$\therefore$ The smallest number is **3647.**

9. Find the least number that is divisible by the first ten natural numbers. JUN-23,JUL-22

The first ten natural numbers are, (2M)
1, 2, 3, 4, 5, 6, 7, 8, 9, 10

Given: The number is divisible by first ten natural numbers.

Thus, LCM of 1,2,3,4,5,6,7,8,9 and 10

$= 1 \times 2^3 \times 3^2 \times 5 \times 7$

$= 8 \times 9 \times 35$

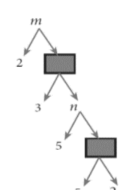
$= 2520$

$\therefore$ The least number is **2520**

For Practice:

5. If $p_1^{x_1} \times p_2^{x_2} \times p_3^{x_3} \times p_4^{x_4} = 113400$ where p_1, p_2, p_3, p_4 are primes in ascending order and x_1, x_2, x_3, x_4 are integers, find the value of p_1, p_2, p_3, p_4 and x_1, x_2, x_3, x_4. **(2M)**

Eg. 2.7: In the given factorisation, find the numbers m and n. **(2M)**



Eg. 2.9: Is $7 \times 5 \times 3 \times 2 + 3$ a composite number? Justify your answer. **(PTA-3) (2M)**

Exercise 2.3

1. Find the least positive value of x such that
(i) $71 \equiv x\ (mod\ 8)$ (2M)

Given: $71 \equiv x(mod\ 8)$

$71 - x = 8k$, for some integer k

$71 - x$ is a multiple of 8.

The least positive value is x must be $\boldsymbol{x = 7}$

Similar Problems (Solve Your Self) (2M)

1. Find the least positive value of x such that
(ii) $78 + x \equiv 3\ (mod\ 5)$ (iii) $89 \equiv (x + 3)(mod\ 4)$
(iv) $96 \equiv \frac{x}{7}(mod\ 5)$ (v) $5x \equiv 4\ (mod\ 6)$

Eg. 2.12: Determine the value of d such that
$15 \equiv 3(mod\ \ d)$

Eg. 2.13: Find the least positive value of x such that
(i) $67 + x \equiv 1(mod\ 4)$ (ii) $98 \equiv (x + 4)(mod\ 5)$

2. If x is congruent to 13 modulo 17 then $7x-3$ is congruent to which number modulo 17 ? PTA-2 2M

Given: $x \equiv 13\ (mod 17)$ [If $a \equiv b(mod\ m)$ then $a \times c \equiv b \times c\ (mod\ m)$]

Multiply by 7,

$7x = 91\ (mod\ 17)$

$7x - 3 \equiv 91 - 3\ (mod\ 17)$

$7x - 3 \equiv 88(mod\ 17)$

$7x - 3 \equiv 3\ (mod\ 17)$ [$\because 88 \equiv 3\ (mod\ 17)$]

$$\begin{array}{r|l} & 5 \\ 17 & 88 \\ & 85 \\ \hline & 3 \end{array}$$

$\therefore 7x-3$ is congruent to **3 modulo 17.**

3. Solve $5x \equiv 4\ (mod\ 6)$ 2M

Given: $5x \equiv 4\ (mod\ 6)$

$5x - 4 \equiv 6k$, for some integer k

$5x \equiv 6k+4$

$x = \frac{6k+4}{5}$

$6k+4$ is divided by 5, $k = 1,6,11,16 \ldots$

$k = 1 \Rightarrow x = \frac{6(1)+4}{5} = \frac{6+4}{5} = \frac{10}{5} = 2$

$k = 6 \Rightarrow x = \frac{6(6)+4}{5} = \frac{36+4}{5} = \frac{40}{5} = 8$

$k = 11 \Rightarrow x = \frac{6(11)+4}{5} = \frac{66+4}{5} = \frac{70}{5} = 14$

$\therefore \boldsymbol{x = 2, 8, 14} \ldots$

Similar Problems (Solve Your Self) 2M

4. Solve $3x - 2 \equiv 0\ (mod\ 11)$

Eg. 2.14: Solve $8x \equiv 1\ (mod\ 11)$

Eg. 2.15: Compute x, such that $10^4 \equiv x\ (mod\ 19)$ **(MDL)**

Eg. 2.16: Find the number of integer solutions of $3x \equiv 1\ (mod\ 15)$ **(SEP-21)**

5. What is the time 100 hours after 7 a.m? 2M

Starting time 7a.m,

To find, the time 100 hours after 7a.m.

Here, we use modulo 24.

$100 + 7\ (mod\ 24) = 107\ (mod\ 24)$

$= 11\ (mod24)$

$$\begin{array}{r|l} & 4 \\ 24 & 107 \\ & 96 \\ \hline & 11 \end{array}$$

$\therefore$ The time 100 hours after 7 a.m is **11.00 am.**

Similar Problems (Solve Your Self) 2M

6. What is the time 15 hours before 11p.m?

10. The duration of flight travel from Chennai to London through British Airlines is approximately 11 hours. The airplane begins its journey on Sunday at 23.30 hours. If the time at Chennai is four and half hours ahead to that of london's time, then find the time at London, when will the flight lands at London Airport.

7. Today is Tuesday. My uncle will come after 45 days. In which day my uncle will be coming? 2M

Let us associate the numbers 0,1,2,3,4,5 and 6 to represent the week days from Sunday to Saturday respectively.

Given condition,To day is Tuesday.

My uncle will come after 45 days.

We have, to Added 45 and 2 and take modulo 7.

$$\begin{array}{r|l} & 6 \\ 7 & 47 \\ & 42 \\ \hline & 5 \end{array}$$

$2 + 45\ (mod\ 7) = 47(mod\ 7)$

$= 5\ (mod\ 7)\ (\because 47 \equiv 5\ mod\ 7)$

$\therefore$ My uncle will coming on **Friday.**

Similar Problems (Solve Your Self) 2M

Eg. 2.17: A man starts his journey from Chennai to Delhi by train. He starts at 22.30 hours on Wednesday. If it takes 32 hours of travelling time and assuming that the train is not late, when will he reach Delhi?

Eg. 2.18: Kala and Vani are friends. Kala says, "Today is my birthday" and she asks Vani, "When will you celebrate your birthday?" Vani replies, "Today is Monday and I celebrated my birthday 75 days ago". Find the day when Vani celebrated her birthday.

9. Find the remainder when 2^{81} is divided by 17. (2M)

We know that,

$512 \equiv 2 \ (mod\ 17)$

$$\begin{array}{r|l} & 30 \\ 17 & 512 \\ & 510 \\ \hline & 2 \end{array}$$

$2^9 \equiv 2 \ (mod\ 17)$

$(2^9)^9 \equiv 2^9 \ (mod 17)$

$2^{81} \equiv 512 (mod 17)$ $[\because 512 = 2(mod 17)]$

$2^{81} \equiv 2 \ (mod 17)$

$\therefore 2^{81} - 2 = 17k$ for some integer k.

$2^{81} = 17k + 2.$

By Euclid's division lemma, $a = bq + r, 0 \le r < b$.

$\therefore$ The Remainder is **2.**

For Practice:

8. Prove that $2^n + 6 \times 9^n$ is always divisible by 7 for any positive integer n. **(5M)**

Eg. 2.11: Find the remainders when 70004 and 778 is divided by 7. **(2M)**

Exercise 2.4

1. Find the next three terms of the following sequence. (i) 8,24,72,... (2M)

In the above sequence, each term is multiply by 3.

$8 \times 3 = 24$

$24 \times 3 = 72$

$72 \times 3 = 216$

$216 \times 3 = 648$

$648 \times 3 = 1944$

$\therefore$ The next three terms are $\mathbf{216, 648, 1944}$.

Similar Problems (Solve Your Self) (2M)

1. Find the next three terms of the following sequence.
   (ii) $5, 1, -3, \ldots$ (iii) $\frac{1}{4}, \frac{2}{9}, \frac{3}{16}, \ldots$

Eg. 2.19: Find the next three terms of the sequences
(i) $\frac{1}{2}, \frac{1}{6}, \frac{1}{10}, \ldots\ldots$ (ii) $5, 2, -1, -4, \ldots.$ (iii) $1, 0.1, 0.01, \ldots\ldots$

2. Find the first four terms of the sequences whose n^{th} terms are given by. (2M)

(i) $a_n = n^3 - 2$

$n = 1 \Rightarrow a_1 = (1)^3 - 2$
$= 1 - 2 = -1$

$n = 2 \Rightarrow a_2 = 2^3 - 2$
$= 8 - 2 = 6$

$n = 3 \Rightarrow a_3 = 3^3 - 2$
$= 27 - 2 = 25$

$n = 4 \Rightarrow a_4 = 4^3 - 2 = 64 - 2 = 62.$

$\therefore$ The first four terms are, $\mathbf{-1, 6, 25, 62}$.

Similar Problems (Solve Your Self) (2M)

2. Find the first four terms of the sequences whose n^{th} terms are given by.
   (ii) $a_n = (-1)^{n+1} n(n+1)$ (iii) $a_n = 2n^2 - 6$

6. If $a_1 = 1, a_2 = 1$ and $a_n = 2a_{n-1} + a_{n-2}$, $n \ge 3$, $n \in N$, then find the first six terms of the sequence.

Eg. 2.22: Find the first five terms of the following sequence. $a_1 = 1,\ a_2 = 1,\ a_n = \frac{a_{n-1}}{a_{n-2}+3}; n \ge 3, n \in N$

3. Find the n^{th} term of the following sequences. (2M)

(i) 2, 5, 10, 17, ...

$1^2 + 1 = 2$

$2^2 + 1 = 5$

$3^2 + 1 = 10$

$4^2 + 1 = 17$

$\vdots$

$n^2 + 1 = a_n$

$\therefore$ The n^{th} term is $\mathbf{n^2 + 1}$

Similar Problems (Solve Your Self) (2M)

3. Find the n^{th} term of the following sequences.
   (ii) $0, \frac{1}{2}, \frac{2}{3}, \ldots$ (iii) 3,8,13,18, ...

Eg. 2.20: Find the general term for the following sequences
(i) 3, 6, 9,..... (ii) $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots\ldots$ (iii) $5, -25, 125, \ldots\ldots$

5. Find a_8 and a_{15} whose n^{th} term is $a_n = \begin{cases} \frac{n^2-1}{n+3}; n \text{ is even}, n \in N \\ \frac{n^2}{2n+1}; n \text{ is odd}, n \in N \end{cases}$ (2M)

$n = 8$ (even)

$a_n = \frac{n^2-1}{n+3}$

$a_8 = \frac{8^2-1}{8+3} = \frac{64-1}{11} = \frac{63}{11}$

$n = 15$, (odd)

$a_n = \frac{n^2}{2n+1}$

$a_{15} = \frac{(15)^2}{2(15)+1} = \frac{225}{30+1} = \frac{225}{31}$

$\therefore \boldsymbol{a_8 = \frac{63}{11}}, \quad \boldsymbol{a_{15} = \frac{225}{31}}$

Similar Problems (Solve Your Self) (2M)

4. Find the indicated terms of the sequences whose n^{th} terms are given by.

(i) $a_n = \frac{5n}{n+2}$; a_6 and a_{13} (ii) $a_n = -(n^2-4)$; a_4 and a_{11}

Eg. 2.21: The general term of a sequence is defined as

$a_n = \begin{cases} n(n+3); n \in N \text{ is odd} \\ n^2+1 \quad ; n \in N \text{ is even} \end{cases}$

Find the eleventh and eighteenth terms.

CQ: Find the 3rd and 4th terms of a sequence, if

$a_n = \begin{cases} n^2, & \text{if } n \text{ is odd} \\ \frac{n^2}{2}, & \text{if } n \text{ is even} \end{cases}$ **(SEP-20)**

Exercise 2.5

1. Check whether the following sequences are in $A.P$.

(i) $a-3, a-5, a-7, \ldots$ (2M)

$d = t_2 - t_1 = a - 5 - (a-3) = \cancel{a} - 5 - \cancel{a} + 3$

$d = -2$....................(1)

$d = t_3 - t_2 = a - 7 - (a-5) = \cancel{a} - 7 + 5 - \cancel{a}$

$d = -2$ ....(2)

From (1) & (2), $t_2 - t_1 = t_3 - t_2$

$\therefore$ Given sequences is an $\boldsymbol{A.P}$.

Similar Problems (Solve Your Self) (2M)

1. Check whether the following sequences are in $A.P$. (ii) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$ (iii) $9, 13, 17, 21, 25, \ldots$

(iv) $\frac{-1}{3}, 0, \frac{1}{3}, \frac{2}{3}, \ldots$ (v) $1, -1, 1, -1, 1, -1, \ldots$

Eg.2.23: Check whether the following sequences are in A.P. or not? (2M)

(i) $x+2,\ 2x+3,\ 3x+4, \ldots.$

(ii) $2, 4, 8, 16, \ldots$ (iii) $3\sqrt{2},\ 5\sqrt{2},\ 7\sqrt{2}, 9\sqrt{2}, \ldots.$

2. First term a and common difference d are given below. Find the corresponding $A.P$.

(i) $a = 5, d = 6$

In general term of $A.P.$ $a, a+d, a+2d, a+3d, \ldots$

$5, (5+6), (5+(2\times 6)), 5+3(6), \ldots$

$5, 11,\ 5+12,\ 5+18, \ldots$

$5, 11, 17, 23, \ldots$

Similar Problems (Solve Your Self) (2M)

2. First term a and common difference d are given below. Find the corresponding $A.P$.

(ii) $a = 7, d = -5$ (iii) $a = \frac{3}{4}, d = \frac{1}{2}$

Eg. 2.24: Write an A.P. Whose first term is 20 and common difference is 8.

3. Find the first term and common difference of the Arithmetic progressions whose n^{th} terms are given below. (i) $t_n = -3 + 2n$ (2M)

Given: $t_n = -3 + 2n$

First term $a = t_1$

$= -3 + 2(1)$

$= -3 + 2$

$\boldsymbol{a = -1}$

$n = 2 \Rightarrow t_2 = -3 + 2(2)$

$= -3 + 4$

$t_2 = 1$

Common difference

$d = t_2 - t_1$

$= 1 - (-1)$

$= 1 + 1$

$\boldsymbol{d = 2}$

Similar Problems (2M)

Solve Your Self

3. Find the first term and common difference of the Arithmetic progressions whose n^{th} terms are given below. (ii) $t_n = 4 - 7n$

CQ: Find the common difference of an A.P. in which $t_{18} - t_{14} = 32$ **(PTA-6)**

4. Find the 19^{th} term of an $A.P.$ $-11, -15, -19, \dots$ MDL, JUL-22 2M

Given, $A.P$ is $-11, -15, -19, \dots$

$a = -11, d = t_2 - t_1 = -15 + 11$

$d = -4$

n^{th} term of $A.P$ $t_n = a + (n-1)d$

$n = 19 \Rightarrow t_{19} = -11 + (19-1)(-4)$

$= -11 + 18(-4) = -11 - 72$

$t_{19} = -83$

Similar Problems (Solve Your Self) 5M

Eg. 2.25: Find the 15^{th}, 24^{th} and n^{th} term (general term) of an A.P. given by $3, 15, 27, 39, \dots\dots$

Eg. 2.27: Determine the general term of an A.P. whose 7^{th} term is -1 and 16^{th} term is 17.

UE-6: Find the 12^{th} term from the last term of the A.P $-2, -4, -6, \dots. -100$

5. Which term of an A. P. $16, 11, 6, 1, \dots$ is -54 ? MAY-22 2M

Given: $A.P$ is $16,11,6,1,\dots$

$t_n = -54$, $a = 16$,

$d = t_2 - t_1 = 11 - 16 = -5$

$n = \left(\frac{l-a}{d}\right) + 1$

$= \left(\frac{-54-16}{-5}\right) + 1 = \left(\frac{-70}{-5}\right) + 1$

$n = 14 + 1 = 15$

$\therefore \boldsymbol{t_{15} = -54}$

Similar Problems (Solve Your Self) 2M

Eg. 2.26: Find the number of terms in the A.P. $3, 6, 9, 12, \dots\dots 111$. **(SEP-21)**

CQ: Which term of the A.P 21, 18, 15, … is -81? State with reason is there any term 0 in this A.P.? **(PTA-5)**

6. Find the middle term(s) of an $A.P.$ $9, 15, 21, 27, \dots, 183$. PTA-1 2M

Given: $A.P$ is $9,15,21,27,\dots 183$.

$a = 9,\quad d = t_2 - t_1 = 15 - 9$

$l = 183,\ d = 6$

The number of term in $A.P.$

$n = \frac{l-a}{d} + 1$

$= \frac{183-9}{6} + 1 = \frac{174}{6} + 1 = 29 + 1$

$n = 30$

$n = 30$ even,

The middle term $= \frac{n}{2}^{th}$ term and $\left(\frac{n}{2}+1\right)^{th}$ term

$= \frac{30}{2}$ term and $\frac{30}{2} + 1$ term

$= 15^{th}$ term and 16^{th} term

$t_n = a + (n-1)d$

$n = 15 \Rightarrow t_{15} = 9 + (15-1)(6) = 9 + (14)(6)$

$t_{15} = 93$

$n = 16 \Rightarrow t_{16} = 9 + (16-1)(6) = 9 + (15)(6)$

$t_{16} = 99$

$\therefore$ The middle terms are $\boldsymbol{t_{15} = 93}$, $\boldsymbol{t_{16} = 9}$

7. If nine times ninth term is equal to the fifteen times fifteenth term, show that six times twenty fourth term is zero. 5M JUL-22

Given: $9t_9 = 15t_{15}$ ($\because t_n = a + (n-1)d$)

$9[a + (9-1)]d = 15[a + (15-1)d]$

$9(a + 8d) = 15(a + 14d)$

$9a + 72d = 15a + 210d$

$9a - 15a + 72d - 210d = 0$

$-6a - 138d = 0$

$-6(a + 23d) = 0$

$6[a + (24-1)d] = 0$

Hence proved. $6t_{24} = 0$

8. If $3 + k, 18 - k, 5k + 1$ are in $A.P$ then find k. 2M SEP-21, PTA-3, 5

Given: $3 + k, 18 - k, 5k + 1$ are in $A.P.$

i.e., $d = t_2 - t_1 = t_3 - t_2$

$18 - k - (3 + k) = 5k + 1 - (18 - k)$

$18 - k - 3 - k = 5k + 1 - 18 + k$

$15 - 2k = 6k - 17$

$15 + 17 = 6k + 2k$

$32 = 8k$

$k = 4$

Similar Problems (Solve Your Self) 2M

9. Find x, y & z given that the numbers $x, 10, y, 24, z$ are in $A.P.$

10. In a theatre, there are 20 seats in the front row and 30 rows were allotted. Each, successive row contains two additional seats than its front row. How many seats are there in the last row?

2M PTA-4

Given: 30 rows were allotted in the theatre

$n = 30$

20 seats in the front row then $a = 20$

2 seats increased in each row.

Thus, $1^{st}, 2^{nd}, 3^{rd}, \ldots$ 30 rows are 20,22,24, … respectively.

It is an $A.P.$ $d = t_2 - t_1 = 22 - 20 = 2$

To find: t_{30}

$t_n = a + (n-1)d$

$t_{30} = 20 + (30-1)2$

$= 20 + (29)(2)$

$= 20 + 58 = 78$

78 seats in the last row.

Similar Problems (Solve Your Self) 5M

UE-5: If $(m+1)^{th}$ term of an A.P is twice the $(n+1)^{th}$ term, then prove that $(3m+1)^{th}$ term is twice the $(m+n+1)^{th}$ term.

UE-7: Two A.P.'s have the same common difference. The first term of one A.P. is 2 and that of the other is 7. Show that the difference between their 10^{th} terms is the same as the difference between their 21^{st} terms, Which is the same as the difference between any two corresponding terms.

11. The sum of three consecutive terms that are in $A.P.$ is 27 and their product is 288. Find the three terms.

SEP-21 5M

Let the three consecutive terms be

$a-d,\ a,\ a+d$

Given: $a - d + a + a + d = 27$

$3a = 27 \Rightarrow \quad a = 9$

Also, $(a-d)(a)(a+d) = 288$

$(a^2 - d^2)a = 288$

$(9^2 - d^2) = \frac{288}{9}$

$81 - d^2 = 32$

$-d^2 = 32 - 81$

$-d^2 = -49 \Rightarrow\ d = \pm 7$

When $a = 9, d = 7$ the $A.P$ is

$9-7,\ 9,\ 9+7$

$\mathbf{2},\ \mathbf{9},\ \mathbf{16}$

When $a = 9, d = -7$

$9+7,\ 9,\ 9-7$

$\mathbf{16},\ \mathbf{9},\ \mathbf{2}.$

Similar Problems (Solve Your Self) 5M

Eg. 2.29: In an A.P., sum of four consecutive terms is 28 and their sum of their squares is 276. Find the four numbers.

12. The ratio of 6^{th} and 8^{th} term of an $A.P.$ is $7:9$. Find the ratio of 9^{th} term to 13^{th} term.

MDL MAY-22 5M

Given: $t_6 : t_8 = 7:9 \Rightarrow \frac{t_6}{t_8} = \frac{7}{9}$

$\frac{a+(6-1)d}{a+(8-1)d} = \frac{7}{9} \quad [\because t_n = a + (n-1)d]$

$9(a+5d) = 7(a+7d)$

$9a + 45d = 7a + 49d$

$9a - 7a = 49d - 45d$

$2a = 4d$

$a = 2d$ ……………(1)

To find, $\quad t_9 : t_{13} = \frac{t_9}{t_{13}}$

$= \frac{a+(9-1)d}{a+(13-1)d}$

$= \frac{a+8d}{a+12d}$

$= \frac{2d+8d}{2d+12d}$

$= \frac{10d}{14d} = \frac{5}{7}$

$\therefore \boldsymbol{t_9 : t_{13} = 5:7}$

Similar Problems (Solve Your Self) 5M

Eg. 2.28: If l^{th}, m^{th} and n^{th} terms of an A.P. are x, y, z respectively, then show that

(i) $x(m-n) + y(n-l) + z(l-m) = 0$ (ii) $(x-y)n + (y-z)l + (z-x)m = 0$ **(MAY-22)**

CQ: If the sum of the first p terms of an A.P. is $ap^2 + bq$. Find its common difference. **(PTA-6)**

13. In a winter season let us take the temperature of Ooty from Monday to Friday to be in $A.P$. The sum of temperatures from Monday to Wednesday is $0°C$ & the sum of the temperatures from Wednesday to Friday is $18°C$. Find the temperature on each of the five days. (5M)

Let $a, a+d, a+2d, a+3d, a+4d$ be the temperature of Ooty from Monday to Friday respectively.

Given: the sum of temperatures from Monday to Wednesday,

$$a+a+d+a+2d = 0$$
$$3a+3d = 0$$
$$a+d = 0$$
$$a = -d \text{ ....(1)}$$

Also, the sum of Temperatures from Wednesday to Friday.

$$a+2d+a+3d+a+4d = 18$$
$$3a+9d = 18$$
$$\text{(Sub } a=-d) \quad -3d+9d = 18$$
$$6d = 18$$
$$d = 3$$

Sub $d=3$ in (1), $a=-3$, The temperatures on each day, $a+d, a+2d, a+3d, a+4d$

$(-3), (-3+3), (-3+2(3)), (-3+3(3)), (-3+3(4))$

$-3, 0, -3+6, -3+9, -3+12,$

$\mathbf{-3°C, 0°C, 3°C, 6°C, 9°C}$

> **Similar Problems (Solve Your Self)** (5M)
>
> 14. Priya earned ₹15,000 in the first month. Thereafter her salary increased by ₹ 1500 per year. Her expenses are ₹ 13,000 during the first month and the expenses increases by ₹ 900 per year. How long will it take for her to save ₹ 20,000 per months.
>
> **Eg. 2.30:** A mother divides ₹ 207 into three parts such that the amount are in A.P. and gives it to her three children. The product of the two least amounts that the children had ₹ 4623. Find the amount received by each child.

Exercise 2.6

1. Find the sum of the following.

(i) 3,7,11, ... upto 40 terms (5M)

Given: 3,7,11, ... is an $A.P$.

$$a=3, d=t_2-t_1=7-3=4.\ n=40$$
$$S_n = \frac{n}{2}[2a+(n-1)d]$$
$$S_{40} = \frac{40}{2}[2(3)+(40-1)4]$$
$$= 20[6+39(4)]$$
$$= 20(6+156)$$
$$= 20(162) = \mathbf{3240}$$

> **Similar Problems (Solve Your Self)** (5M)
>
> 1. Find the sum of the following. (ii) 102,97,92, ... upto 27 terms.
>
> **Eg. 2.31:** Find the sum of first 15 terms of the A.P. $8, 7\frac{1}{4}, 6\frac{1}{2}, 5\frac{3}{4}, \ldots\ldots$

(iii) $6+13+20+\cdots+97$ (5M)

Given: $6+13+20+\cdots+97$.

$$a=6$$
$$d=t_2-t_1=13-6=7,\ l=97$$
$$n=\frac{l-a}{d}+1$$
$$n=\frac{97-6}{7}+1=\frac{91}{7}+1=13+1=14$$
$$S_n=\frac{n}{2}[a+l]$$
$$S_{14}=\frac{14}{2}[6+97] \quad =7[103]$$
$$\mathbf{S_{14}=721}$$

> **Similar Problems (Solve Your Self)** (5M)
>
> **Eg. 2.32:** Find the sum of $0.40+0.43+0.46+\ldots\ldots+1$

2. How many consecutive odd integers beginning with 5 will sum to 480? (5M)

Given: consecutive odd integers beginning with 5.

Then the series, $5+7+9+\cdots$

$$a=5, \quad d=t_2-t_1=7-5=2$$
$$S_n=480$$
$$\frac{n}{2}[2a+(n-1)d]=480$$
$$\frac{n}{2}[2(5)+(n-1)2]=480$$
$$\frac{2n}{2}[5+n-1]=480$$
$$n(4+n)=480$$
$$n^2+4n-480=0$$
$$(n-20)(n+24)=0$$
$$n-20=0 \text{ (or) } n+24=0$$
$$n=20 \text{ (or) } n=-24$$

$n=-24$ is not admissible, then $n=20$.

∴ The sum of **20** consecutive odd integers is 480

$$S_{20}=480$$

> **Similar Problems (Solve Your Self)** (2M)
>
> **Eg. 2.33:** How many terms of the series $1+5+9+\ldots$.. must be taken so that their sum is 190?

3. Find the sum of first 28 terms of an $A.P.$ whose n^{th} term is $4n-3$.

Given, $t_n = 4n-3$ (2M)

$n=1 \Rightarrow t_1 = 4(1)-3 = 4-3$

$t_1 = a = 1$

$n=2 \Rightarrow t_2 = 4(2)-3 = 8-3$

$t_2 = 5$

$n=3 \Rightarrow t_3 = 4(3)-3 = 12-3$

$t_3 = 9$

$a=1, d = t_2 - t_1 = 5-1$

$d=4, \quad n=28$

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{28} = \frac{28}{2}[2(1)+(28-1)4]$

$= 14[2+27(4)]$

$= 14[2+108]$

$= 14(110)$

$\boldsymbol{S_{28} = 1540}$

$\therefore$ The sum of first 28 terms is 1540.

Similar Problems (Solve Your Self)

4. The sum of first n terms of a certain series is given as $2n^2-3n$. Show that the series is an $A.P.$ (5M)

Eg. 2.35: In a A.P. the sum of first n terms is $\frac{5n^2}{2}+\frac{3n}{2}$. Find the 17^{th} term. (2M)

5. The 104^{th} term and 4^{th} term of an $A.P.$ are 125 and 0. Find the sum of first 35 terms. (5M)

Given: $t_{104} = 125, t_4 = 0$

$t_n = a+(n-1)d$

$t_4 = 0$

$a+(4-1)d = 0$

$a+3d = 0$

$a = -3d \quad ...(1)$

$t_{104} = 125$

$a+(104-1)d = 125$

$a+103d = 125$

Sub $a = -3d$

$-3d+103d = 125$

$100d = 125 \Rightarrow d = \frac{125}{100} \Rightarrow d = \frac{5}{4}$

Similar Problems (Solve Your Self)

Eg. 2.34: The 13^{th} term of an A.P. is 3 and the sum of first 13 terms is 234. Find the common difference and the sum of first 21 terms. (5M)

Sub $d = \frac{5}{4}$ in equation (1)

$a = -3\left(\frac{5}{4}\right) = -\frac{15}{4}$

To find, the sum of first 35 terms, $n = 35$.

$S_n = \frac{n}{2}[2a+(n-1)d]$

$S_{35} = \frac{35}{2}\left[2\left(\frac{-15}{4}\right)+(35-1)\frac{5}{4}\right]$

$= \frac{35}{2}\left[\frac{-15}{2}+34\left(\frac{5}{4}\right)\right] = \frac{35}{2}\left[\frac{-15+85}{2}\right]$

$= \frac{35}{2}\left(\frac{70}{2}\right)$

$S_{35} = \frac{1225}{2}$

$\boldsymbol{S_{35} = 612.5}$

7. Find the sum of all natural numbers between 602 and 902 which are not divisible by 4. (5M)

Given series is between 602 and 902

$603+604+605+\cdots+901$

$a = 603, d = 1, l = 901$

$n = \frac{l-a}{d}+1$

$= \frac{901-603}{1}+1 = 298+1$

$n = 299$

$S_n = \frac{n}{2}[a+l]$

$S_{299} = \frac{299}{2}(603+901)$

$= \frac{299}{2}(1504) = 299(752)$

$S_{299} = 224848$

The sum of the numbers between 602 and 902 which are divisible by 4.

$604+608+612+\cdots+900$

$a = 604, d = 4, l = 900.$

$n = \frac{l-a}{d}+1 = \frac{900-604}{4}+1$

$n = \frac{296}{4}+1$

$= 74+1 = 75$

$S_n = \frac{n}{2}(a+l)$

$S_{75} = \frac{75}{2}(604+900)$

$= \frac{75}{2}(1504)$

$S_{75} = 56400$

The sum of the number which are not divisible by 4.

$= S_{299} - S_{75}$

$= 224848 - 56400$

$= \mathbf{168448}$

Similar Problems (Solve Your Self)

6. Find the sum of all odd positive integers less than 450. (5M)

Eg. 2.36: Find the sum of all natural numbers between 300 and 600 which are divisible by 7.

CQ: Find the sum of all natural numbers between 100 and 1000 which are divisible by 11? **(SEP-20)**

CQ: Find the sum of all 3 digit natural numbers which are divisible by 9 **(PTA-3)**

8. Raghu wish to buy a laptop. He can by it by paying ₹ 40,000 cash or by giving it in 10 installments as ₹ 4800 in the first month, ₹ 4750 in the second month, ₹ 4700 in the third month and so on. If he pays the money in this fashion, find (i) total amount paid in 10 installments. (ii) How much extra amount that he has to pay than the cost ? (5M)

Given, The cost of laptop ₹40000. He wants to buy it in 10 installments. 1st, 2nd, 3rd,…10 installments are ₹ 4800, ₹ 4750, ₹ 4700… respectively.

$ie, 4800 + 4750 + 4700 + \cdots$ is an A. P

$a = 4800,$

$d = t_2 - t_1 = 4750 - 480 = -50$

(i) $n = 10$, $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{10} = \frac{10}{2}[2(4800) + (10-1)(-50)]$

$= \frac{10}{2} \times 2[4800 + (9)(-25)]$

$= 10[4800 - 225] = 10[4575] = 45750.$

$\therefore$ He paid in 10 installments is **₹ 45750.**

(ii) He paid the extra amount $= 45750 - 40000 =$ **₹ 5750**

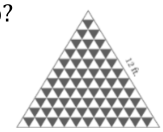
Similar Problems (Solve Your Self) (5M)

UE-8: A man saved ₹ 16500 in ten years. In each year after the first he saved ₹ 100 more than he did in the preceding year. How much did he save in the first year? **(PTA-4)**

9. A man repays a loan of ₹ 65,000 by paying ₹ 400 in the first month and then increasing the payment by ₹ 300 every month. How long will it take for him to clear the loan?

10. A brick staircase has a total of 30 steps. The bottom step requires 100 bricks. Each successive step requires two bricks less than the previous step. (i) How many bricks are required for the top most step? (ii) How many bricks are required to build the stair case?

Eg. 2.37: A mosaic is designed in the shape of an equilateral triangle, 12 ft on each side. Each tile in the mosaic is in the shape of an equilateral triangle of 12 inch side. The tiles are alternate in colour as shown in the figure. Find the number of tiles of each colour and total number of tiles in the mosaic.



Eg. 2.38: The houses of a street are numbered from 1 to 49. Senthil's house is numbered such that the sum of number of the houses prior to Senthil's house is equal to the sum of numbers of the houses following Senthil's house. Find Senthil's house number? **(APR-23, PTA-2)**

11. If $S_1, S_2, S_3, \ldots S_m$ are the sums of n terms of m A.P.'s whose first terms are $1, 2, 3, \ldots m$ and whose common differences are $1, 3, 5, \ldots, (2m-1)$ respectively, then show that $S_1 + S_2 + S_3 + \cdots + S_m = \frac{1}{2}mn(mn+1)$ (5M)

$S_1 = 1 + 2 + 3 + \cdots + n, \quad a = 1, \quad d = 1$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_1 = \frac{n}{2}[2(1) + (n-1)(1)] = \frac{n}{2}[2 + n - 1]$

$S_1 = \frac{n}{2}[n+1]$

$S_2 \Rightarrow a = 2, d = 3$

$S_2 = \frac{n}{2}[2(2) + (n-1)3] = \frac{n}{2}[4 + 3n - 3]$

$S_2 = \frac{n}{2}[3n+1]$

$S_3 \Rightarrow a = 3, d = 5$

$S_3 = \frac{n}{2}[2(3) + (n-1)5] = \frac{n}{2}[6 + 5n - 5]$

$S_3 = \frac{n}{2}[5n+1]$

$S_1 + S_2 + S_3 + S_4 + \cdots + S_m$

$= \frac{n}{2}[n+1] + \frac{n}{2}[3n+1] + \frac{n}{2}[5n+1] + \cdots + m$ terms

$t_2 - t_1 = \frac{n}{2}(3n+1) - \frac{n}{2}(n+1)$

$= \frac{n}{2}[3n + 1 - n - 1] = \frac{n}{2}[2n] = n^2$

$t_3 - t_2 = \frac{n}{2}(5n+1) - \frac{n}{2}(3n+1)$

$= \frac{n}{2}[5n + 1 - 3n - 1] = \frac{n}{2} \times 2n = n^2$

$\therefore$ It forms an A.P

$a = \frac{n}{2}(n+1), d = n^2$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_m = \frac{m}{2}\left[2\left(\frac{n}{2}\right)(n+1) + (m-1)n^2\right]$

$= \frac{mn}{2}[n + 1 + (m-1)n]$

$= \frac{mn}{2}[n + 1 + mn - n]$

$= \frac{mn}{2}[mn+1] = \frac{1}{2}mn[mn+1]$

Similar Problems (Solve Your Self) (5M)

12. Find the sum $\left[\frac{a-b}{a+b} + \frac{3a-2b}{a+b} + \frac{5a-3b}{a+b} + \cdots \text{to 12 terms}\right]$

Eg. 2.39: The sum first n, $2n$ and $3n$ terms of an A.P. are S_1, S_2 and S_3 respectively. Prove that $S_3 = 3(S_2 - S_1)$. **(MDL)**

Exercise 2.7

1. Which of the following sequences are in G. P? 2M

(i) 3,9,27,81, …

$a = 3, r = \frac{t_2}{t_1} = \frac{9}{3} = 3$

$r = \frac{t_3}{t_2} = \frac{27}{9} = 3$

$r = \frac{t_4}{t_3} = \frac{81}{27} = 3$

$\frac{t_2}{t_1} = \frac{t_3}{t_2} = \frac{t_4}{t_3} = r$

Given sequence is **a G.P**

(ii) 4,44,444,4444, …

$r = \frac{t_2}{t_1} = \frac{44}{4} = 11$

$\frac{t_3}{t_2} = \frac{444}{44} = \frac{111}{11}$

$\frac{t_4}{t_3} = \frac{4444}{444} = \frac{1111}{111}$

$\frac{t_2}{t_1} \neq \frac{t_3}{t_2} \neq \frac{t_4}{t_3}$

Given sequence is **not a G.P**

Similar Problems (Solve Your Self) 2M

1. Which of the following sequences are in G. P?
   (iii) 0.5,0.05,0.005, … (iv) $\frac{1}{3}, \frac{1}{6}, \frac{1}{12}$, ….
   (v) 1, −5,25, −125, … (vi) 120,60,30,18, …
   (vii) 16,4,1, $\frac{1}{4}$, …

Eg. 2.40: Which of the following sequences form a Geometric Progression?
(i) 7, 14, 21, 28,……… (ii) $\frac{1}{2}$, 1, 2, 4,……
(iii) 5, 25, 50, 75,……

2. Write the first three terms of the *G. P.* whose first term and the common ratio are given below. (i) $a = 6, r = 3$ 2M

The First three terms of $G.P$ are a, ar, ar^2

$6, 6(3), 6(3)^2$

First 3 terms **6, 18, 54**

Similar Problems (Solve Your Self) 2M

2. Write the first three terms of the $G.P.$ whose first term and the common ratio are given below.
   (ii) $a = \sqrt{2}, r = \sqrt{2}$ (iii) $a = 1000, r = \frac{2}{5}$

Eg. 2.41: Find the geometric progression whose first term and common ratios are given by
(i) $a = -7,\ r = 6$ **(PTA-5)** (ii) $a = 256,\ r = 0.5$

3. In a G. P 729, 243, 81, … find t_7. 2M

Given: G. P is 729,243,81, …

$a = 729, r = \frac{243}{729} = \frac{1}{3},\ n = 7$

$t_n = ar^{n-1}$

$t_7 = 729\left(\frac{1}{3}\right)^{7-1}$

$= 729\left(\frac{1}{3}\right)^{6}$

$= 729\left(\frac{1}{729}\right)$

$\boldsymbol{t_7 = 1}$

Similar Problems (Solve Your Self) 2M

Eg. 2.42: Find the 8^{th} term of the G.P. 9, 3, 1,…… **(JUN-23)**

CQ: In a G.P $\frac{1}{4}, -\frac{1}{2}, 1, -2, \ldots$ find t_{10} **(PTA - 4)**

4. Find x so that $x + 6, x + 12$ and $x + 15$ are consecutive terms of a Geometric progression. APR-23 2M

Given: $x + 6, x + 12, x + 15$ are in $G.P$

$\frac{t_2}{t_1} = \frac{t_3}{t_2}$

$\frac{x+12}{x+6} = \frac{x+15}{x+12}$

$(x + 12)(x + 12) = (x + 15)(x + 6)$

$x^2 + 12x + 12x + 144 = x^2 + 6x + 15x + 90$

$x^2 + 24x + 144 = x^2 + 21x + 90$

$x^2 + 24x - 21x + 144 - 90 - x^2 = 0$

$3x + 54 = 0$

$3x = -54$

$\boldsymbol{x = -18}$

5. Find the number of terms in the following *G. P.* (i) 4, 8, 16, … , 8192 ? 2M

Given 4,8,16, … ,8192

$a = 4, r = \frac{t_2}{t_1} = \frac{8}{4} = 2, t_n = 8192$

$t_n = ar^{n-1}$

$8192 = 4(2)^{n-1}$

$\frac{8192}{4} = (2)^{n-1}$

$2048 = 2^{n-1}$

$2^{11} = 2^{n-1}$

$11 = n - 1$

$n = 12$

$t_{12} = 8192$

∴ Number of terms = **12**

Similar Problems (Solve Your Self) 2M

5. Find the number of terms in the following $G.P.$
   (ii) $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots, \frac{1}{2187}$

6. In a $G.P$ the 9th term is 32805 and 6th term is 1215. Find the 12th term. (5M)

Given: $t_9 = 32805,\ t_6 = 1215$

To find t_{12}

$t_n = a(r)^{n-1}$

$t_9 = ar^{9-1} = 32805$

$ar^8 = 32805$ ....(1)

$t_6 = ar^{6-1} = 1215$

$t_6 = ar^5 = 1215$ ....(2)

$\frac{t_9}{t_6} = \frac{ar^8}{ar^5}$

$r^3 = \frac{32805}{1215}$

$r^3 = 27$

$r = 3$

Sub (2) $\Rightarrow\ a(3)^5 = 1215 \Rightarrow a = \frac{1215}{243} = 5$

$t_{12} = a\,r^{12-1} \Rightarrow\ \boldsymbol{t_{12} = 5(3^{11})}$

7. Find the 10th term of a $G.P$ whose 8th term is 768 and the common ratio is 2. (2M)

Given: $t_8 = 768$ and $r = 2$

$ar^{8-1} = 768$ $(\because t_n = ar^{n-1})$

$a(2)^7 = 768$ $(\because 2^7 = 128)$

$a = \frac{768}{128}$

$a = 6$

To find $t_{10} = ar^{10-1}$

$= 6(2)^9$

$= 6(512)$

$\boldsymbol{t_{10} = 3072}$

Similar Problems (Solve Your Self) (5M)

Eg.2.43: In a Geometric progression, the 4^{th} term is $\frac{8}{9}$ and the 7^{th} term is $\frac{64}{243}$. Find the G.P.

UE-9: Find the G.P in which the 2nd term is $\sqrt{6}$ and the 6th term is $9\sqrt{6}$

9. In a $G.P$. the product of three consecutive term is 27 and the sum of the product of two terms taken at a time is $\frac{57}{2}$. Find the three terms. (5M)

Let $\frac{a}{r}, a, ar$ be the three consecutive term is G.P

Given, $\frac{a}{r} \times a \times ar = 27$

$a^3 = 27$

$a = 3$

Also, $\frac{a}{r} \times a + a \times ar + ar \times \frac{a}{r} = \frac{57}{2}$

$a^2\left(\frac{1}{r} + r + 1\right) = \frac{57}{2}$

$(3)^2\left(\frac{1+r^2+r}{r}\right) = \frac{57}{2}$

$2\left(\frac{r^2+r+1}{r}\right) = \frac{57}{9}$

$2\left(\frac{r^2+r+1}{r}\right) = \frac{19}{3}$

$6[r^2 + r + 1] = 19r$

$6r^2 + 6r - 19r + 6 = 0$

$6r^2 - 13r + 6 = 0$

$\left(r - \frac{3}{2}\right)\left(r - \frac{2}{3}\right) = 0$

$r = \frac{3}{2},\ r = \frac{2}{3}$

When $a = 3, r = \frac{3}{2}$

$\frac{3}{3/2}, 3, 3\left(\frac{3}{2}\right)$

$2,\ 3, \frac{9}{2}$

When $a = 3, r = \frac{2}{3}$

$\frac{3}{2/3}, 3, 3\left(\frac{2}{3}\right)$

$\frac{9}{2}, 3, 2$

$\therefore$ The three terms are $\frac{9}{2}$, 3,2, (or)2,3, $\frac{9}{2}$

Similar Problems (Solve Your Self) (5M)

Eg. 2.44: The product of three consecutive terms of a Geometric progression is 343 and their sum is $\frac{91}{3}$. Find the three terms.

10. A man joined a company as Assistant Manager. The company gave him a starting salary of ₹ 60,000 & agreed to increase his salary 5% annually. What will be his salary after 5 years?

5M PTA-6

The starting salary of man is ₹ 60,000.

His salary increased 5% annually.

$P = 60000$, $r = 5\%$, $n = 5$ years

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$= 60000\left(1+\frac{5}{100}\right)^5$$

$$= 60000\left(\frac{21}{20}\right)^5$$

$$= 60000\left(\frac{21\times21\times21\times21\times21}{20\times20\times20\times20\times20}\right)$$

$$= \frac{12252303}{160} = 76576.89$$

$A =$ ₹ 76577

His salary will be after 5 years is ₹ 76577

Similar Problems (Solve Your Self) 5M

11. Sivamani is attending an interview for a job and the company gave two offers to him. Offer A: ₹ 20,000 to start with followed by a guaranteed annual increase of 6% for the first 5 years. Offer B: ₹ 22,000 to start with followed by a guaranteed annual increase of 3% for the first 5 years. What is his salary in the 4th year with respect to the offers A and B ?

Eg. 2.45: The present value of a machine is ₹40,000 and its value depreciates each year by 10%. Find the estimated value of the machine in the 6^{th} year.

UE-10: The value of motor cycle depreciates at the rate of 15% per year. What will be the value of the motor cycle 3 year hence, which is now purchased for ₹ 45000?

12. If a, b, c are three consecutive terms of an A.P and x, y, z are three consecutive terms of a G.P then prove that $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$.

5M

Given: a, b, c are in A.P $\Rightarrow 2b = a + c$

x, y, z are in G.P then $\frac{y}{x} = \frac{z}{y} = k$

$y^2 = xz$

$\frac{y}{x} = k$ $\qquad$ $\frac{z}{y} = k$

$y = xk$ $\qquad$ $z = yk$

$z = (xk)k$

$z = xk^2$

Similar Problems (Solve Your Self) 2M

8. If a, b, c are in $A.P$. then show that $3^a, 3^b, 3^c$ are in $G.P$.

To prove: $x^{b-c} \times y^{c-a} \times z^{a-b} = 1$

LHS : $x^{b-c} \times y^{c-a} \times z^{a-b}$

$$= x^{b-c} \times (xk)^{c-a} \times (xk^2)^{a-b}$$

$$= x^{b-c} \times x^{c-a} \times k^{c-a} \times x^{a-b} \times (k^2)^{a-b}$$

$$= x^{b-c+c-a+a-b} \times k^{c-a} \times k^{2a-2b}$$

$$= x^0\, k^{c-a+2a-2b}$$

$$= (1)k^{c+a-(c+a)}$$

$$= k^{c+a-c-a} = k^0 = 1 \quad \text{RHS}$$

$$\boldsymbol{x^{b-c} \times y^{c-a} \times z^{a-b} = 1}$$

Hence proved.

Exercise 2.8

1. Find the sum of first n terms of the G.P 5M

(i) $5, -3, \frac{9}{5}, -\frac{27}{25}, \ldots$

Given: $5, -3, \frac{9}{5}, -\frac{27}{25}, \ldots$

$a = 5, r = \frac{t_2}{t_1} = -\frac{3}{5}, r < 1$

$$S_n = a\left[\frac{1-r^n}{1-r}\right] = 5\left[\frac{1-\left(-\frac{3}{5}\right)^n}{1-\left(-\frac{3}{5}\right)}\right] = 5\left[\frac{1-\left(-\frac{3}{5}\right)^n}{\frac{5+3}{5}}\right]$$

$$= 5\times\frac{5}{8}\left[1-\left(-\frac{3}{5}\right)^n\right]$$

$$S_n = \frac{25}{8}\left[1-\left(-\frac{3}{5}\right)^n\right]$$

Similar Problems (Solve Your Self) 5M

1. Find the sum of first n terms of the G.P (ii) 256, 64, 16,....
2. Find the sum of first six terms of the G.P 5, 15, 45, ...

Eg. 2.46: Find the sum of 8 terms of the G.P. $1, -3, 9, -27, \ldots$.

3. **Find the first term of the G.P whose common ratio 5 and whose sum to first 6 terms is 46872.**

Given: In a G.P (5M)

$r = 5$ and $S_6 = 46872$

$S_n = a\left[\frac{r^n-1}{r-1}\right], r > 1$

$S_6 = a\left[\frac{5^6-1}{5-1}\right]$

$46872 = a\left[\frac{15625-1}{4}\right]$

$46872 = a\left[\frac{15624}{4}\right]$

$46872 = a(3906) \Rightarrow a = \frac{46872}{3906} = \mathbf{12}$

Similar Problems
(Solve Your Self)
Eg. 2.47: Find the first term of a G.P. in which $S_6 = 4095$ & $r = 4$. (2M)
Eg. 2.53: A person saved money every year, half as much as he could in the previous year. If he had totally saved ₹ 7875 in 6 years then how much did he save in the first year? (5M)

4. **Find the sum to infinity of (i) $9 + 3 + 1 + \cdots$** (5M)

Given infinity series is $9 + 3 + 1 + \cdots$

$S_n = \frac{a}{1-r}, -1 < r < 1$

$a = 9,\ r = \frac{t_2}{t_1} = \frac{3}{9}$

$S_\infty = \frac{9}{1-\frac{3}{9}} = \frac{9}{\frac{9-3}{9}} = 9 \times \frac{9}{6} = \mathbf{\frac{27}{2}}$

Similar Problems (Solve Your Self) (2M)
4. Find the sum to infinity of (ii) $21 + 14 + \frac{28}{3} + \cdots$
5. If the first term of an infinite G.P is 8 and its sum to infinity is $\frac{32}{3}$ then find the common ratio.
Eg. 2.49: Find the sum $3 + 1 + \frac{1}{3} + \cdots \infty$

6. **Find the sum to n terms of the series** (5M)

(i) $0.4 + 0.44 + 0.444 + \cdots$ to n terms

Given series is $0.4 + 0.44 + 0.444 + \cdots + n$ terms

$S_n = 4(0.1 + 0.11 + 0.111 + \cdots + n \text{ terms})$

Multiply and divide by 9

$= \frac{4}{9}[0.9 + 0.99 + 0.999 + \cdots + n \text{ terms}]$

$= \frac{4}{9}[(1-0.1) + (1-0.01) + (1-0.001) + \cdots + n \text{ terms}]$

$= \frac{4}{9}[(1 + 1 + \cdots + n \text{ terms}) - (0.1 + 0.01 + 0.001 + \cdots + n \text{ terms})]$

$= \frac{4}{9}\left[(1 + 1 + \cdots + n \text{ terms}) - \left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots + n \text{ terms}\right)\right]$

Similar Problems (Solve Your Self) (5M)
6. Find the sum to n terms of the series
(ii) $3 + 33 + 333 + \cdots$ to n terms **(JUN-23)**
Eg. 2.51: Find the sum to n terms of the series $5 + 55 + 555 + \ldots$ **(APR-23, PTA-4)**
Eg. 2.48: How many terms of the series $1 + 4 + 16 + \ldots$ make the sum 1365 ?

Consider $1 + 1 + \cdots + n$ terms	Consider $\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \cdots + n\ terms$
$a = 1, r = \frac{t_2}{t_1} = 1$ $S_n = na$ $1 + 1 + \cdots + n$ terms $= n(1) = n$	$a = \frac{1}{10}, r = \frac{t_2}{t_1} = \frac{\frac{1}{100}}{\frac{1}{10}} = \frac{1}{100} \times \frac{10}{1} = \frac{1}{10} \Rightarrow r = \frac{1}{10}$ $S_n = a\left(\frac{1-r^n}{1-r}\right) = \frac{1}{10}\left[\frac{1-\left(\frac{1}{10}\right)^n}{1-\frac{1}{10}}\right]$

$S_n = \frac{4}{9}\left[n - \frac{1}{10}\left(\frac{1-\left(\frac{1}{10}\right)^n}{1-\frac{1}{10}}\right)\right]$

$= \frac{4}{9}\left[n - \frac{1}{10}\left(\frac{1-\left(\frac{1}{10}\right)^n}{\frac{9}{10}}\right)\right]$

$= \frac{4}{9}\left[n - \left\{\frac{1}{10} \times \frac{10}{9}\left(1 - \left(\frac{1}{10}\right)^n\right)\right\}\right]$

$S_n = \mathbf{\frac{4n}{9} - \frac{4}{81}\left(1 - \left(\frac{1}{10}\right)^n\right)}$

7. Find the sum of the Geometric series $3+6+12+\cdots+1536$. PTA-3 5M

Given geometric series

$$3+6+12+\cdots+1536$$

$a=3, r=\frac{t_2}{t_1}=\frac{6}{3}=2, l=1536$

$t_n=ar^{n-1}$ (n term is 1536)

$1536=3(2)^{n-1}$

$\frac{1536}{3}=2^{n-1}$

$512=2^{n-1}$

$2^9=2^{n-1}$

$9=n-1$

$n=10$

$S_n=a\left[\frac{r^n-1}{r-1}\right], r>1$

$S_{10}=3\left[\frac{2^{10}-1}{2-1}\right]=3(1024-1)=3(1023)$

$= \mathbf{3069}$

Similar Problems (Solve Your Self) 5M

Eg. 2.50: Find the rational form of the number 0.6666 ....

9. Find the rational form of the number $0.\overline{123}$ 5M

$0.\overline{123}=0.123123123\ldots$

$=0.123+0.000123+0.000000123+\cdots$

$=\frac{123}{1000}+\frac{123}{1000000}+\frac{123}{1000000000}+\cdots$

$=\frac{123}{1000}\left[1+\frac{1}{1000}+\frac{1}{1000000}+\cdots\right]$

$1+\frac{1}{1000}+\frac{1}{1000000}+\cdots$ is an infinite series

$S_n=\frac{a}{1-r}, a=1, r=\frac{1}{1000}$

$0.\overline{123}=\frac{123}{1000}\left[\frac{1}{1-\frac{1}{1000}}\right]=\frac{123}{1000}\left[\frac{1}{\frac{1000-1}{1000}}\right]$

$=\frac{123}{1000}\left(\frac{1000}{999}\right)=\frac{123}{999}$

$0.\overline{123}=\mathbf{\frac{41}{333}}$

8. Kumar writes a letter to four of his friends. He asks each one of them to copy the letter and mail to four different persons with the instruction that they continue the process similarly. Assuming that the process is unaltered and it costs ₹ 2 to mail one letter, find the amount spent on postage when 8th set of letters is mailed. 5M

Given, Kumar writes a letter to four of his friends. He asks copy the letter to four different person. Continues the way

$4+16+64+\cdots$ is of the form an G.P

$a=4, r=\frac{t_2}{t_1}=\frac{16}{4}=4$

$n=8$

$S_n=a\left[\frac{r^n-1}{r-1}\right], r>1$

$S_8=4\left[\frac{(4)^8-1}{4-1}\right]$

$=4\left[\frac{65536-1}{3}\right]=4\left[\frac{65535}{3}\right]=4[21845]$

$=87380$

$\therefore$ 87380 letters are postage when 8th sets.

The cost of each mail is ₹ 2

Total cost $=2\times 87380=$ ₹ **174760**

Similar Problems (Solve Your Self) 5M

Eg. 2.52: Find the least positive integer n such that

$$1+6+6^2+\cdots+6^n>5000$$

CQ: An organization plans to plant saplings in 25 streets in a town in such a way that one sapling for the first street, three for the second, nine for the third and so on. How many saplings are need to complete the work? **(MDL, PTA-1)**

10. If $S_n = (x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + \cdots n$ terms then prove that $(x-y)S_n = \left[\frac{x^2(x^n-1)}{x-1} - \frac{y^2(y^n-1)}{y-1}\right]$ PTA-1

Given: $S_n = (x+y) + (x^2+xy+y^2) + (x^3+x^2y+xy^2+y^3) + \cdots + n$ terms 5M

Multiply by $(x-y)$

$(x-y)S_n = [(x-y)(x+y) + (x-y)(x^2+xy+y^2) + (x-y)(x^3+x^2y+xy^2+y^3) + \cdots + n$

$= [(x^2-y^2) + (x^3-y^3) + (x^4-y^4) + \cdots n \text{ terms}]$

$(x-y)S_n = [(x^2+x^3+x^4+\cdots+n \text{ terms}) - (y^2+y^3+y^4+\cdots n \text{ terms})]$

$x^2+x^3+x^4+\cdots+n$ terms	$y^2+y^3+y^4+\cdots n$ terms
Here $a = x^2, r = \frac{x^3}{x^2} = x$	Here $a = y^2, r = \frac{y^3}{y} = y$

$S_n = \frac{a(r^n-1)}{r-1}$

$(x-y)S_n = \left[\frac{x^2(x^n-1)}{x-1}\right] - \left[\frac{y^2(y^n-1)}{y-1}\right]$

$= \left[\frac{x^2(x^n-1)}{x-1} - \frac{y^2(y^n-1)}{y-1}\right]$ Hence proved.

Exercise 2.9

1. **Find the sum of the following series** 2M

(i) $1+2+3+\cdots+60$

Given $1+2+3+\cdots+60$, Here $n = 60$

$$\boxed{1+2+3+\cdots+n = \frac{n(n+1)}{2}}$$

$1+2+3+\cdots+60 = \frac{60(60+1)}{2}$

$= 30(61) = \mathbf{1830}$

> **Similar Problems (Solve Your Self)**
> 1. Find the sum of the following series
> (ii) $3+6+9+\cdots+96$ 2M (iii) $51+52+53+\cdots+92$ 5M
> **Eg. 2.54:** Find the value of (i) $1+2+3+\cdots+50$
> (ii) $16+17+18+\cdots+75$ 2M
> **Eg. 2.55:** Find the sum of (i) $1+3+5+\cdots$to 40 terms 2M
> (ii) $2+4+6+\cdots+80$ (iii) $1+3+5+\cdots+55$ **(PTA-6)**

1. (iv) $1+4+9+16+\cdots+225$ 2M

$1+4+9+16+\cdots+225$

$= 1^2+2^2+3^2+4^2+\cdots+15^2$

$$\boxed{1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}}$$

Here, $n = 15$

$= \frac{15(15+1)(2\times15+1)}{6} = \frac{5(16)(31)}{2} = \mathbf{1240}$

> **Similar Problems (Solve Your Self)** 2M
> 1. Find the sum of the following series
> (v) $6^2+7^2+8^2+\cdots+21^2$
> **Eg. 2.56:** Find the sum of (i) $1^2+2^2+\cdots+19^2$
> (ii) $5^2+10^2+15^2+\cdots+105^2$ (iii) $15^2+16^2+17^2+\cdots 28^2$
> **CQ:** Find the value of $1^2+2^2+3^2+\cdots 10^2$ and hence deduce $2^2+4^2+6^2+\cdots+20^2$ **(SEP-20)**

(vi) $10^3 + 11^3 + 12^3 + \cdots + 20^3$ 5M PTA-5

$10^3 + 11^3 + 12^3 + \cdots + 20^3$

$= (1^3 + 2^3 + 3^3 + \cdots + 20^3) - (1^3 + 2^3 + \cdots + 9^3)$

$$\boxed{1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2}$$

$= \left[\frac{20(20+1)}{2}\right]^2 - \left[\frac{9(9+1)}{2}\right]^2$

$= \left[\frac{20(21)}{2}\right]^2 - \left[\frac{9(10)}{2}\right]^2$

$= [10(21)]^2 - [9(5)]^2$

$= (210)^2 - (45)^2$

$= 44100 - 2025 = \mathbf{42075}$

Similar Problems (Solve Your Self) 5M

Eg. 2.57: Find the sum of

(i) $1^3 + 2^3 + 3^3 + \cdots + 16^3$

(ii) $9^3 + 10^3 + \cdots + 21^3$

(vii) $1 + 3 + 5 + \cdots + 71$ 5M

Given: $1 + 3 + 5 + \cdots + 71$

$$\boxed{\text{Sum } n \text{ odd natural numbers} = n^2}$$

$a = 1, d = t_2 - t_1 = 3 - 1 = 2, l = 71$

$n = \frac{l-a}{d} + 1$

$= \frac{71-1}{2} + 1$

$= \frac{70}{2} + 1 = 35 + 1 = 36$

$S_n = n^2$

$S_{36} = (36)^2 = \mathbf{1296}$

2. If $1 + 2 + 3 + \cdots + k = 325$, then find $1^3 + 2^3 + 3^3 + \cdots + k^3$ 2M

Given: $1 + 2 + 3 + \cdots + k = 325$

$$\boxed{1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}}$$

$\frac{k(k+1)}{2} = 325$

$\left[\frac{k(k+1)}{2}\right]^2 = (325)^2$

$1^3 + 2^3 + 3^3 + \cdots + k^3 = \mathbf{105625}$

Similar Problems (Solve Your Self) 2M

Eg. 2.58: If $1 + 2 + 3 + \cdots + n = 666$ then find n. **(PTA-2)**

CQ: If $1^3 + 2^3 + 3^3 + \cdots k^3 = 16900$ then find $1 + 2 + 3 + \cdots + k$ **(PTA-3)**

3. If $1^3 + 2^3 + 3^3 + \cdots + k^3 = 44100$ then find $1 + 2 + 3 + 4 + \cdots + k$

Given: $1^3 + 2^3 + 3^3 + \cdots + k^3 = 44100$

$$\boxed{\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2, \sum_{k=1}^{n} k = \frac{n(n+1)}{2}}$$

$\left[\frac{k(k+1)}{2}\right]^2 = 44100$

$\frac{k(k+1)}{2} = \sqrt{44100}$

$1 + 2 + 3 + 4 + \cdots + k = \sqrt{210^2}$

$= \mathbf{210}$

4. How many terms of the series $1^3 + 2^3 + 3^3 + \cdots$ should be taken to get the sum 14400? 2M

Given: $\sum_{k=1}^{n} k^3 = 14400$

To find n, $1^3 + 2^3 + 3^3 + \cdots + n^3 = 14400$

$\left[\frac{n(n+1)}{2}\right]^2 = 14400$

$\frac{n(n+1)}{2} = \sqrt{14400}$

$n^2 + n = 2(\sqrt{120^2})$

$= 2(120)$

$n^2 + n = 240$

$n^2 + n - 240 = 0$

$(n + 16)(n - 15) = 0$

$n = -16$ or $n = 15$

$n = -16$ is not admissible

$\therefore \mathbf{n = 15}$

5. The sum of the cubes of the first n natural numbers is 2025. Find the value of n. 5M

Given: Sum of cube of 1st n natural number is

$\sum_{k=1}^{n} k^3 = 2025$

$\left[\frac{n(n+1)}{2}\right]^2 = 2025$

$\frac{n(n+1)}{2} = \sqrt{2025}$

$\frac{n(n+1)}{2} = 45$ ....(1)

$n(n + 1) = 90$

$n^2 + n - 90 = 0$

$(n + 10)(n - 9) = 0$

$n = -10$ or $n = 9$

$\therefore \mathbf{n = 9}$ ($n = -10$ is not possible)

6. **Rekha has 15 square colour papers of sizes 10cm, 11cm, 12cm, … ,24cm. How much area can be decorated with these colour papers?** JUN-23,PTA-1

5M

Given: The size of 15 square colour papers are 10cm, 11cm, 12cm, … 24cm

The area of square $= a^2$

The colour paper decorated area

$= 10^2 + 11^2 + 12^2 + \cdots + 24^2$

$= (1^2 + 2^2 + \cdots + 24^2) \quad -(1^2 + 2^2 + 3^2 + \cdots + 9^2)$

$$\boxed{1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}}$$

$= \frac{24(24+1)(24\times2+1)}{6} - \frac{9(9+1)(2\times9+1)}{6}$

$= 4(25)(49) - 3(5)(19)$

$= 4900 - 285$

$= \mathbf{4615\ cm^2}$

7. **Find the sum of the series $(2^3 - 1^3) + (4^3 - 3^3) + (6^3 - 5^3) + \cdots$ to (i) n terms (ii) 8 terms**

5M

Given: $(2^3 - 1^3) + (4^3 - 3^3) + (6^3 - 5^3) + \cdots$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$
$$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$
$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

(i) n terms

$(2^3 - 1^3) + (4^3 - 3^3) + (6^3 - 5^3) + \cdots n$ terms

$[(2n)^3 - (2n-1)^3]$

$= \Sigma[(2n)^3 - (2n-1)^3] \Rightarrow a = 2n, \ b = 2n - 1$

$= \Sigma([2n - (2n-1)][(2n)^2 + (2n)(2n-1) + (2n-1)^2])$

$= \Sigma([\cancel{2n} - \cancel{2n} + 1][4n^2 + 4n^2 - 2n + 4n^2 - 4n + 1])$

$= \Sigma(12n^2 - 6n + 1)$

$= 12\Sigma n^2 - 6\Sigma n + \Sigma(1)$ $\quad [\because \Sigma(1) = 1 + 1 + 1 \ldots \text{n times} = n]$

$= 12\left(\frac{n(n+1)(2n+1)}{6}\right) - 6\left(\frac{n(n+1)}{2}\right) + n$

$= 2(n^2 + n)(2n + 1) - 3n^2 - 3n + n$

$= 2(2n^3 + n^2 + 2n^2 + n) - 3n^2 - 2n$

$= 4n^3 + 2n^2 + 4n^2 + \cancel{2n} - 3n^2 - \cancel{2n}$

$\mathbf{S_n = 4n^3 + 3n^2}$

(ii) 8 terms

We know that $S_n = 4n^3 + 3n^2$

$S_8 = 4(8)^3 + 3(8)^2$

$= 4(512) + 3(64)$

$= 2048 + 192 = \mathbf{2240}$

Note for Unit Exercise – 2

Q.No: 1 - Similar to Exercise 2.1 -3rd Question
Q.No: 2 - Similar to Exercise 2.1 – 2nd Question
Q.No: 3 - Similar to Exercise 2.1 -4th Question
Q.No: 4 - Similar to Exercise 2.1 -2nd Question
Q.No: 5 - Similar to Exercise 2.5 -7th Question
Q.No: 6 - Similar to Exercise 2.5 -4th Question
Q.No: 7 - Similar to Exercise 2.5 -7th Question
Q.No: 8 - Similar to Exercise 2.6 -8th Question
Q.No: 9 - Similar to Exercise 2.7 -6th Question
Q.No: 10 - Similar to Exercise 2.7 -10th Question

3. Algebra

Exercise 3.1

1. **Solve the following system of linear equations in three variables** 5M

(i) $x+y+z=5; 2x-y+z=9; x-2y+3z=16$

(ii) $\frac{1}{x}-\frac{2}{y}+4=0; \frac{1}{y}-\frac{1}{z}+1=0; \frac{2}{z}+\frac{3}{x}=14$ (iii) $x+20=\frac{3y}{2}+10=2z+5=110-(y+z)$

SEP-21 PTA-5

(i) $x+y+z=5$ .... (1)

$2x-y+z=9$ .... (2)

$x-2y+3z=16$ .... (3)

Add (1) + (2)

$$\begin{aligned}(1) &\Rightarrow x+\cancel{y}+z=5\\(2) &\Rightarrow 2x-\cancel{y}+z=9\\ \hline &\quad 3x+2z=14 \quad \text{............ (4)}\end{aligned}$$

$$\begin{aligned}(2)\times 2 &\Rightarrow 4x-\cancel{2y}+2z=18\\(3) &\Rightarrow x-\cancel{2y}+3z=16\\ &\quad (-)\ (+)\ (-)\ (-)\\ \hline &\quad 3x-z=2 \quad \text{............ (5)}\end{aligned}$$

$$\begin{aligned}(4) &\Rightarrow \cancel{3x}+2z=14\\(5) &\Rightarrow \cancel{3x}-z=2\\ &\quad (-)\ (+)\ (-)\\ \hline &\quad 3z=12\end{aligned}$$

$3z=12$

$z=\frac{12}{3}=4$

Sub. $z=4$ in (5)

$3x-z=2$

$3x-4=2$

$3x=2+4$

$3x=6 \Rightarrow x=\frac{6}{3} \Rightarrow x=2$

Sub. $x=2, z=4$ in (1)

$x+y+z=5$

$2+y+4=5$

$y=5-2-4$

$y=5-6$

$y=-1$

$\therefore \boldsymbol{x=2, y=-1, z=4}$

(ii) $\frac{1}{x}-\frac{2}{y}+4=0; \frac{1}{y}-\frac{1}{z}+1=0; \frac{2}{z}+\frac{3}{x}=14$

Let $\frac{1}{x}=a,\ \frac{1}{y}=b,\ \frac{1}{z}=c$

$a-2b+4=0$ .... (1)

$b-c+1=0$ .... (2)

$2c+3a-14=0$ .... (3)

From (1) & (2)

$$\begin{aligned}(1) &\Rightarrow a-\cancel{2b}+0c+4=0\\(2)\times 2 &\Rightarrow \cancel{2b}-2c+2=0\\ \hline &\quad a-2c+6=0\end{aligned}$$

$a-2c+6=0$.......... (4)

$$\begin{aligned}(4) &\Rightarrow a-\cancel{2c}+6=0\\(3) &\Rightarrow 3a+\cancel{2c}-14=0\\ \hline &\quad 4a-8=0\\ \hline &\quad 4a=8 \Rightarrow a=\frac{8}{4}=2\end{aligned}$$

Sub. $a=2$ in (4)

$a-2c+6=0$

$2-2c+6=0$

$-2c+8=0$

$-2c=-8$

$c=\frac{-8}{-2}=4$

Sub. $c=4$ in (2)

$b-c+1=0$

$b-4+1=0$

$b-3=0$

$b=3$

$\therefore$ Here $x=\frac{1}{a} \Rightarrow \boldsymbol{x=\frac{1}{2}}$

$y=\frac{1}{b} \Rightarrow \boldsymbol{y=\frac{1}{3}}$

$z=\frac{1}{c} \Rightarrow \boldsymbol{z=\frac{1}{4}}$

Similar Problems

Solve Your Self

Eg. 3.2: Solve $2x-3y=6,\ x+y=1$ 2M

Eg. 3.3: Solve the following system of linear equations in three variables 5M

$3x-2y+z=2,\ 2x+3y-z=5,\ x+y+z=6.$ **(JUN-23)**

CQ: Solve: $6x+2y-5z=13, 3x+3y-2z=13,\ 7x+5y-3z=26$ **(SEP-20)**

CQ: Solve: $2x+y+4z=15, x-2y+3z=13,\ 3x+y-z=2$ **(PTA-1)**

Similar Problems 5M

Solve Your Self

Eg. 3.8: Solve: $\frac{1}{2x}+\frac{1}{4y}-\frac{1}{3z}=\frac{1}{4};$ $\frac{1}{x}=\frac{1}{3y};$ $\frac{1}{x}-\frac{1}{5y}+\frac{4}{z}=2\frac{2}{15}$

(iii) $\boldsymbol{x+20=\frac{3y}{2}+10=2z+5=110-(y+z)}$

$x+20=\frac{3y}{2}+10$ | $2x-3y=2(-10)$

$x-\frac{3y}{2}=10-20$ | $2x-3y=-20$ .... (1)

APR-23

$\frac{3y}{2}+10=2z+5$

$\frac{3y}{2}-2z=5-10$

$3y-4z=2(-5)$

$3y-4z=-10$ .... (2)

$2z+5=110-(y+z)$

$2z+y+z=110-5$

$y+3z=105$ .... (3)

$(2) \Rightarrow \cancel{3y}-4z=-10$

$(3)\times 3 \Rightarrow \cancel{3y}+9z=315$

$(-) \quad (-) \quad (-)$

$-13z=-325$

$z=-\frac{325}{-13}=25$

Sub. $z=25$ in (3)

$y+3z=105$

$y+3(25)=105$

$y+75=105$

$y=105-75$

$y=30$

Sub. $y=30$ in (1)

$2x-3(30)=-20$

$2x-90=-20$

$2x=-20+90$

$2x=70$

$x=\frac{70}{2} \Rightarrow x=35$

$\therefore \boldsymbol{x=35; y=30; z=25}$

Similar Problems (5M)

Solve Your Self

Eg. 3.7 : Solve $\frac{x}{2}-1=\frac{y}{6}+1=\frac{z}{7}+2; \frac{y}{3}+\frac{z}{2}=13$

UE-1: Solve $\frac{1}{3}(x+y-5)=y-z=2x-11=9-(x+2z)$

Similar Problems (5M)

Solve Your Self

Eg. 3.5: Solve $x+2y-z=5;\ x-y+z=-2;$
$-5x-4y+z=-11.$

Eg. 3.6: Solve $3x+y-3z=1;\ -2x-y+2z=1;$
$-x-y+z=2.$

2. Discuss the nature of solutions of the following system of equations (5M)

(i) $\boldsymbol{x+2y-z=6;\ -3x-2y+5z=-12;\ x-2z=3}$

$x+2y-z=6$ .... (1)

$-3x-2y+5z=-12$ .... (2)

$x-2z=3$ .... (3)

$(1)+(2)\Rightarrow$

$(1)\Rightarrow \quad x+\cancel{2y}-z=6$

$(2)\Rightarrow -3x-\cancel{2y}+5z=-12$

$-2x+4z=-6$

$\div 2, \quad -x+2z=-3$ .... (4)

$(3)+(4)\Rightarrow$

$(3)\Rightarrow \quad x-\cancel{2z}=3$

$(4)\Rightarrow -x+\cancel{2z}=-3$

$0=0$

Here $0=0$

The system has an **infinitely many solution.**

(ii) $\boldsymbol{2y+z=3(-x+1);}$

$\boldsymbol{-x+3y-z=-4;\ 3x+2y+z=-\frac{1}{2}}$

$2y+z=3(-x+1)$

$2y+z=-3x+3$

$3x+2y+z=3$ .... (1)

$-x+3y-z=-4$ .... (2)

$3x+2y+z=-\frac{1}{2}$ .... (3)

$(1)+(2)\Rightarrow$

$(1)\Rightarrow \quad 3x+2y+\cancel{z}=3$

$(2)\Rightarrow \ -x+3y-\cancel{z}=-4$

$2x+5y=-1$ .... (4)

$2\times(2)\Rightarrow \ -2x+6y-\cancel{2z}=-8$ ...... (2)

$(3)\Rightarrow \quad 6x+4y+\cancel{2z}=-1$ ...... (3)

$4x+10y=-9$

$\div by\ 2, \quad 2x+5y=-\frac{9}{2}$ ....(5)

$(4)-(5)\Rightarrow$

$(4)\Rightarrow \cancel{2x}+\cancel{5y}=-1$

$(5)\Rightarrow \cancel{2x}+\cancel{5y}=-\frac{9}{2}$ .... (5)

$(-) \quad (-) \quad (+)$

$0=-1-\frac{9}{2}$

$0=\frac{-2+9}{2}$

$0\neq\frac{7}{2}$

This system is Inconsistent and has **no solution.**

(iii) $\frac{y+z}{4}=\frac{z+x}{3}=\frac{x+y}{2}$; $x+y+z=27$

$\frac{y+z}{4}=\frac{z+x}{3}$

$3y+3z=4z+4x$

$4x-3y+4z-3z=0$

$4x-3y+z=0$ …….. (1)

$\frac{z+x}{3}=\frac{x+y}{2}$

$2z+2x=3x+3y$

$3x-2x+3y-2z=0$

$x+3y-2z=0$ …… (2)

$x+y+z=27$ ……….. (3)

(1) + (2) ⇒

(1) ⇒ $4x-\cancel{3y}+z=0$

(2)⇒ $x+\cancel{3y}-2z=0$

$5x \quad -z \quad =0$ ……. (4)

(2) ⇒ $x+\cancel{3y}-2z=0$

$3\times(3)$ ⇒ $3x+\cancel{3y}+3z=81$

(−) (−) (−) (−)

$-2x \quad -5z \quad =-81$ …… (5)

(4) × 5 ⇒ $25x-\cancel{5z}=0$

(5) ⇒ $-2x-\cancel{5z}=-81$

(+) (+) (+)

$27x \quad =81$

$x=\frac{81}{27}=3$

Sub $x=3$ in (4) we get $z=15$

Sub $x=3, z=15$ in (1) we get $y=9$

So the system has **unique solution.**

Similar Problems

Solve Your Self (2M)

Eg. 3.1: The father's age is six times his son's age. Six years hence the age of father will be four times his son's age. Find the present ages (in years) of the son and father.

Eg. 3.9: The sum of thrice the first number, second number and twice the third number is 5. If thrice the second number is subtracted from the sum of first number and thrice the third we get 2. If the third number is subtracted from the sum of twice the first, thrice the second, we get 1. Find the numbers. (5M)

3. Vani, her father and her grand father have an average age of 53. One-half of her grandfather's age plus one-third of her father's age plus one fourth of vani's age is 65. Four years ago if vani's grandfather was four times as old as vani then how old are you they all now? (5M) PTA-2

Vani's age $=x$

Her father's age $=y$

Her grandfather's age $=z$

Average age = 53 ⇒ $\frac{x+y+z}{3}=53$

$x+y+z=53\times 3$

$x+y+z=159$…… (1)

Here $\frac{1}{2}z+\frac{1}{3}y+\frac{1}{4}x=65$

$3x+4y+6z=780$ …… (2)

Four years ago

Vani's age $=x-4$

Her father's age $=y-4$

Grandfather's age $=z-4$

$z-4=4(x-4)$

$z-4=4x-16$

$4x-z-12=0$

$4x-z=12$ ………… (3)

$(1)\times 4$ ⇒ $4x+\cancel{4y}+4z=636$

(2) ⇒ $3x+\cancel{4y}+6z=780$

(−) (−) (−) (−)

$x \quad -2z \quad =-144$ ………… (4)

$(3)\times 2$ ⇒ $8x-\cancel{2z}=24$

$(4)\times 1$ ⇒ $x-\cancel{2z}=-144$

(−) (+) (+)

$7x \quad =168$

$x=\frac{168}{7}=24$

Sub. $x=24$ in (5)

$4x-z=12$

$4(24)-z=12$

$-z=12-96$

$-z=-84$

$z=84$

Sub. $x=24;\ z=84$ in (1)

$x+y+z=159$

$24+y+84=159$

$y+108=159$

$y=159-108$

$y=51$

∴ **Vani's age = 24**

Her father's age = 51

Her grandfather's age = 84

4. The sum of the digits of a three-digit number is 11. If the digits are reversed, the new number is 46 more than five times the former number. If the hundreds digit plus twice the tens digit is equal to the units digit, then find the original three digit number? (5M)

The three digit number $= 100x + 10y + z$

(Unit digit $= z$, 10^{th} digit $= y$, 100^{th} digit $= x$)

Sum of the digits $= 11$, $x + y + z = 11$ ... (1)

If digits reversed then the new number

$= 100z + 10y + x$

$100z + 10y + x = 5[100x + 10y + z] + 46$

$100z + 10y + x = 500x + 50y + 5z + 46$

$499x + 40y - 95z = -46$ .... (2)

$x + 2y = z$

$x + 2y - z = 0$ .... (3)

$(1) + (3) \Rightarrow$

$(1) \Rightarrow x + y + \cancel{z} = 11$

$(3) \Rightarrow x + 2y - \cancel{z} = 0$

$2x + 3y = 11$ ...... (4)

$(1) \times 95 \Rightarrow 95x + 95y + \cancel{95z} = 1045$

$(2) \Rightarrow 499x + 40y - \cancel{95z} = -46$

$594x + 135y = 999$ ... (5)

$(5) \Rightarrow 594x + \cancel{135y} = 999$

$(4) \times 45 \Rightarrow 90x + \cancel{135y} = 495$

$(-) \quad (-) \quad (-)$

$504x = 504$

$x = \frac{504}{504} = 1$

Sub. $x = 1$ in (4)

$2x + 3y = 11$

$2(1) + 3y = 11$

$3y = 11 - 2$

$y = \frac{9}{3}$

$y = 3$

Sub. $x = 1, y = 3$ in (1)

$x + y + z = 11$

$1 + 3 + z = 11$

$4 + z = 11$

$z = 11 - 4$

$z = 7$

$\therefore$ The number $= 100x + 10y + z$

$= 100(1) + 10(3) + 7$

$= 100 + 30 + 7 = \mathbf{137}$

5. There are 12 pieces of five, ten and twenty rupee currencies whose total value is ₹ 105. When first 2 sorts are interchanged in their numbers its value will be increased by ₹ 20. Find the number of currencies in each sort. (5M)

Let number of 5 rupees $= x$

Number of 10 rupees $= y$

Number of 20 rupees $= z$

$x + y + z = 12$...... (1)

Total value $= 105$

$5x + 10y + 20z = 105$

$\div 5 \quad x + 2y + 4z = 21$....... (2)

After interchanging

$10x + 5y + 20z = 105 + 20$

$10x + 5y + 20z = 125$

$\div 5 \quad 2x + y + 4z = 25$...... (3)

$(2) \Rightarrow x + 2y + \cancel{4z} = 21$

$(3) \Rightarrow 2x + y + \cancel{4z} = 25$

$(-) \quad (-) \quad (-) \quad (-)$

$-x + y = -4$

$\times (-1) \quad x - y = 4$ .... (4)

$(1) \times (4) \Rightarrow 4x + 4y + \cancel{4z} = 48$

$(2) \Rightarrow x + 2y + \cancel{4z} = 21$

$(-) \quad (-) \quad (-) \quad (-)$

$3x + 2y = 27$...... (5)

$(5) \Rightarrow \cancel{3x} + 2y = 27$

$(4) \times (3) \Rightarrow \cancel{3x} - 3y = 12$

$(-) \quad (+) \quad (-)$

$5y = 15$

$y = \frac{15}{5} = 3$

Sub. $y = 3$ in (4)

$x - y = 4$

$x - 3 = 4$

$x = 4 + 3$

$x = 7$

Sub. $x = 7;\ y = 3$ in (1)

$x + y + z = 12$

$7 + 3 + z = 12$

$10 + z = 12$

$z = 12 - 10$

$z = 2$

Number of 5 Rupees $= 7$

Number of 10 rupees $= 3$

Number of 20 rupees $= 2$

Similar Problems (5M)

Solve Your Self

UE-3: In a three-digit number, when the tens and the hundreds digit are interchanged the new number is 54 more than three times the original number. If 198 is added to the number, the digits are reversed. The tens digit exceeds the hundreds digit by twice as that of the tens digit exceeds the unit digit. Find the original number.

Similar Problems (5M)

Solve Your Self

Eg. 3.4: In an interschool atheletic meet, with total of 24 individual prizes, securing a total of 56 points, a first place secures 5 points, a second place secures 3 points, and a third place secures 1 point. Having as many third place finishers as first and second place finishers, find how many athletes finished in each place.

UE-2: One hundred and fifty Students are admitted to a school. They are distributed over three sections A, B and C. If 6 students are shifted from section A to section C, the sections will have equal number of students. If 4 times of students of section C exceeds the number of students of section A by the number of students in section B, find the number of students in the three sections.

Exercise 3.2

1. Find the GCD of the given polynomials (i) $\boldsymbol{x^4+3x^3-x-3, x^3+x^2-5x+3}$ SEP-20

(i) $f(x)=x^4+3x^3-x-3$ $\quad g(x)=x^3+x^2-5x+3$

5M

$$\begin{array}{r|l} & x+2 \\ \hline x^3+x^2-5x+3 & \cancel{x^4}+3x^3-0x^2-x-3 \\ & \cancel{x^4}+x^3-5x^2+3x \\ & (-)\ (-)\ (+)\ (-) \\ \hline & \cancel{2x^3}+5x^2-4x-3 \\ & \cancel{2x^3}+2x^2-10x+6 \\ & (-)\ (-)\ (+)\ (-) \\ \hline & 3x^2+6x-9 \\ \hline & 3[x^2+2x-3] \end{array}$$

$3[x^2+2x-3]\neq 0$ here 3 is not a divisor of $g(x)$

$$\begin{array}{r|l} & x-1 \\ \hline x^2+2x-3 & \cancel{x^3}+x^2-5x+3 \\ & \cancel{x^3}+2x^2-3x \\ & (-)\ (-)\ (+) \\ \hline & \cancel{-x^2}-\cancel{2x}+\cancel{3} \\ & \cancel{-x^2}-\cancel{2x}+\cancel{3} \\ & (+)\ (+)\ (-) \\ \hline & 0 \end{array}$$

$\therefore$ GCD of $f(x)$ and $g(x)$ is $\boldsymbol{x^2+2x-3}$

Similar Problems Solve Your Self 5M

1. Find the GCD of the given polynomials (ii) x^4-1, x^3-11x^2+x-11
   (iii) $3x^4+6x^3-12x^2-24x,\ 4x^4+14x^3+8x^2-8x$ (iv) $3x^3+3x^2+3x+3, 6x^3+12x^2+6x+12$

UE-5: Find the GCD of the following by division algorithm $2x^4+13x^3+27x^2+23x+7, x^3+3x^2+3x+1, x^2+2x+1$

Eg. 3.10: Find the GCD of the polynomials x^3+x^2-x+2 and $2x^3-5x^2+5x-3$.

Eg. 3.11: Find the GCD of $6x^3-30x^2+60x-48$ and $3x^3-12x^2+21x-18$.

2. Find the LCM of the given expressions. 2M

(i) $\boldsymbol{4x^2y, 8x^3y^2}$

LCM of 4 & 8 is 8

LCM : $\boldsymbol{8x^3y^2}$

(vi) $\boldsymbol{(2x^2-3xy)^2, (4x-6y)^3, 8x^3-27y^3}$

$(2x^2-3xy^2)^2=\big[x[2x-3y]\big]^2$

$=x^2[2x-3y]^2$

$(4x-6y)^3=\big[2[2x-3y]\big]^3=2^3[2x-3y]^3$

$8x^3-27y^3=(2x)^3-(3y)^3$

$=(2x-3y)(4x^2+9y^2+6xy)$

LCM: $\boldsymbol{2^3x^2(2x-3y)^3(4x^2+6xy+9y^2)}$

Similar Problems Solve Your Self 2M

2. Find the LCM of the given expressions.
   (ii) $9a^3b^2, 12a^2b^2c$
   (iii) $16m, 12m^2n^2, 8n^2$
   (iv) p^2-3p+2, p^2-4
   (v) $2x^2-5x-3, 4x^2-36$

Eg. 3.12: Find the LCM of the following
   (i) $8x^4y^2,\ 48x^2y^4$
   (ii) $5x-10,\ 5x^2-20$
   (iii) $x^4-1,\ x^2-2x+1$
   (iv) $x^3-27,\ (x-3)^2,\ x^2-9$

UE-4: Find the least common multiple of $xy(k^2+1)+k(x^2+y^2)$ and $xy(k^2-1)+k(x^2-y^2)$

Exercise 3.3

1. Find the LCM and GCD for the following and verify that $\boldsymbol{f(x)\times g(x)=LCM\times GCD}$ 2M

(i) $\boldsymbol{21x^2y, 35xy^2}$

$f(x)=21x^2y=7\times 3\times x^2\times y$,

$g(x)=35xy^2=7\times 5\times x\times y^2$

LCM: $3\times 5\times 7\times x^2\times y^2=\boldsymbol{105x^2y^2}$

GCD: $7\times x\times y=\boldsymbol{7xy}$

Verification: $f(x)\times g(x)=LCM\times GCD$

$21x^2y\times 35xy^2=105x^2y^2\times 7xy$

$735x^3y^3=735x^3y^3$

Hence $f(x)\times g(x)=LCM\times GCD$ verified.

Similar Problems Solve Your Self 2M

1. Find the LCM and GCD for the following and verify that $f(x)\times g(x)=LCM\times GCD$
   (ii) $(x^3-1)(x+1), (x^3+1)$ (iii) $(x^2y+xy^2), (x^2+xy)$

CQ: If two positive integers p and q are written as $p=a^2b^3$ and $q=a^3b; a, b$ are prime numbers, then verify $LCM(p,q)\times HCF(p,q)=pq$ **(PTA-2)**

2. Find the LCM of each pair of the following polynomials (i) $a^2+4a-12, a^2-5a+6$ Whose GCD is $a-2$ (2M) PTA-6

$f(x) = a^2+4a-12 = (a+6)(a-2)$

$g(x) = a^2-5a+6 = (a-3)(a-2)$

$GCD: a-2$

$LCM = \frac{f(x)\times g(x)}{GCD} = \frac{(a+6)\cancel{(a-2)}\times(a-3)(a-2)}{\cancel{(a-2)}}$

$LCM = (\boldsymbol{a+6})(\boldsymbol{a-3})(\boldsymbol{a-2})$

> **Similar Problems**
> Solve Your Self
> 2(ii). Find the LCM of each pair of the following polynomials $x^4-27a^3x, (x-3a)^2$ Whose GCD is $(x-3a)$ (2M)

3. Find the GCD of each pair of the following polynomials: (i) $12(x^4-x^3), 8(x^4-3x^3+2x^2)$ (2M)

$12(x^4-x^3), 8(x^4-3x^3+2x^2)$

Whose LCM is $24x^3(x-1)(x-2)$

$f(x) = 12(x^4-x^3)$

$= 12x^3[x-1]$

$g(x) = 8(x^4-3x^3+2x^2)$

$= 8x^2[x^2-3x+2]$

$= 8x^2(x-1)(x-2)$

$LCM = 24x^3(x-1)(x-2)$

$GCD = \frac{f(x)\times g(x)}{LCM}$

$= \frac{12x^3(x-1)\times 8x^2(x-1)(x-2)}{24x^3(x-1)(x-2)}$

$GCD = \boldsymbol{4x^2(x-1)}$

> **Similar Problems**
> Solve Your Self
> 3(ii). Find the GCD of each pair of the following polynomials: (2M)
> $(x^3+y^3), (x^4+x^2y^2+y^4)$ Whose LCM is $(x^3+y^3)(x^2+xy+y^2)$

4. Given the LCM and GCD of the two polynomials $p(x)$ and $q(x)$ find the unknown polynomial in the following table (5M)

S.No	LCM	GCD	$p(x)$	$q(x)$
(i)	$a^3-10a^2+11a+70$	$a-7$	$a^2-12a+35$	?

$LCM: \ a^3-10a^2+11a+70$

$GCD: \ a-7$

$p(x) = a^2-12a+35$

$q(x) = ?$

$q(x) = \frac{LCM\times GCD}{p(x)}$

$= \frac{a^3-10a^2+11a+70\times a-7}{a^2-12a+35}$

$= \frac{(a^3-10a^2+11a+70)\times \cancel{a-7}}{\cancel{(a-7)}(a-5)}$

$q(x) = a^2-5a-14 = (\boldsymbol{a+2})(\boldsymbol{a-7})$

$$
\begin{array}{r|l}
 & a^2-5a-14 \\
\hline
a-5 & \cancel{a^3}-10a^2+11a+70 \\
 & \cancel{a^3}-5a^2 \\
 & (-) \quad (+) \\
\hline
 & -\cancel{5a^2}+11a \\
 & -\cancel{5a^2}+25a \\
 & (+) \quad (-) \\
\hline
 & -\cancel{14a}+70 \\
 & -\cancel{14a}+70 \\
 & (+) \quad (-) \\
\hline
 & 0
\end{array}
$$

> **Similar Problems**
> Solve Your Self (5M)
> 4. Given the LCM and GCD of the two polynomials $p(x)$ and $q(x)$ find the unknown polynomial in the following table

S.No	LCM	GCD	$p(x)$	$q(x)$
(ii)	$(x^4-y^4)(x^4+x^2y^2+y^4)$	(x^2-y^2)	?	$(x^4-y^4)(x^2+y^2-xy)$

Exercise 3.4

1. Reduce each of the following rational expressions to its lowest form. (2M)

(i) $\frac{x^2-1}{x^2+x} = \frac{x^2-1^2}{x(x+1)} = \frac{\cancel{(x+1)}(x-1)}{x\cancel{(x+1)}} = \boldsymbol{\frac{x-1}{x}}$

(ii) $\frac{x^2-11x+18}{x^2-4x+4} = \frac{(x-9)\cancel{(x-2)}}{(x-2)\cancel{(x-2)}} = \boldsymbol{\frac{x-9}{x-2}}$

> **Similar Problems**
> Solve Your Self (2M)
> **Eg. 3.13**: Reduce the rational expression to its lowest form
> (i) $\frac{x-3}{x^2-9}$ (ii) $\frac{x^2-16}{x^2+8x+16}$

(iii) $\frac{9x^2+81x}{x^3+8x^2-9x} = \frac{9x[x+9]}{x(x^2+8x-9)} = \frac{9(x+9)}{(x+9)(x-1)} = \mathbf{\frac{9}{x-1}}$

(iv) $\frac{p^2-3p-40}{2p^3-24p^2+64p} = \frac{(p-8)(p+5)}{2p(p^2-12p+32)} = \frac{\cancel{(p-8)}(p+5)}{2p\cancel{(p-8)}(p-4)}$

$= \mathbf{\frac{p+5}{2p(p-4)}}$

Similar Problems (2M)
Solve Your Self
UE-6: Reduce the given rational expressions to its lowest form (i) $\frac{x^{3a}-8}{x^{2a}+2x^a+4}$ (ii) $\frac{10x^3-25x^2+4x-10}{-4-10x^2}$

2. Find the excluded values, if any of the following expressions. (iv) $\frac{x^3-27}{x^3+x^2-6x}$ (2M)

(iv) $\frac{x^3-27}{x^3+x^2-6x} = \frac{x^3-3^3}{x[x^2+x-6]}$

$= \frac{(x-3)(x^2+9-3x)}{x(x^2+x-6)}$

$\frac{p(x)}{q(x)} = \frac{(x-3)(x^2-3x+9)}{x(x+3)(x-2)}$ for

-6
1
+3 -2

excluded value $q(x) = 0$

$x(x+3)(x-2) = 0$

$x = 0$ | $x + 3 = 0$, $x = -3$ | $x - 2 = 0$, $x = 2$

$\therefore x = 0, -3, 2$

Similar Problems (2M)
Solve Your Self
2. Find the excluded values, if any of the following expressions.
(i) $\frac{y}{y^2-25}$ (ii) $\frac{t}{t^2-5t+6}$ (iii) $\frac{x^2+6x+8}{x^2+x-2}$

Eg. 3.14: Find the excluded values of the following expressions (if any).
(i) $\frac{x+10}{8x}$ (ii) $\frac{7p+2}{8p^2+13p+5}$ **(MAY-22)** (iii) $\frac{x}{x^2+1}$

Exercise 3.5

1. Simplify (2M)

(i) $\frac{4x^2y}{2z^2} \times \frac{6xz^3}{20y^4} = \frac{4x^2y}{2} \times \frac{6xz}{20y^4} = \frac{3}{5}\frac{x^3 \times z}{y^3} = \mathbf{\frac{3x^3z}{5y^3}}$

2. Simplify (ii) $\frac{x^3-y^3}{3x^2+9xy+6y^2} \times \frac{x^2+2xy+y^2}{x^2-y^2}$ (2M)

$x^3 - y^3 = (x-y)(x^2+y^2+xy)$

$3x^2+9xy+6y^2 = 3(x^2+3xy+2y^2) = 3(x+2y)(x+y)$
$x^2+2xy+y^2 = (x+y)^2$
$x^2-y^2 = (x+y)(x-y)$

$= \frac{\cancel{(x-y)}(x^2+y^2+xy)}{3(x+2y)\cancel{(x+y)}} \times \frac{\cancel{(x+y)^2}}{\cancel{(x+y)}\cancel{(x-y)}}$

$= \mathbf{\frac{x^2+xy+y^2}{3(x+2y)}}$

Similar Problems (2M)
Solve Your Self
1. Simplify
(ii) $\frac{p^2-10p+21}{p-7} \times \frac{p^2+p-12}{(p-3)^2}$ (iii) $\frac{5t^3}{4t-8} \times \frac{6t-12}{10t}$
2. Simplify (i) $\frac{x+4}{3x+4y} \times \frac{9x^2-16y^2}{2x^2+3x-20}$

Eg. 3.15: (i) Multiply $\frac{x^3}{9y^2}$ by $\frac{27y}{x^5}$
(ii) Multiply $\frac{x^4b^2}{x-1}$ by $\frac{x^2-1}{a^4b^3}$

UE-7: Simplify $\frac{\frac{1}{p}+\frac{1}{q+r}}{\frac{1}{p}-\frac{1}{q+r}} \times \left[1+\frac{q^2+r^2-p^2}{2qr}\right]$

3. Simplify (i) $\frac{2a^2+5a+3}{2a^2+7a+6} \div \frac{a^2+6a+5}{-5a^2-35a-50}$ (5M)

$= \frac{2a^2+5a+3}{2a^2+7a+6} \times \frac{-5a^2-35a-50}{a^2+6a+5}$

$2a^2+5a+3 = (a+1)(2a+3)$
$2a^2+7a+6 = (2a+3)(a+2)$
$-5a^2-35a-50 = -5[a^2+7a+10]$
$= -5(a+2)(a+5)$
$a^2+6a+5 = (a+1)(a+5)$

$= \frac{\cancel{(a+1)}\cancel{(2a+3)}}{\cancel{(2a+3)}\cancel{(a+2)}} \times -\frac{5\cancel{(a+2)}\cancel{(a+5)}}{\cancel{(a+5)}\cancel{(a+1)}}$

$= \mathbf{-5}$

Similar Problems Solve Your Self
3. Simplify
(ii) $\frac{b^2+3b-28}{b^2+4b+4} \div \frac{b^2-49}{b^2-5b-14}$ **(JUL-22)**
(iii) $\frac{x+2}{4y} \div \frac{x^2-x-6}{12y^2}$ **(APR-23)**
(iv) $\frac{12t^3-22t+8}{3t} \div \frac{3t^2+2t-8}{2t^2+4t}$

Eg. 3.16: Find (i) $\frac{14x^4}{y} \div \frac{7x}{3y^4}$ (ii) $\frac{x^2-16}{x+4} \div \frac{x-4}{x+4}$
(iii) $\frac{16x^2-2x-3}{3x^2-2x-1} \div \frac{8x^2+11x+3}{3x^2-11x-4}$

4. If $x=\frac{a^2+3a-4}{3a^2-3}$ and $y=\frac{a^2+2a-8}{2a^2-2a-4}$ find the value of x^2y^{-2} 5M PTA-3

$$x=\frac{a^2+3a-4}{3a^2-3}=\frac{(a+4)(a-1)}{3(a^2-1^2)}=\frac{(a+4)\cancel{(a-1)}}{3(a+1)\cancel{(a-1)}}$$

$$x=\frac{a+4}{3(a+1)}$$

$$y=\frac{a^2+2a-8}{2a^2-2a-4}$$

$$=\frac{a^2+2a-8}{2(a^2-a-2)}=\frac{(a+4)\cancel{(a-2)}}{2[\cancel{(a-2)}(a+1)]}$$

$$y=\frac{a+4}{2(a+1)}$$

$$x^2y^{-2}=\frac{x^2}{y^2}=\left(\frac{x}{y}\right)^2$$

$$\frac{x}{y}=\frac{\cancel{a+4}}{3\cancel{(a+1)}}\times\frac{2\cancel{(a+1)}}{\cancel{a+4}}$$

$$\frac{x}{y}=\frac{2}{3}$$

$$\left(\frac{x}{y}\right)^2=\left(\frac{2}{3}\right)^2=\frac{4}{9}$$

$$\therefore x^2y^{-2}=\frac{4}{9}$$

5. If a polynomial $p(x)=x^2-5x-14$ is divided by another polynomial $q(x)$ we get $\frac{x-7}{x+2}$, find $q(x)$. 2M PTA-2

$$\frac{p(x)}{q(x)}=\frac{x-7}{x+2}$$

$$\frac{x^2-5x-14}{q(x)}=\frac{x-7}{x+2}$$

$$q(x)=\frac{x^2-5x-14}{x-7}\times x+2$$

$$=\frac{\cancel{(x-7)}(x+2)}{\cancel{x-7}}\times(x+2)$$

$$q(x)=(x+2)(x+2)$$

$$q(x)=x^2+4x+4$$

Creative Questions

1. Simplify : $\frac{a^2-16}{a^3-8}\times\frac{2a^2-3a-2}{2a^2+9a+4}\div\frac{3a^2-11a-4}{a^2+2a+4}$ PTA-4 5M

$$\frac{a^2-16}{a^3-8}\times\frac{2a^2-3a-2}{2a^2+9a+4}\div\frac{3a^2-11a-4}{a^2+2a+4}$$

$$=\frac{a^2-16}{a^3-8}\times\frac{2a^2-3a-2}{2a^2+9a+4}\times\frac{a^2+2a+4}{3a^2-11a-4}$$

$$=\frac{\cancel{(a+4)}\cancel{(a-4)}}{\cancel{(a-2)}\cancel{(a^2+2a+4)}}\times\frac{\cancel{(2a+1)}\cancel{(a-2)}}{\cancel{(2a+1)}\cancel{(a+4)}}\times\frac{\cancel{a^2+2a+4}}{(3a+1)\cancel{(a-4)}}$$

$$=\frac{1}{(3a+1)}$$

Exercise 3.6

1. Simplify (i) $\frac{x(x+1)}{x-2}+\frac{x(1-x)}{x-2}$ 2M

$$\frac{x(x+1)}{x-2}+\frac{x(1-x)}{x-2}=\frac{\cancel{x^2}+x+x-\cancel{x^2}}{x-2}$$

$$=\frac{2x}{x-2}$$

Similar Problems 2M
Solve Your Self
1. Simplify (ii) $\frac{x+2}{x+3}+\frac{x-1}{x-2}$ (iii) $\frac{x^3}{x-y}+\frac{y^3}{y-x}$
Eg. 3.18: Simplify $\frac{1}{x^2-5x+6}+\frac{1}{x^2-3x+2}-\frac{1}{x^2-8x+15}$

2. Simplify (i) $\frac{(2x+1)(x-2)}{x-4}-\frac{(2x^2-5x+2)}{x-4}$ 2M

$$\frac{(2x+1)(x-2)}{x-4}-\frac{(2x^2-5x+2)}{x-4}$$

$$=\frac{\cancel{2x^2}-3x-2-\cancel{2x^2}+5x-2}{x-4}$$

$$=\frac{2x-4}{x-4}=\frac{2(x-2)}{x-4}$$

Similar Problems 2M
Solve Your Self
2. Simplify (ii) $\frac{4x}{x^2-1}-\frac{x+1}{x-1}$
Eg.3.17: Find $\frac{x^2+20x+36}{x^2-3x-28}-\frac{x^2+12x+4}{x^2-3x-28}$

3. Subtract $\frac{1}{x^2+2}$ from $\frac{2x^3+x^2+3}{(x^2+2)^2}$ 2M

$$\frac{2x^3+x^2+3}{(x^2+2)^2}-\frac{1}{x^2+2}=\frac{2x^3+x^2+3-(x^2+2)}{(x^2+2)^2}$$

$$=\frac{2x^3+x^2+3-x^2-2}{(x^2+2)^2}$$

$$=\frac{2x^3+1}{(x^2+2)^2}$$

4. Which rational expression should be subtracted from $\frac{x^2+6x+8}{x^3+8}$ to get $\frac{3}{x^2-2x+4}$ 2M PTA-4

$$\frac{x^2+6x+8}{x^3+8}-p(x)=\frac{3}{x^2-2x+4}$$

$$\frac{x^2+6x+8}{x^3+2^3}-\frac{3}{x^2-2x+4}=p(x)$$

$$\frac{(x+4)\cancel{(x+2)}}{\cancel{(x+2)}(x^2-2x+4)}-\frac{3}{x^2-2x+4}=p(x)$$

$$\frac{(x+4)}{(x^2-2x+4)}-\frac{3}{x^2-2x+4}=p(x)$$

$$\frac{x+4-3}{x^2-2x+4}=p(x)$$

$$p(x)=\frac{x+1}{x^2-2x+4}$$

5. If $A = \frac{2x+1}{2x-1}, B = \frac{2x-1}{2x+1}$ find $\frac{1}{A-B} - \frac{2B}{A^2-B^2}$ (5M)

$$\frac{1}{A-B} - \frac{2B}{A^2-B^2} = \frac{1}{A-B} - \frac{2B}{(A+B)(A-B)}$$

$$= \frac{A+B-2B}{(A+B)(A-B)}$$

$$= \frac{\cancel{(A-B)}}{(A+B)\cancel{(A-B)}}$$

$$= \frac{1}{A+B}$$

Here $A = \frac{2x+1}{2x-1}, \quad B = \frac{2x-1}{2x+1}$

$$A + B = \frac{2x+1}{2x-1} + \frac{2x-1}{2x+1}$$

$$= \frac{(2x+1)^2+(2x-1)^2}{(2x-1)(2x+1)}$$

$$\therefore A + B = \frac{2(4x^2+1)}{4x^2-1}$$

$$\frac{1}{A+B} = \frac{1}{\frac{2(4x^2+1)}{4x^2-1}}$$

$$= \frac{\mathbf{4x^2-1}}{\mathbf{2(4x^2+1)}}$$

$(2x+1)^2 = 4x^2 + 1 + 4x$
$(2x-1)^2 = 4x^2 + 1 - 4x$
$(2x+1)^2 + (2x-1)^2$
$= 8x^2 + 2$
$= 2(4x^2 + 1)$
$(2x-1)(2x+1) = 4x^2 - 1$

Similar Problems (5M)
Solve Your Self

6. If $A = \frac{x}{x+1}, B = \frac{1}{x+1}$, prove that $\frac{(A+B)^2+(A-B)^2}{A \div B} = \frac{2(x^2+1)}{x(x+1)^2}$

CQ: $P = \frac{x}{x+y}, Q = \frac{y}{x+y}$ then find $\frac{1}{P^2-Q^2}$ **(MAY-22)**

8. Iniya bought 50kg of fruits consisting of apples and bananas. She paid twice as much per kg for the apple as she did for the banana. If Iniya bought ₹ 1800 worth of apples and ₹ 600 worth bananas, then how many kgs of each fruit did she buy? (5M)

Weight of apples $= x$
Weight of bananas $= y$
Total weight $= 50$
$x + y = 50$ ...... (1)
Cost of banana $= z/kg$
Cost of apple $= 2z/kg$
Total amount of apples =₹1800
$2z \times x = 1800$
$2zx = 1800$
$x = \frac{\cancel{1800}}{\cancel{2z}}$
$x = \frac{900}{z}$..................... (2)
Total amount of bananas = ₹600
$yz = 600$
$y = \frac{600}{z}$..................... (3)
$x = \frac{900}{z}, \; y = \frac{600}{z}$

Substituting in (1)
$x + y = 50$
$\frac{900}{z} + \frac{600}{z} = 50$
$\frac{1500}{z} = 50$
$= \frac{\cancel{1500}}{\cancel{50}}$
$z = 30$
$z = 30$ Substituting in (2) & (3)
$x = \frac{900}{z} = \frac{\cancel{900}}{\cancel{30}} = 30$
$x = 30$
$y = \frac{600}{z} = \frac{600}{30} = 20$
$y = 20$
Weight of apple $=$ **30** kgs
Weight of banana $=$ **20** kgs

Similar Problems (5M)
Solve Your Self

7. Pari needs 4 hours to complete a work. His friend Yuvan needs 6 hours to complete the same work. How long will it take to complete if they work together?

UE-8: Arul, Madan and Ram working together can clean a store in 6 hours. Working alone, Madan takes twice as long to clean the store as Arul does. Ram needs three times as long as Arul does. How long would it take each if they are working alone?

Exercise 3.7

1. Find the square root of the following rational expressions.

(i) $\frac{\mathbf{400x^4y^{12}z^{16}}}{\mathbf{100x^8y^4z^4}}$ (2M) JUL-22

$$\sqrt{\frac{400x^4y^{12}z^{16}}{100x^8y^4z^4}} = \left[\frac{4x^4y^{12}z^{16}}{x^8y^4z^4}\right]^{\frac{1}{2}}$$

$$= \left[\frac{4y^8z^{12}}{x^4}\right]^{\frac{1}{2}}$$

$$= 2\left|\frac{y^4z^6}{x^2}\right|$$

Similar Problems (2M)
Solve Your Self

1. Find the square root of the following rational expressions.
(ii) $\frac{7x^2+2\sqrt{14}x+2}{x^2-\frac{1}{2}x+\frac{1}{16}}$ (iii) $\frac{121(a+b)^8(x+y)^8(b-c)^8}{81(b-c)^4(a-b)^{12}(b-c)^4}$

Eg. 3.19: Find the square root of the following expressions
(i) $256(x-a)^8 (x-b)^4 (x-c)^{16}(x-d)^{20}$ (ii) $\frac{144a^8b^{12}c^{16}}{81f^{12}g^4h^{14}}$ **(PTA-5)**

2. Find the square root of the following

(i) $4x^2+20x+25$ (2M)

$4x^2+20x+25$

$=\sqrt{4x^2+20x+25}$

$=\sqrt{(2x+5)(2x+5)}$

$=|\mathbf{2x+5}|$

100

20

5 $\frac{\cancel{10}}{\cancel{4}}$ 2 $\qquad$ $\frac{\cancel{10}}{\cancel{4}}$ 5 2

Similar Problems (5M)
Solve Your Self

2. Find the square root of the following
(ii) $9x^2-24xy+30xz-40yz+25z^2+16y^2$
(iii) $(4x^2-9x+2)(7x^2-13x-2)(28x^2-3x-1)$
(iv) $\left(2x^2+\frac{17}{6}x+1\right)\left(\frac{3}{2}x^2+4x+2\right)\left(\frac{4}{3}x^2+\frac{11}{3}x+2\right)$

Eg. 3.20: Find the square root of the following expressions
(i) $16x^2+9y^2-24xy+24x-18y+9$
(ii) $(6x^2+x-1)(3x^2+2x-1)(2x^2+3x+1)$
(iii) $[\sqrt{15}x^2+(\sqrt{3}+\sqrt{10})x+\sqrt{2}]$
$[\sqrt{5}x^2+(2\sqrt{5}+1)x+2][\sqrt{3}x^2+(\sqrt{2}+2\sqrt{3})x+2\sqrt{2}]$

Exercise 3.8

1. Find the square root of the following polynomials by division method

(i) $x^4-12x^3+42x^2-36x+9$ JUL-22 (5M)

$$\begin{array}{r|lllll}
 & 1 & -6 & +3 & & \\
\hline
1 & \cancel{1} & -12 & +42 & -36 & +9 \\
 & \cancel{1} & & & & \\
 & (-) & & & & \\
\hline
2-6 & & -\cancel{12} & +42 & & \\
 & & -\cancel{12} & +36 & & \\
 & & (+) & (-) & & \\
\hline
2-12+3 & & & 6 & -\cancel{36} & +\cancel{9} \\
 & & & 6 & -\cancel{36} & +\cancel{9} \\
 & & & (-) & (+) & (-) \\
\hline
 & & & & 0 & \\
\end{array}$$

$\sqrt{x^4-12x^3+42x^2-36x+9}\quad=|x^2-6x+3|$

Similar Problems Solve Your Self (5M)

1. Find the square root of the following polynomials by division method
(ii) $37x^2-28x^3+4x^4+42x+9$ (iii) $16x^4+8x^2+1$
(iv) $121x^4-198x^3-183x^2+216x+144$ **(JUN-23)**

Eg. 3.21: Find the square root of $64x^4-16x^3+17x^2-2x+1$ **(SEP-21)**

UE-9: Find the square root of $289x^4-612x^3+970x^2-684x+361$

CQ: Find the square root of the expression $\frac{x^2}{y^2}-\frac{10x}{y}+27-\frac{10y}{x}+\frac{y^2}{x^2}$ **(SEP-20)**

2. Find the values of a and b if the following polynomials are perfect squares

(i) $4x^4-12x^3+37x^2+bx+a$ PTA-4 (5M)

$$\begin{array}{r|lllll}
 & 2 & -3 & +7 & & \\
\hline
2 & \cancel{4} & -12 & +37 & +b & +a \\
 & \cancel{4} & & & & \\
 & (-) & & & & \\
\hline
4-3 & & -\cancel{12} & +37 & & \\
 & & -\cancel{12} & +9 & & \\
 & & (+) & (-) & & \\
\hline
4-6+7 & & & \cancel{28} & +b & +a \\
 & & & \cancel{28} & -42 & +49 \\
 & & & (-) & (+) & (-) \\
\hline
 & & & & & 0 \\
\end{array}$$

The given polynomial is perfect square then $a=49, b=-42$

3. Find the values of m and n if the following polynomials are perfect squares (5M) MAY-22

(i) $36x^4-60x^3+61x^2-mx+n$

$$\begin{array}{r|lllll}
 & 6 & -5 & +3 & & \\
\hline
6 & \cancel{36} & -60 & +61 & -m & +n \\
 & \cancel{36} & & & & \\
 & (-) & & & & \\
\hline
12-5 & & -\cancel{60} & +61 & & \\
 & & -\cancel{60} & +25 & & \\
 & & (+) & (-) & & \\
\hline
12-10+3 & & & \cancel{36} & -m & +n \\
 & & & \cancel{36} & -30 & 9 \\
 & & & (-) & (+) & (-) \\
\hline
 & & & & 0 & \\
\end{array}$$

Similar Problems (Solve Your Self) (5M)

2. Find the values of a and b if the following polynomials are perfect squares (ii) $ax^4+bx^3+361x^2+220x+100$
3. Find the values of m and n if the following polynomials are perfect squares (ii) $x^4-8x^3+mx^2+nx+16$

Eg. 3.22: If $9x^4+12x^3+28x^2+ax+b$ is a perfect square, find the values of a and b. **(PTA-5)**

CQ: Find the values of a and b if $16x^4-24x^3+(a-1)x^2+(b+1)x+49$ is a perfect square. **(PTA-2)**

The given polynomial is perfect square $-m+30=0\Rightarrow -m=-30\Rightarrow \boldsymbol{m=30}$, $n-9=0\Rightarrow \boldsymbol{n=9}$

Exercise 3.9

1. Determine the quadratic equations, whose sum and product of roots are

(i) $-9, 20$ SEP-21

$\alpha+\beta=-9, \alpha\beta=20$ 2M

The general form

$x^2-(\alpha+\beta)x+\alpha\beta=0$

$\boldsymbol{x^2+9x+20=0}$

> **Similar Problems** 2M
> **Solve Your Self**
> 1. Determine the quadratic equations, whose sum and product of roots are
> (ii) $\frac{5}{3}, 4$ (iii) $-\frac{3}{2}, -1$ **(PTA-4)** (iv) $-(2-a)^2, (a+5)^2$
> **Eg. 3.24:** Write down the quadratic equation in general form for which sum and product of the roots are given below.
> (i) $9, 14$ (ii) $-\frac{7}{2}, \frac{5}{2}$ (iii) $-\frac{3}{5}, -\frac{1}{2}$

2. Find the sum and product of the roots for each of the following quadratic equations

(i) $x^2+3x-28=0$ 2M

Compare with

$ax^2+bx+c=0$

$a=1,\ b=3,\ c=-28$

$\alpha+\beta=-\frac{b}{a}=-\frac{3}{1}=\boldsymbol{-3}$

$\alpha\beta=\frac{c}{a}=-\frac{28}{1}=\boldsymbol{-28}$

> **Similar Problems (Solve Your Self)** 2M
> 2. Find the sum and product of the roots for each of the following quadratic equations (ii) $x^2+3x=0$ (iii) $3+\frac{1}{a}=\frac{10}{a^2}$ (iv) $3y^2-y-4=0$
> **Eg. 3.23:** Find the zeroes of the quadratic expression $x^2+8x+12$.
> **Eg.3.25:** Find the sum & product of the roots for each of the following quadratic equation:
> (i) $x^2+8x-65=0$ (ii) $2x^2+5x+7=0$ (iii) $kx^2-k^2x-2k^3=0$
> **CQ:** Find the sum and product of the roots of equation $8x^2-25=0$ **(PTA-4)**

Exercise 3.10

1. Solve the following quadratic equations by factorization method 2M

(i) $4x^2-7x-2=0$

$(4x+1)(x-2)=0$

-8
-7
$\frac{1}{4}$ $\frac{-8}{4}$ 2

$4x+1=0$ | $x-2=0$
$\boldsymbol{x=-\frac{1}{4}}$ | $\boldsymbol{x=2}$

$\therefore \boldsymbol{x=\left\{-\frac{1}{4}, 2\right\}}$

> **Similar Problems** Solve Your Self 2M
> 1. Solve the following quadratic equations by factorization method
> (ii) $3(p^2-6)=p(p+5)$ (iii) $\sqrt{a(a-7)}=3\sqrt{2}$
> (iv) $\sqrt{2}x^2+7x+5\sqrt{2}=0$ (v) $2x^2-x+\frac{1}{8}=0$
> **Eg. 3.26:** Solve $2x^2-2\sqrt{6}x+3=0$ **(PTA-6)**
> **Eg. 3.27:** Solve $2m^2+19m+30=0$
> **Eg. 3.28:** Solve $x^4-13x^2+42=0$ **(PTA-1)**
> **Eg.3.29:** Solve $\frac{x}{x-1}+\frac{x-1}{x}=2\frac{1}{2}$
> **UE-10:** Solve $\sqrt{y+1}+\sqrt{2y-5}=3$

2. The number of volleyball games that must be scheduled in a league with n teams is given by $G(n)=\frac{n^2-n}{2}$ where each team plays with every other team exactly once. A league schedules 15 games. How many teams are in the league? 2M

$G(n)=\frac{n^2-n}{2}$

Total number of games $=15$

$\frac{n^2-n}{2}=15$

$n^2-n=30$

$n^2-n-30=0$

$(n-6)(n+5)=0$

-30
-1
-6 +5

$n-6=0$ | $n+5=0$
$n=6$ | $n=-5$

Here $n\neq-5$ because n must be positive

$n=6$

Total number of teams $=6$

Exercise 3.11

1. Solve the following quadratic equations by completing the square method. 5M

(ii) $\frac{5x+7}{x-1} = 3x+2$ PTA-3

$$5x+7 = (3x+2)(x-1)$$
$$5x+7 = 3x^2-3x+2x-2$$
$$5x+7 = 3x^2-x-2$$
$$3x^2-6x-9=0$$
$$\div 3,\quad x^2-2x-3=0$$
$$x^2-2x=3$$
$$x^2-2(1)(x)=3$$

Adding 1 on both side

$$x^2-2(x)+1=3+1$$

$\left(-\frac{b}{2}\right)^2 = \left(\frac{-(-2)}{2}\right)^2$
Adding $(1)^2 = 1$

$$x^2-2(x)+1=4$$
$$(x-1)^2=2^2$$
$$(x-1)=\pm 2$$

$x-1=+2$	$x-1=-2$
$x=2+1$	$x=-2+1$
$x=3$	$x=-1$

$\therefore \boldsymbol{x = \{3, -1\}}$

Similar Problems (Solve Your Self) 5M
1. Solve the following quadratic equations by completing the square method(i) $9x^2-12x+4=0$
Eg.3.30: Solve $x^2-3x-2=0$ 2M
Eg.3.31: Solve $2x^2-x-1=0$ 2M

2. Solve the following quadratic equations by formula method (i) $2x^2-5x+2=0$ 5M

Here $a=2, b=-5, c=2$

$$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$= \frac{5\pm\sqrt{(-5)^2-4(2)(2)}}{2(2)}$$
$$= \frac{5\pm\sqrt{25-16}}{4}$$
$$= \frac{5\pm\sqrt{9}}{4}$$
$$= \frac{5\pm 3}{4}$$

$x=\frac{5+3}{4}$	$x=\frac{5-3}{4}$
$=\frac{8}{4}$	$x=\frac{2}{4}$
$x=2$	$x=\frac{1}{2}$

$\therefore \boldsymbol{x = \left\{2, \frac{1}{2}\right\}}$

Similar Problems (Solve Your Self)
2. Solve the following quadratic equations by formula method (ii) $\sqrt{2}f^2-6f+3\sqrt{2}=0$ 5M
(iii) $3y^2-20y-23=0$ (iv) $36y^2-12ay+(a^2-b^2)=0$
Eg. 3.32: Solve $x^2+2x-2=0$ by formula method **(JUL-22)**
Eg. 3.33: Solve $2x^2-3x-3=0$ by formula method.
Eg. 3.34: Solve $3p^2+2\sqrt{5}p-5=0$ by formula method.
Eg. 3.35: Solve $pqx^2-(p+q)^2x+(p+q)^2=0$
CQ: Find the value of x in $x^2-4x+12$ **(JUL-22)**
CQ: Solve the equation $\frac{1}{x+1}+\frac{2}{x+2}=\frac{4}{x+4}$ where $x+1\neq 0$, $x+2\neq 0$ & $x+4\neq 0$ using quadratic formula. **(PTA-3)** 2M

3. A ball rolls down a slope and travels a distance $d = t^2-0.75t$ feet in t seconds. Find the time when the distance travelled by the ball is 11.25 feet. 5M

Total distance $(d) = t^2-0.75t$

Total time $= t\ sec$

But $d = 11.25$ feet

$$t^2-0.75t=11.25$$
$$t^2-0.75t-11.25=0$$
$$t^2-\frac{75}{100}t-\frac{1125}{100}=0$$
$$\frac{100t^2-75t-1125}{100}=0$$
$$100t^2-75t-1125=0$$

$\div\ by\ 25,$

$$\frac{100}{25}t^2-\frac{75}{25}t-\frac{1125}{25}=0$$
$$4t^2-3t-45=0$$

Compare with

$$ax^2+bx+c=0$$
$$a=4, b=-3, c=-45$$
$$t=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$=\frac{-(-3)\pm\sqrt{(-3)^2-4(4)(-45)}}{2(4)}$$
$$=\frac{3\pm\sqrt{9+(16\times 45)}}{8}$$
$$=\frac{3\pm\sqrt{9+720}}{8}$$
$$=\frac{3\pm\sqrt{729}}{8}$$
$$=\frac{3\pm 27}{8}$$

$t=\frac{3+27}{8}$	$t=\frac{3-27}{8}$
$t=\frac{30}{8}$	$t=\frac{-24}{8}=-3$
$t=3.75$	$t\neq -3$

Because t must be positive

$\therefore$ Time $= \boldsymbol{3.75}$ **seconds**

Exercise 3.12

1. If the difference between a number and its reciprocal is $\frac{24}{5}$, find the number. (2M) PTA-6

First number $= x$,

It's reciprocal $= \frac{1}{x}$

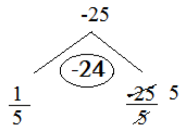
Difference $= \frac{24}{5}$

$$x - \frac{1}{x} = \frac{24}{5}$$

$$\frac{x^2-1}{x} = \frac{24}{5}$$

$$5x^2 - 5 = 24x$$

$$5x^2 - 24x - 5 = 0$$



$$(5x+1)(x-5) = 0$$

$\therefore x - 5 = 0$ | $5x + 1 = 0$

$x = 5$ | $5x = -1$

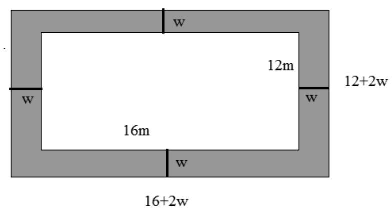
$x = -\frac{1}{5}$

If the number is **5** and its reciprocal $\frac{1}{5}$

If the number is $-\frac{1}{5}$ and its reciprocal -5

> **Similar Problems**
> (Solve Your Self)
> **CQ:** Find two consecutive positive integers, sum of whose squares is 365. **(MDL)** (5M)

2. A garden measuring $12\ m$ by $16\ m$ is to have a pedestrian pathway that is $'w'$ meters wide installed all the way around so that it increases the total area to $285m^2$. What is the width of the pathway? (5M)



For garden

Length $= 16m$,

Breadth $= 12m$

Width of path $= w$

Length $= (16+2w)m$

Breadth $= (12+2w)m$

$$\therefore (16+2w)(12+2w) = 285$$

$$192 + 56w + 4w^2 = 285$$

$$4w^2 + 56w + 192 - 285 = 0$$

$$4w^2 + 56w - 93 = 0$$

$a = 4, b = 56, c = -93$

$$w = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-56 \pm \sqrt{56^2 - 4(4)(-93)}}{2(4)}$$

$$= \frac{-56 \pm \sqrt{3136+1488}}{8} = \frac{-56 \pm 68}{8}$$

$w = -\frac{56+68}{8}$ | $w = \frac{-56-68}{8}$

$= \frac{12}{8}$ | $w = -\frac{124}{8}$

$= 1.5m$

Here w must be positive $w \neq -\frac{124}{8}$

$\therefore width = \mathbf{1.5m}$

> **Similar Problems**
> Solve Your Self (5M)
> 8. There is a square field whose side 10m. A square flower bed is prepared in its centre leaving a gravel path all-round the flower bed. The total cost of laying the flower bed and gravelling the path at ₹3 and ₹ 4 per square metre respectively is ₹ 364. Find the width of the gravel path.
> **UE-12:** Is it possible to design a rectangular park of perimeter 320 m and area $4800m^2$? If so find its length and breadth.

3. A bus covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hour more it would have taken 30 minutes less for the journey. Find the original speed of the bus. (5M)

Speed of a bus $= x\ km/hr$

Total distance $= 90km$

Time taken $= T_1 = \frac{\text{distance}}{\text{speed}} = \frac{90}{x}$ hours

Increasing speed $= 15\ km/hr$

Speed of a bus $= (x+15)km/hr$

Total distance $= 90km$

$$T_2 = \frac{90}{(x+15)} hours$$

Time difference $= 30 minutes$

$$T_1 - T_2 = \frac{1}{2} hours$$

$$\frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$$

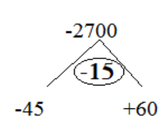
$$\frac{90(x+15) - 90x}{x(x+15)} = \frac{1}{2}$$

$$90x + 1350 - 90x = \frac{1}{2}(x^2 + 15x)$$

$$x^2 + 15x = +1350 \times 2$$

$$x^2 + 15x - 2700 = 0$$

$$(x-45)(x+60) = 0$$



$$x = 45 \text{ (or) } -60$$

x must be positive $\therefore \boldsymbol{x} = \mathbf{45}\ km/hr$.

Similar Problems 5M

Solve Your Self

Eg. 3.39: A passenger train takes 1 hr more than an express train to travel a distance of 240 km from Chennai to Virudhachalam. The speed of express train is more than that of an passenger train by 20 km per hour. Find the average speed of both the trains.

UE-11: A boat takes 1.6 hours longer to go 36 *kms* up a river than down the river. If the speed of the water current is 4 *km* per hours. What is the speed of the boat in still water?

CQ: A motor boat whose speed is 18 km/hr in still water takes 1 hour more to go to 24 km upstream than to return downstream to the same spot. Find the speed of the stream. **(MDL, PTA-2)**

CQ: A train covered a certain distance at a uniform speed. If the train would have been 10km/hr faster it would have taken 2 hour less than the scheduled time and if the train were slower by 10km/hr, it would have taken 3 hour more than the scheduled time. Find the distance covered by the train. **(PTA-5)**

CQ: A car left 30 minutes later than the scheduled time. In order to reach its destination 150km away in time, it has to increase its speed by 25km/hr from its usual speed. Find its usual speed. **(PTA-6)**

4. A girl is twice as old as her sister. Five years hence, the product of their ages (in years) will be 375. Find their present ages. 5M PTA-4

Sister's age $= x$

Girl's age $= 2x$

After 5 years

Sister's age $= x + 5$

Girl's age $= 2x + 5$

Product $= 375$

$(x+5)(2x+5) = 375$

$2x^2 + 5x + 10x + 25 = 375$

Similar Problems 5M

Solve Your Self

Eg. 3.36: The product of Kumaran's age (in years) two years ago and his age four years from now is one more than twice his present age. What is his present age? **(PTA-1)**

$2x^2 + 15x - 350 = 0$

-700

$\frac{35}{2}$ $\frac{\cancel{-20}}{2}$ -10

$(2x+35)(x-10) = 0$

$2x + 35 = 0$

$2x = -35$

$x = -\frac{35}{2}$

$x - 10 = 0$

$x = 10$

$\therefore x = 10$ [Because x must be positive]

Sister's age $= x =$ **10 years**

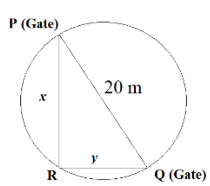
Girl's age $= 2x = 2(10) =$ **20 years**

5. A pole has to be erected at a point on the boundary of a circular ground of diameter 20m in such a way that the difference of its distances from two diametrically opposite fixed gates P and Q on the boundary is 4m. Is it possible to do so? If answer is yes at what distance from the two gates should the pole be erected? 5M

From the given data

$x - y = 4$

$x = 4 + y$



From the figure,

ΔPQR is right angled triangle

$PQ^2 = PR^2 + RQ^2$

$20^2 = x^2 + y^2$

$(4+y)^2 + y^2 = 20^2$

$16 + y^2 + 8y + y^2 = 400$

$2y^2 + 8y - 384 = 0$

$\div$ by 2

$y^2 + 4y - 192 = 0$

$y^2 + 16y - 12y - 192 = 0$

-192

+16 -12

$(y+16)(y-12) = 0$

$y = -16$(or)12

y must be positive

$\therefore y = 12$

$x = 4 + y \quad = 4 + 12 = 16$

$x = 16$

Yes, it is possible.

$16 - 12 = 4$

The pole should be erected at the distance of P from **16m** & Q from **12m.**

6. **From a group of $2x^2$ black bees, square root of half of the group went to a tree. Again eight-ninth of the bees went to the same tree. The remaining two got caught up in a fragrant lotus. How many bees were there in total ?** (5M)

Total bees $= 2x^2$

Square root of half of the group

$= \sqrt{x^2} = x$

Eight ninth of the bees $= \frac{8}{9}(2x^2)$

$= \frac{16x^2}{9}$

Remaining bees $= 2$

$\therefore 2x^2 - \left[x + \frac{16x^2}{9}\right] = 2$

$2x^2 - \left[\frac{9x+16x^2}{9}\right] = 2$

$18x^2 - 9x - 16x^2 = 18$

$2x^2 - 9x - 18 = 0$

$(x-6)(2x+3) = 0$

$x - 6 = 0 \quad | \quad 2x + 3 = 0$

$x = 6 \quad | \quad 2x = -3$

$x = -\frac{3}{2}$

$x = 6$ [Because x must be positive]

$\therefore$ Total bees $= 2x^2$

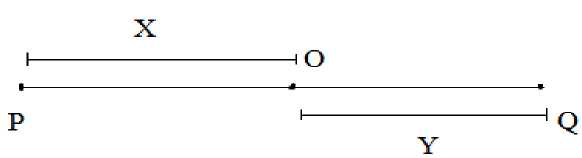
$= 2(6)^2$

$= 2(36)$

$= 72$ bees

Similar Problems (5M)
Solve Your Self
Eg. 3.38: A flock of swans contained x^2 members. As the clouds gathered, $10x$ went to a lake and one-eighth of the members flew away to a garden. The remaining three pairs played about in the water. How many swans were there in total?

7. **Music is been played in two opposite galleries with certain group of people. In the first gallery a group of 4 singers were singing and in the second gallery 9 singers were singing. The two galleries are separated by the distance of 70m. Where should a person stand for hearing the same intensity of the singer's voice? (Hint: The ratio of the sound intensity is equal to the square of the ratio of their corresponding distances)** (5M)



P – First gallery

Q – Second gallery

Distance between galleries $= 70m$

O is the point of a person standing between galleries.

Number of singers in the first gallery (P) $= 4$

Number of singer in the second gallery (Q) $= 9$

Distance between P and O $= x\ m$

Distance between Q and O $= y\ m$

$x + y = 70$ ……….. (1)

By using hint $\frac{4}{9} = \frac{x^2}{y^2}$

Taking square root both sides

$\frac{2}{3} = \frac{x}{y}$

$x = \frac{2y}{3}$ ……………………… (2)

Substituting (2) in (1)

$x + y = 70$

$\frac{2y}{3} + y = 70$

$2y + 3y = 70 \times 3$

$5y = 210$

$y = \frac{210}{5}$

$y = 42\ m$

Substituting y in (1), $\quad x + y = 70$

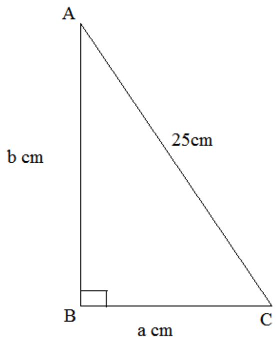
$x + 42 = 70$

$x = 70 - 42 = 28$

$x = 28\ m$

A person standing **28 m** from P and **42m** from Q.

9. The hypotenuse of a right angled triangle is 25 cm and its perimeter 56 cm. find the length of the smallest side. (5M)



$b = a\ cm, \quad h = b\ cm$

Hypotenuse $= 25cm$

Perimeter $= 56cm$

$a + b + 25 = 56$

$a + b = 56 - 25$

$a + b = 31$

$b = 31 - a$

$a^2 + b^2 = 25^2$

$a^2 + (31 - a)^2 = 25^2$

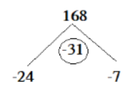
$a^2 + 961 + a^2 - 62a = 625$

$2a^2 - 62a + 961 - 625 = 0$

$2a^2 - 62a + 336 = 0$

$\div$ By 2

$a^2 - 31a + 168 = 0$



$(a - 24) \mid (a - 7) = 0$

$a = 24 \mid a = 7$

$\therefore$ The length of the smallest side is **7cm.**

Similar Problems (2M)
Solve Your Self

Eg. 3.37: A ladder 17 feet long is leaning against a wall. If the ladder, vertical wall and the floor from the bottom of the wall to the ladder form a right triangle, find the height of the wall where the top of the ladder meets if the distance between bottom of the wall to bottom of the ladder is 7 feet less than the height of the wall?

Similar Problems (Solve Your Self) (5M)

UE-13: At t minutes past 2 pm, the time needed to 3pm is 3 minutes less than $\frac{t^2}{4}$. Find t.

UE-14: The number of seats in a row is equal to the total number of rows in a hall. The total number of seats in the hall will increase by 375 if the number of rows is doubled and the number of seats in each row is reduced by 5. Find the number of rows in the hall at the beginning.

Exercise 3.13

1. Determine the nature of the roots for the following quadratic equations (2M)

(i) $15x^2 + 11x + 2 = 0$

Compare with $ax^2 + bx + c = 0$

$a = 15\ , b = 11\ , c = 2$ (JUN-23, SEP-21)

$\Delta = b^2 - 4ac$

$= 11^2 - 4(15)(2)$

$= 121 - 120$

$\Delta = 1,\ $ Here $\Delta > 0$

$\therefore$ The roots are **real and unequal**

Similar Problems (2M)
Solve Your Self

1. Determine the nature of the roots for the following quadratic equations
(ii) $x^2 - x - 1 = 0$ (iii) $\sqrt{2}t^2 - 3t + 3\sqrt{2} = 0$ (iv) $9y^2 - 6\sqrt{2}y + 2 = 0$
(v) $9a^2b^2x^2 - 24abcdx + 16c^2d^2 = 0, a \neq 0, b \neq 0$

CQ: Determine the nature of roots for the following quadratic equation.
$2x^2 - x - 1 = 0$ **(APR-23)**

Eg. 3.40: Determine the nature of roots for the following quadratic equations
(i) $x^2 - x - 20 = 0$ (ii) $9x^2 - 24x + 16 = 0$ (iii) $2x^2 - 2x + 9 = 0$

2. Find the value(s) of $'k'$ for which the roots of the following equations are real and equal.

(i) $(5k - 6)x^2 + 2kx + 1 = 0$ (2M)

$\Delta = 0$

$a = 5k - 6\ , b = 2k, \quad c = 1$

$\Delta = b^2 - 4ac$

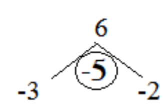
$b^2 - 4ac = 0$

$(2k)^2 - 4(5k - 6)(1) = 0$

$4k^2 - 20k + 24 = 0$

$\div$ by 4 $\quad k^2 - 5k + 6 = 0$

$(k - 3)(k - 2) = 0$



$k - 3 = 0 \mid k - 2 = 0$

$k = 3 \mid k = 2$

$\therefore \boldsymbol{k = 2,\ 3}$

Similar Problems Solve Your Self (2M)

2. Find the value(s) of $'k'$ for which the roots of the equations are real & equal. (ii) $kx^2 + (6k + 2)x + 16 = 0$

Eg. 3.41 : (i) Find the values of 'k', for which the quadratic equation $kx^2 - (8k + 4)x + 81 = 0$ has real and equal roots?
(ii) Find the values of 'k' such that quadratic equation $(k + 9)x^2 + (k + 1)x + 1 = 0$ has no real roots?

CQ: Find the value of k for which the equation $9x^2 + 3kx + 4 = 0$ has real and equal roots. **(SEP-20)**

5. **If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + b^2 - ac = 0$ are real and equal prove that either $a = 0$ (or) $a^3 + b^3 + c^3 = 3abc$.** PTA-6 5M

$\Delta = 0$

$a = c^2 - ab, b = -2(a^2 - bc), c = b^2 - ac$

$b^2 - 4ac = 0$

$$\begin{aligned} b^2 - 4ac &= \left(-2(a^2 - bc)\right)^2 - 4(c^2 - ab)(b^2 - ac) \\ &= 4(a^4 + b^2c^2 - 2a^2bc) - 4(c^2b^2 - ac^3 - ab^3 + a^2bc) \\ &= 4[a^4 + b^2c^2 - 2a^2bc - c^2b^2 + ac^3 + ab^3 - a^2bc] \\ &= 4[a^4 + ac^3 + ab^3 - 3a^2bc] \\ &= 4a[a^3 + b^3 + c^3 - 3abc] \end{aligned}$$

$4a(a^3 + b^3 + c^3 - 3abc) = 0$ $\quad \boxed{\because b^2 - 4ac = 0}$

$4a = 0$ | $a^3 + b^3 + c^3 - 3abc = 0$

$\boldsymbol{a = 0}$ | $\boldsymbol{a^3 + b^3 + c^3 = 3abc}$ $\quad \therefore$ Hence proved

Similar Problems
Solve Your Self 5M

3. If the roots of $(a-b)x^2 + (b-c)x + (c-a) = 0$ are real and equal, then prove that b, a, c are in arithmetic progression.
4. If a, b are real then show that the roots of the equation $(a-b)x^2 - 6(a+b)x - 9(a-b) = 0$ are real and unequal.

Eg. 3.42: Prove that the equation $x^2(p^2+q^2) + 2x(pr+qs) + r^2 + s^2 = 0$ has no real roots. If $ps = pr$, then show that the roots are real and equal.

CQ: If the equation $(1+m^2)x^2 + 2mcx + (c^2 - a^2) = 0$ has equal roots, then prove that $c^2 = a^2(1+m^2)$ **(SEP-21)**

Exercise 3.14

1. **Write each of the following expression in terms of $\alpha + \beta$ and $\alpha\beta$.** 2M

(i) $\dfrac{\alpha}{3\beta} + \dfrac{\beta}{3\alpha}$

$$\frac{\alpha}{3\beta} + \frac{\beta}{3\alpha} = \frac{\alpha^2+\beta^2}{3\alpha\beta} = \frac{\alpha^2+\beta^2}{3\alpha\beta} = \boldsymbol{\frac{(\alpha+\beta)^2 - 2\alpha\beta}{3\alpha\beta}}$$

(ii) $\dfrac{1}{\alpha^2\beta} + \dfrac{1}{\beta^2\alpha}$

$$\frac{1}{\alpha^2\beta} + \frac{1}{\beta^2\alpha} = \frac{\beta+\alpha}{\alpha^2\beta^2} = \frac{\alpha+\beta}{\alpha^2\beta^2} = \boldsymbol{\frac{\alpha+\beta}{(\alpha\beta)^2}}$$

(iii) $(3\alpha - 1)(3\beta - 1)$

$$\begin{aligned}(3\alpha - 1)(3\beta - 1) &= 9\alpha\beta - 3\alpha - 3\beta + 1 \\ &= \boldsymbol{9\alpha\beta - 3(\alpha+\beta) + 1}\end{aligned}$$

(iv) $\dfrac{\alpha+3}{\beta} + \dfrac{\beta+3}{\alpha}$

$$\frac{\alpha+3}{\beta} + \frac{\beta+3}{\alpha} = \frac{\alpha^2+3\alpha+\beta^2+3\beta}{\alpha\beta} = \frac{\alpha^2+\beta^2+3(\alpha+\beta)}{\alpha\beta} = \boldsymbol{\frac{(\alpha+\beta)^2 - 2\alpha\beta + 3(\alpha+\beta)}{\alpha\beta}}$$

2. **The roots of the equation $2x^2 - 7x + 5 = 0$ are α and β. without solving for the roots, find** 5M

$2x^2 - 7x + 5 = 0$

$a = 2, \ b = -7, c = 5$

$\alpha + \beta = -\frac{b}{a} \Rightarrow \alpha + \beta = \boldsymbol{\frac{7}{2}}$,

$\alpha\beta = \frac{c}{a} \Rightarrow \alpha\beta = \boldsymbol{\frac{5}{2}}$

(i) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta+\alpha}{\alpha\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{7}{2}}{\frac{5}{2}} = \frac{7}{2} \times \frac{2}{5} = \boldsymbol{\frac{7}{5}}$$

(ii) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$

$$\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^2+\beta^2}{\alpha\beta}=\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$=\frac{\left(\frac{7}{2}\right)^2-2\left(\frac{5}{2}\right)}{\frac{5}{2}}=\frac{\frac{49}{4}-5}{\frac{5}{2}}=\frac{49-20}{4}\times\frac{2}{5}=\mathbf{\frac{29}{10}}$$

(iii) $\frac{\alpha+2}{\beta+2}+\frac{\beta+2}{\alpha+2}$

$$\frac{\alpha+2}{\beta+2}+\frac{\beta+2}{\alpha+2}=\frac{(\alpha+2)^2+(\beta+2)^2}{(\alpha+2)(\beta+2)}$$

$$=\frac{\alpha^2+4\alpha+4+\beta^2+4\beta+4}{\alpha\beta+2\alpha+2\beta+4}$$

$$=\frac{\alpha^2+\beta^2+4(\alpha+\beta)+8}{\alpha\beta+2(\alpha+\beta)+4}$$

$$=\frac{(\alpha+\beta)^2-2\alpha\beta+4(\alpha+\beta)+8}{\alpha\beta+2(\alpha+\beta)+4}$$

$$=\frac{\left(\frac{7}{2}\right)^2-2\left(\frac{5}{2}\right)+4\left(\frac{7}{2}\right)+8}{\frac{5}{2}+2\left(\frac{7}{2}\right)+4}$$

$$=\frac{\frac{49}{4}-5+14+8}{\frac{5}{2}+7+4}$$

$$=\frac{\frac{49}{4}+17}{\frac{5}{2}+11}=\frac{\frac{49+68}{4}}{\frac{5+22}{2}}$$

$$=\frac{117}{4}\times\frac{2}{27}=\frac{117}{54}=\mathbf{\frac{13}{6}}$$

Similar Problems
Solve Your Self (5M)

3. The roots of the equation $x^2+6x-4=0$ are α,β. Find the quadratic equation whose roots are (i) α^2 and β^2 (ii) $\frac{2}{\alpha}$ and $\frac{2}{\beta}$ (iii) $\alpha^2\beta$ and $\beta^2\alpha$

UE-15: If α and β are the roots of the polynomial $f(x)=x^2-2x+3$, find the polynomial whose roots are
(i) $\alpha+2,\beta+2$ (ii) $\frac{\alpha-1}{\alpha+1},\frac{\beta-1}{\beta+1}$

Eg. 3.44: If α and β are the roots of $x^2+7x+10=0$ find the values of (i) $(\alpha-\beta)$ (ii) $\alpha^2+\beta^2$ (iii) $\alpha^3-\beta^3$ (iv) $\alpha^4+\beta^4$ (v) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ (vi) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

Eg. 3.45: If α,β are the roots of the equation $3x^2+7x-2=0$,find the values of (i) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ (ii) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

Eg. 3.46: If α, β are the roots of the equation $2x^2-x-1=0$, then form the equation whose roots are (i) $\frac{1}{\alpha},\frac{1}{\beta}$ (ii) $\alpha^2\beta$, $\beta^2\alpha$ (iii) $2\alpha+\beta$, $2\beta+\alpha$ **(MDL)**

4. If α,β are the roots of $7x^2+ax+2=0$ and if $\beta-\alpha=-\frac{13}{7}$. Find the values of a.

PTA-6, MAY-22

5M

$$7x^2+ax+2=0$$
$$a=7, b=a, c=2$$
$$\alpha+\beta=\frac{-b}{a}=\frac{-a}{7}$$
$$\alpha\beta=\frac{c}{a}=\frac{2}{7}$$
$$(\beta-\alpha)=-\frac{13}{7}$$
$$\text{Here } (\beta-\alpha)^2=\left(-\frac{13}{7}\right)^2$$
$$\beta^2+\alpha^2-2\beta\alpha=\frac{169}{49}$$
$$(\alpha+\beta)^2-2\alpha\beta-2\alpha\beta=\frac{169}{49}$$
$$(\alpha+\beta)^2-4\alpha\beta=\frac{169}{49}$$
$$\left(-\frac{a}{7}\right)^2-4\left(\frac{2}{7}\right)=\frac{169}{49}$$
$$\frac{a^2}{49}-\frac{8}{7}=\frac{169}{49}$$
$$\frac{a^2}{49}=\frac{169}{49}+\frac{8}{7}$$
$$\frac{a^2}{49}=\frac{169+56}{49}$$
$$a^2=\frac{225}{49}\times 49$$
$$a^2=225$$
$$a=\pm 15$$
$$\boldsymbol{a=15,-15}$$

Similar Problems
Solve Your Self

5. If one root of the equation $2y^2-ay+64=0$ is twice the other then find the values of a. (5M)
6. If one root of the equation $3x^2+kx+81=0$ (having real roots) is the square of the other then find k. (5M)

UE-16: If – 4 is a root of the equation $x^2+px-4=0$ and if the equation $x^2+px+q=0$ has equal roots, find the values of p and q. (5M)

Eg. 3.43: If the difference between the roots of the equation $x^2-13x+k=0$ is 17 find k. (5M)

CQ: Find the value of p, when $px^2+(\sqrt{3}-\sqrt{2})x-1=0$ and $x=\frac{1}{\sqrt{3}}$ is one root of the equation. **(PTA-5)** (2M)

Exercise 3.17

1. In the matrix $A = \begin{bmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{bmatrix}$, **write (i) The number of elements (ii) The order of the matrix (iii) Write the elements** $a_{22}, a_{23}, a_{24}, a_{34}, a_{43}, a_{44}$. 2M

$$A = \begin{bmatrix} 8 & 9 & 4 & 3 \\ -1 & \sqrt{7} & \frac{\sqrt{3}}{2} & 5 \\ 1 & 4 & 3 & 0 \\ 6 & 8 & -11 & 1 \end{bmatrix} \qquad \text{Here } A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

(i) Number of elements $= \mathbf{16}$ (ii) $\mathbf{4 \times 4}$

(iii) $a_{22} = \sqrt{\mathbf{7}}, a_{23} = \frac{\sqrt{3}}{2}, a_{24} = \mathbf{5}, a_{34} = \mathbf{0}, a_{43} = \mathbf{-11}, a_{44} = \mathbf{1}$

2. If a matrix has 18 elements, what are the possible orders it can have? What if it has 6 elements? 2M

We know that a matrix of order $m \times n$, has mn elements.

So here total elements 18 then possible orders

$\Rightarrow 1 \times 18, 2 \times 9, 3 \times 6, 6 \times 3, 9 \times 2, 18 \times 1$

similarly for total elements 6, Orders may be $1 \times 6, 2 \times 3, 3 \times 2, 6 \times 1$

> **Similar Problems** 2M
> Solve Your Self
> **Eg. 3.57:** If a matrix has 16 elements, what are the possible orders it can have?

3. Construct a 3×3 matrix whose elements are given by (i) $a_{ij} = |i - 2j|$ 2M

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_{3\times3}$$

$a_{11} = |1 - 2(1)| = |1 - 2| = |-1| = 1$

$a_{12} = |1 - 2(2)| = |1 - 4| = |-3| = 3$

$a_{13} = |1 - 2(3)| = |1 - 6| = |-5| = 5$

$a_{21} = |2 - (2)(1)| = |2 - 2| = |0| = 0$

$a_{22} = |2 - 2(2)| = |2 - 4| = |-2| = 2$

$a_{23} = |2 - 2(3)| = |2 - (6)| = |-4| = 4$

$a_{31} = |3 - 2(1)| = |3 - 2| = |1| = 1$

$a_{32} = |3 - 2(2)| = |3 - 4| = |-1| = 1$

$a_{33} = |3 - 2(3)| = |3 - 6| = |-3| = 3$

$$\therefore A = \begin{bmatrix} 1 & 3 & 5 \\ 0 & 2 & 4 \\ 1 & 1 & 3 \end{bmatrix}$$

4. If $A = \begin{bmatrix} 5 & 4 & 3 \\ 1 & -7 & 9 \\ 3 & 8 & 2 \end{bmatrix}$ **then find the transpose of A.** PTA-2 2M

$$\text{Transpose of } A = A^T = \begin{bmatrix} 5 & 1 & 3 \\ 4 & -7 & 8 \\ 3 & 9 & 2 \end{bmatrix}$$

5. If $A = \begin{bmatrix} \sqrt{7} & -3 \\ -\sqrt{5} & 2 \\ \sqrt{3} & -5 \end{bmatrix}$ **then find the transpose of $-A$.** 2M SEP-20

$$-A = \begin{bmatrix} -\sqrt{7} & 3 \\ \sqrt{5} & -2 \\ -\sqrt{3} & 5 \end{bmatrix}$$

$$\text{Transpose of } -A = \begin{bmatrix} -\sqrt{7} & \sqrt{5} & -\sqrt{3} \\ 3 & -2 & 5 \end{bmatrix}$$

> **Similar Problems** 2M
> Solve Your Self
> 3. Construct a 3×3 matrix whose elements are given by (ii) $a_{ij} = \frac{(i+j)^3}{3}$
> **Eg. 3.58:** Construct a 3×3 matrix whose elements are $a_{ij} = i^2 j^2$

6. If $A = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix}$ then verify $(A^T)^T = A$ (2M)

JUN-23

$$A^T = \begin{bmatrix} 5 & -\sqrt{17} & 8 \\ 2 & 0.7 & 3 \\ 2 & \frac{5}{2} & 1 \end{bmatrix}$$

$$(A^T)^T = \begin{bmatrix} 5 & 2 & 2 \\ -\sqrt{17} & 0.7 & \frac{5}{2} \\ 8 & 3 & 1 \end{bmatrix} = A$$

Here $(A^T)^T = A$

Hence proved.

Similar Problems
Solve Your Self

7. Find the values of x, y and z from the following equations

(i) $\begin{bmatrix} 12 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 3 & 5 \end{bmatrix}$ (2M) (iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$ (5M)

Eg.3.59: Find the value of a, b, c, d from the equation $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ 0 & 2 \end{bmatrix}$ (2M)

7. Find the values of x, y and z from the following equations (ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$ (5M)

$$\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$$

$x + y = 6$ .... (1)

$xy = 8$ .... (2)

$5 + z = 5$

$z = 5 - 5$

$z = 0$

From (1), $x = 6 - y$

$xy = 8$

$(6 - y)y = 8$

$6y - y^2 = 8$

$6y - y^2 = 8$

$\therefore y^2 - 6y + 8 = 0$

$(y - 4)(y - 2) = 0$

$y = 4$ and 2

8

-4 -2

If $y = 4$	If $y = 2$
$x = 6 - y$	$x = 6 - y$
$x = 6 - 4$	$x = 6 - 2$
$x = 2$	$x = 4$

$\boldsymbol{x = 2 \text{ and } 4, \quad y = 4 \text{ and } 2, \quad z = 0}$

For Practice:

Eg. 3.56: Consider the following information regarding the number of men and women workers in three factories I, II & III.

Factory	Men	Women
I	23	18
II	47	36
III	15	16

Represent the above information in the form of a matrix. What does the entry in the second row & first column represent? **(2M)**

UE-17: Two farmers Thilagan and Kausigan cultivates three varieties of grains namely rice, wheat and ragi. If the sale (in ₹) of three varieties of grains by both the farmers in the month of April is given by the matrix.

$$\text{April sale in ₹} \quad A = \begin{array}{c} \begin{array}{ccc} \text{Rice} & \text{Wheat} & \text{Ragi} \end{array} \\ \begin{bmatrix} 500 & 1000 & 1500 \\ 2500 & 1500 & 500 \end{bmatrix} \end{array} \begin{array}{l} \text{Thilagan} \\ \text{Kausigan} \end{array}$$

And the May month sale (in ₹) is exactly twice as that of the April month sale for each variety.

(i) What is the average sales of the months April and May. (ii) If the sales continues to increase in the same way in the successive months, what will be sales in the month of August? **(5M)**

Exercise 3.18

1. If $A = \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix}, B = \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix}$ then verify that (i) $A + B = B + A$ (2M)

$$A = \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix}, B = \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix} + \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{bmatrix} \text{...... (1)}$$

$$B + A = \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 9 \\ 3 & 4 \\ 8 & -3 \end{bmatrix} = \begin{bmatrix} 6 & 16 \\ 6 & 7 \\ 9 & -3 \end{bmatrix} \text{...... (2)}$$

$(1) = (2)$

$\therefore \boldsymbol{A + B = B + A}$

Similar Problems (Solve Your Self)

1. If $A=\begin{bmatrix}1 & 9\\3 & 4\\8 & -3\end{bmatrix}, B=\begin{bmatrix}5 & 7\\3 & 3\\1 & 0\end{bmatrix}$ then verify that (ii) $A+(-A)=(-A)+A=O$. (2M)
2. If $A=\begin{bmatrix}4 & 3 & 1\\2 & 3 & -8\\1 & 0 & -4\end{bmatrix}, B=\begin{bmatrix}2 & 3 & 4\\1 & 9 & 2\\-7 & 1 & -1\end{bmatrix}$ and $C=\begin{bmatrix}8 & 3 & 4\\1 & -2 & 3\\2 & 4 & -1\end{bmatrix}$ then verify that $A+(B+C)=(A+B)+C$. (5M)

Eg. 3.60: If $A=\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix}$, $B=\begin{bmatrix}1 & 7 & 0\\1 & 3 & 1\\2 & 4 & 0\end{bmatrix}$, find $A+B$. (2M)

Eg.3.62: If $A=\begin{bmatrix}1 & 3 & -2\\5 & -4 & 6\\-3 & 2 & 9\end{bmatrix}$, $B=\begin{bmatrix}1 & 8\\3 & 4\\9 & 6\end{bmatrix}$, find $A+B$. (2M)

3. Find X and Y if $X+Y=\begin{bmatrix}7 & 0\\3 & 5\end{bmatrix}$ and $X-Y=\begin{bmatrix}3 & 0\\0 & 4\end{bmatrix}$ (2M)

$$X+Y=\begin{bmatrix}7 & 0\\3 & 5\end{bmatrix}\text{..........} (1)$$

$$X-Y=\begin{bmatrix}3 & 0\\0 & 4\end{bmatrix}\text{..........} (2)$$

$(1)+(2)\Rightarrow$

$$2X=\begin{bmatrix}7 & 0\\3 & 5\end{bmatrix}+\begin{bmatrix}3 & 0\\0 & 4\end{bmatrix}$$

$$2X=\begin{bmatrix}10 & 0\\3 & 9\end{bmatrix}\Rightarrow X=\begin{bmatrix}\frac{10}{2} & 0\\\frac{3}{2} & \frac{9}{2}\end{bmatrix}\Rightarrow \therefore X=\begin{bmatrix}\mathbf{5} & \mathbf{0}\\\mathbf{3/2} & \mathbf{9/2}\end{bmatrix}$$

$$(1)-(2)\Rightarrow 2Y=\begin{bmatrix}7 & 0\\3 & 5\end{bmatrix}-\begin{bmatrix}3 & 0\\0 & 4\end{bmatrix}$$

$$=\begin{bmatrix}4 & 0\\3 & 1\end{bmatrix}$$

$$Y=\begin{bmatrix}\frac{4}{2} & 0\\\frac{3}{2} & \frac{1}{2}\end{bmatrix}$$

$$Y=\begin{bmatrix}2 & 0\\3/2 & 1/2\end{bmatrix}$$

4. If $A=\begin{bmatrix}0 & 4 & 9\\8 & 3 & 7\end{bmatrix}, B=\begin{bmatrix}7 & 3 & 8\\1 & 4 & 9\end{bmatrix}$ find the value of (i) $B-5A$ (ii) $3A-9B$ (2M) PTA-5

(i) $B-5A$

$$B=\begin{bmatrix}7 & 3 & 8\\1 & 4 & 9\end{bmatrix},$$

$$5A=5\begin{bmatrix}0 & 4 & 9\\8 & 3 & 7\end{bmatrix}=\begin{bmatrix}0 & 20 & 45\\40 & 15 & 35\end{bmatrix}$$

$$B-5A=\begin{bmatrix}7 & 3 & 8\\1 & 4 & 9\end{bmatrix}-\begin{bmatrix}0 & 20 & 45\\40 & 15 & 35\end{bmatrix}$$

$$=\begin{bmatrix}\mathbf{7} & \mathbf{-17} & \mathbf{-37}\\\mathbf{-39} & \mathbf{-11} & \mathbf{-26}\end{bmatrix}$$

(ii) $3A-9B$

$$3A=3\begin{bmatrix}0 & 4 & 9\\8 & 3 & 7\end{bmatrix}=\begin{bmatrix}0 & 12 & 27\\24 & 9 & 21\end{bmatrix}$$

$$9B=9\begin{bmatrix}7 & 3 & 8\\1 & 4 & 9\end{bmatrix}=\begin{bmatrix}63 & 27 & 72\\9 & 36 & 81\end{bmatrix}$$

$$3A-9B=\begin{bmatrix}0 & 12 & 27\\24 & 9 & 21\end{bmatrix}-\begin{bmatrix}63 & 27 & 72\\9 & 36 & 81\end{bmatrix}$$

$$=\begin{bmatrix}\mathbf{-63} & \mathbf{-15} & \mathbf{-45}\\\mathbf{15} & \mathbf{-27} & \mathbf{-60}\end{bmatrix}$$

Similar Problems
Solve Your Self

Eg.3.63: If $A=\begin{bmatrix}7 & 8 & 6\\1 & 3 & 9\\-4 & 3 & -1\end{bmatrix}$, $B=\begin{bmatrix}4 & 11 & -3\\-1 & 2 & 4\\7 & 5 & 0\end{bmatrix}$ then Find $2A+B$. **(PTA-3)** (2M)

Eg. 3.64: If $A=\begin{bmatrix}5 & 4 & -2\\\frac{1}{2} & \frac{3}{4} & \sqrt{2}\\1 & 9 & 4\end{bmatrix}$, $B=\begin{bmatrix}-7 & 4 & -3\\\frac{1}{4} & \frac{7}{2} & 3\\5 & -6 & 9\end{bmatrix}$, find $4A-3B$. (2M)

Eg. 3.66: If $A=\begin{bmatrix}1 & 8 & 3\\3 & 5 & 0\\8 & 7 & 6\end{bmatrix}, B=\begin{bmatrix}8 & -6 & -4\\2 & 11 & -3\\0 & 1 & 5\end{bmatrix}, C=\begin{bmatrix}5 & 3 & 0\\-1 & -7 & 2\\1 & 4 & 3\end{bmatrix}$ compute the following: (i) $3A+2B-c$ (ii) $\frac{1}{2}A-\frac{3}{2}B$ (2M)

CQ: If $A=\begin{bmatrix}1 & 2\\3 & 4\end{bmatrix}, B=\begin{bmatrix}0 & 3\\-1 & 5\end{bmatrix}, C=\begin{bmatrix}-1 & 5\\1 & 3\end{bmatrix}$, prove that $A(BC)=(AB)C$. **(PTA-6)** (5M)

5. Find the values of x, y, z if (ii) $(x \quad y-z \quad z+3) + (y \quad 4 \quad 3) = (4 \quad 8 \quad 16)$ PTA-5 5M

$(x+y \quad y-z+4 \quad z+6) = (4 \quad 8 \quad 16)$

$z+6=16$	$y-z+4=8$	$x+y=4$
$z=16-6$	$y-10=8-4$	$x+14=4$
$z=10$	$y-10=4$	$x=4-14$
	$y=4+10$	$x=-10$
	$y=14$	

$\therefore \boldsymbol{x=-10, y=14, z=10}$

Similar Problems
Solve Your Self

5. Find the values of x, y, z if 2M
(i) $\begin{bmatrix} x-3 & 3x-z \\ x+y+7 & x+y+z \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 6 \end{bmatrix}$

Eg. 3.65: Find the value of $a,\ b,\ c,\ d$ from the following matrix equation. 5M
$\begin{bmatrix} d & 8 \\ 3b & a \end{bmatrix} + \begin{bmatrix} 3 & a \\ -2 & -4 \end{bmatrix} = \begin{bmatrix} 2 & 2a \\ b & 4c \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -5 & 0 \end{bmatrix}$

6. Find x and y if $x\begin{bmatrix} 4 \\ -3 \end{bmatrix} + y\begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$ 2M

$$\begin{bmatrix} 4x \\ -3x \end{bmatrix} + \begin{bmatrix} -2y \\ 3y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 4x-2y \\ -3x+3y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$4x-2y=4$

$\div\ by\ 2,\quad 2x-y=2$……… (1)

$-3x+3y=6$

$\div\ by\ 3,\quad -x+y=2$……… (2)

$(1) \Rightarrow 2x-\cancel{y}=2$

$(2) \Rightarrow -x+\cancel{y}=2$

$x \quad = 4$

Sub. $x=4$ in (2)

$-x+y=2$

$-4+y=2$

$y=2+4$

$y=6$

$\therefore \boldsymbol{x=4\ , y=6}$

7. Find the non-zero values of x satisfying the matrix equation PTA-4

$\boldsymbol{x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 24 \\ 10 & 6x \end{bmatrix}}$ 5M

$$x\begin{bmatrix} 2x & 2 \\ 3 & x \end{bmatrix} + 2\begin{bmatrix} 8 & 5x \\ 4 & 4x \end{bmatrix} = 2\begin{bmatrix} x^2+8 & 24 \\ 10 & 6x \end{bmatrix}$$

$$\begin{bmatrix} 2x^2 & 2x \\ 3x & x^2 \end{bmatrix} + \begin{bmatrix} 16 & 10x \\ 8 & 8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

$$\begin{bmatrix} 2x^2+16 & 12x \\ 3x+8 & x^2+8x \end{bmatrix} = \begin{bmatrix} 2x^2+16 & 48 \\ 20 & 12x \end{bmatrix}$$

Comparing the elements

$12x=48$

$\Rightarrow x = \frac{48}{12}$

$\boldsymbol{x=4}$

Similar Problems
Solve Your Self

8. Solve for x, y: $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} + 2\begin{bmatrix} -2x \\ -y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ 2M

UE-18: If $\cos\theta\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta\begin{bmatrix} x & -\cos\theta \\ \cos\theta & x \end{bmatrix} = I_2$, find x. 5M

For Practice:

Eg. 3.61: Two examinations were conducted for three groups of students namely group 1, group 2, group 3 and their data on average of marks for the subjects Tamil, English, Science and Mathematics are given below in the form of matrices A and B. Find the total marks of both the examinations for all the three groups.

$$A = \begin{array}{c} \\ \text{Group 1} \\ \text{Group 2} \\ \text{Group 3} \end{array}\begin{array}{c} \begin{array}{cccc} \text{Tamil} & \text{English} & \text{Science} & \text{Mathematics} \end{array} \\ \begin{bmatrix} 22 & 15 & 14 & 23 \\ 50 & 62 & 21 & 30 \\ 53 & 80 & 32 & 40 \end{bmatrix} \end{array}$$

$$B = \begin{array}{c} \\ \text{Group 1} \\ \text{Group 2} \\ \text{Group 3} \end{array}\begin{array}{c} \begin{array}{cccc} \text{Tamil} & \text{English} & \text{Science} & \text{Mathematics} \end{array} \\ \begin{bmatrix} 20 & 38 & 15 & 40 \\ 18 & 12 & 17 & 80 \\ 81 & 47 & 52 & 18 \end{bmatrix} \end{array}$$

(2M)

Exercise 3.19

1. Find the order of the product matrix AB if (2M)

	(i)
Orders of A	3×3
Orders of B	3×3

i) Order of $A =$ 3×3

Order of $B =$ 3×3

$\therefore$ Order of $= AB = \mathbf{3\times 3}$

Similar Problems

Solve Your Self (2M)

1. Find the order of the product matrix AB if

	(ii)	(iii)	(iv)	(v)
Orders of A	4×3	4×2	4×5	1×1
Orders of B	3×2	2×2	5×1	1×3

2. If A is of order $p\times q$ and B is of order $q\times r$ what is the order of AB and BA? PTA-1 (2M)

Order of $A =$ $p\times q$

Order of $B =$ $q\times r$

Order of $AB = \boldsymbol{p\times r}$

Order of BA is not defined because

Order of $B = q\times r$

Order of $A = p\times q$

Column of $B \neq$ row of A.

3. A has 'a' rows and '$a+3$' columns. B has 'b' rows and '$17-b$' columns, and if both products AB and BA exist, find a, b? (2M)

If AB exists

Column of $A =$ row of B

$a+3=b$

$a-b=-3$...... (1)

If BA exists

Column of $B =$ row of A

$17-b=a$

$a+b=17$....... (2)

$(1)\Rightarrow a-\cancel{b}=-3$

$(2)\Rightarrow a+\cancel{b}=17$

$2a=14$

$a=7$

Sub $a=7$ in (1) $\Rightarrow 7-b=-3$

$-b=-3-7=-10$

$b=10$

$\therefore \boldsymbol{a=7, b=10}$

5. Given that $A=\begin{bmatrix}1 & 3\\ 5 & -1\end{bmatrix}$, $B=\begin{bmatrix}1 & -1 & 2\\ 3 & 5 & 2\end{bmatrix}$, $C=\begin{bmatrix}1 & 3 & 2\\ -4 & 1 & 3\end{bmatrix}$ verify that $A(B+C)=AB+AC$. (5M)

LHS: $B+C=\begin{bmatrix}1 & -1 & 2\\ 3 & 5 & 2\end{bmatrix}+\begin{bmatrix}1 & 3 & 2\\ -4 & 1 & 3\end{bmatrix}$

$=\begin{bmatrix}2 & 2 & 4\\ -1 & 6 & 5\end{bmatrix}$

$A(B+C)=\begin{bmatrix}1 & 3\\ 5 & -1\end{bmatrix}\begin{bmatrix}2 & 2 & 4\\ -1 & 6 & 5\end{bmatrix}$

$=\begin{bmatrix}2-3 & 2+18 & 4+15\\ 10+1 & 10-6 & 20-5\end{bmatrix}$

$=\begin{bmatrix}-1 & 20 & 19\\ 11 & 4 & 15\end{bmatrix}$........ (1)

RHS: $AB=\begin{bmatrix}1 & 3\\ 5 & -1\end{bmatrix}\begin{bmatrix}1 & -1 & 2\\ 3 & 5 & 2\end{bmatrix}$

$=\begin{bmatrix}1+9 & -1+15 & 2+6\\ 5-3 & -5-5 & 10-2\end{bmatrix}=\begin{bmatrix}10 & 14 & 8\\ 2 & -10 & 8\end{bmatrix}$

$AC=\begin{bmatrix}1 & 3\\ 5 & -1\end{bmatrix}\begin{bmatrix}1 & 3 & 2\\ -4 & 1 & 3\end{bmatrix}$

$=\begin{bmatrix}1-12 & 3+3 & 2+9\\ 5+4 & 15-1 & 10-3\end{bmatrix}$

$=\begin{bmatrix}-11 & 6 & 11\\ 9 & 14 & 7\end{bmatrix}$

$AB+AC=\begin{bmatrix}10 & 14 & 8\\ 2 & -10 & 8\end{bmatrix}$

$+\begin{bmatrix}-11 & 6 & 11\\ 9 & 14 & 7\end{bmatrix}$

$=\begin{bmatrix}-1 & 20 & 19\\ 11 & 4 & 15\end{bmatrix}$.............. (2)

(1) = (2), $\boldsymbol{A(B+C)=AB+AC}$

Hence proved.

6. Show that the matrices $\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{3} & \mathbf{1} \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} \mathbf{1} & \mathbf{-2} \\ \mathbf{-3} & \mathbf{1} \end{bmatrix}$ satisfy commutative property $\boldsymbol{AB} = \boldsymbol{BA}$ (5M)

LHS: $AB = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1-6 & -2+2 \\ 3-3 & -6+1 \end{bmatrix}$

$= \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$.......... (1)

RHS: $BA = \begin{bmatrix} 1 & -2 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1-6 & 2-2 \\ -3+3 & -6+1 \end{bmatrix}$

$= \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}$............ (2)

$(1) = (2)$

$\therefore AB = BA, A \& B$ Satisfies the commutative property.

Similar Problems (2M)
Solve Your Self

4. If $A = \begin{bmatrix} 2 & 5 \\ 4 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & -3 \\ 2 & 5 \end{bmatrix}$ find AB, BA and verify $AB = BA$?

Eg. 3.67: If $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 1 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 8 & 3 & 1 \\ 2 & 4 & 1 \\ 5 & 3 & 1 \end{bmatrix}$, find AB.

Eg. 3.68: $A = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ 1 & 3 \end{bmatrix}$ find AB and BA. Verify $AB = BA$.

Eg. 3.69: If $A = \begin{bmatrix} 2 & -2\sqrt{2} \\ \sqrt{2} & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2\sqrt{2} \\ -\sqrt{2} & 2 \end{bmatrix}$ Show that A and B satisfy commutative property with respect to matrix multiplication.

7. Let $\boldsymbol{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} \\ \mathbf{1} & \mathbf{3} \end{bmatrix}, \boldsymbol{B} = \begin{bmatrix} \mathbf{4} & \mathbf{0} \\ \mathbf{1} & \mathbf{5} \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} \mathbf{2} & \mathbf{0} \\ \mathbf{1} & \mathbf{2} \end{bmatrix}$ show that (i) $\boldsymbol{A}(\boldsymbol{BC}) = (\boldsymbol{AB})\boldsymbol{C}$ (5M)

$A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

$\boldsymbol{A}(\boldsymbol{BC}) = (\boldsymbol{AB})\boldsymbol{C}$

LHS: $BC = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 8+0 & 0+0 \\ 2+5 & 0+10 \end{bmatrix}$

$= \begin{bmatrix} 8 & 0 \\ 7 & 10 \end{bmatrix}$

$A(BC) = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 8 & 0 \\ 7 & 10 \end{bmatrix}$

$= \begin{bmatrix} 8+14 & 0+20 \\ 8+21 & 0+30 \end{bmatrix}$

$= \begin{bmatrix} 22 & 20 \\ 29 & 30 \end{bmatrix}$......... (1)

RHS: $AB = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}$

$= \begin{bmatrix} 4+2 & 0+10 \\ 4+3 & 0+15 \end{bmatrix}$

$= \begin{bmatrix} 6 & 10 \\ 7 & 15 \end{bmatrix}$

$(AB)C = \begin{bmatrix} 6 & 10 \\ 7 & 15 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 12+10 & 0+20 \\ 14+15 & 0+30 \end{bmatrix}$

$= \begin{bmatrix} 22 & 20 \\ 29 & 30 \end{bmatrix}$................ (2)

$(1) = (2) \Rightarrow \boldsymbol{A}(\boldsymbol{BC}) = (\boldsymbol{AB})\boldsymbol{C}$

Hence proved.

Similar Problems
Solve Your Self

7. Let $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 4 & 0 \\ 1 & 5 \end{bmatrix}, C = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$ show that (ii) $(A-B)C = AC - BC$ (iii) $(A-B)^T = A^T - B^T$ (5M)

Eg. 3.71: If $A = [1 \ -1 \ 2]$, $B = \begin{bmatrix} 1 & -1 \\ 2 & 1 \\ 1 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ show that $(AB)C = A(BC)$. (5M)

Eg. 3.72: If $A = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ -4 & 2 \end{bmatrix}$, $C = \begin{bmatrix} -7 & 6 \\ 3 & 2 \end{bmatrix}$ verify that $A(B+C) = AB + AC$. **(PTA-1)** (5M)

UE-19: Given $A = \begin{bmatrix} p & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 & -q \\ 1 & 0 \end{bmatrix} C = \begin{bmatrix} 2 & -2 \\ 2 & 2 \end{bmatrix}$ and if $BA = C^2$, find p and q. (5M)

UE-20: $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 6 & 3 \\ 8 & 5 \end{bmatrix}, C = \begin{bmatrix} 3 & 6 \\ 1 & 1 \end{bmatrix}$ find the matrix D, such that $CD - AB = 0$ (5M)

12. If $A = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix}, B = \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$ verify that $(AB)^T = B^T A^T$ APR-23,PTA-3 (5M)

$A = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix}, B = \begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$

LHS: $AB = \begin{bmatrix} 5 & 2 & 9 \\ 1 & 2 & 8 \end{bmatrix}\begin{bmatrix} 1 & 7 \\ 1 & 2 \\ 5 & -1 \end{bmatrix}$

$AB = \begin{bmatrix} 5+2+45 & 35+4-9 \\ 1+2+40 & 7+4-8 \end{bmatrix} = \begin{bmatrix} 52 & 30 \\ 43 & 3 \end{bmatrix}$

$(AB)^T = \begin{bmatrix} 52 & 43 \\ 30 & 3 \end{bmatrix}$............ (1)

RHS: $A^T = \begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{bmatrix}, B^T = \begin{bmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{bmatrix}$

$B^T A^T = \begin{bmatrix} 1 & 1 & 5 \\ 7 & 2 & -1 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 2 & 2 \\ 9 & 8 \end{bmatrix}$

$= \begin{bmatrix} 5+2+45 & 1+2+40 \\ 35+4-9 & 7+4-8 \end{bmatrix} = \begin{bmatrix} 52 & 43 \\ 30 & 3 \end{bmatrix}$... (2)

$(1) = (2) \Rightarrow (\boldsymbol{AB})^{\boldsymbol{T}} = \boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{A}^{\boldsymbol{T}}$, Hence proved.

Similar Problems

Solve Your Self

9. If $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ prove that $AA^T = I$. (2M)

Eg. 3.73: If $A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ -1 & 4 \\ 0 & 2 \end{bmatrix}$ show that $(AB)^T = B^T A^T$ **(SEP-20)** (5M)

13. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ show that $A^2 - 5A + 7I_2 = 0$ JUN-23 (5M)

$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$

$5A = 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix}$

$7I_2 = 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$A^2 - 5A + 7I_2$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix}$

$= \begin{bmatrix} -7+7 & 0+0 \\ 0+0 & -7+7 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$= 0$

$\therefore \boldsymbol{A^2 - 5A + 7I_2 = 0}$ Hence proved.

Similar Problems

Solve Your Self

8. If $A = \begin{bmatrix} \cos\theta & 0 \\ 0 & \cos\theta \end{bmatrix}, B = \begin{bmatrix} \sin\theta & 0 \\ 0 & \sin\theta \end{bmatrix}$ then show that $A^2 + B^2 = I$. (5M)

10. Verify that $A^2 = I$ when $A = \begin{bmatrix} 5 & -4 \\ 6 & -5 \end{bmatrix}$ (2M)

11. If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ show that $A^2 - (a+d)A = (bc - ad)I_2$ (5M)

CQ: If $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, show that $A^2 - 4A + 5I_2 = 0$ **(PTA-5)** (5M)

Note for Unit Exercise – 3

Q.No: 1 – Similar to Exercise 3.1 – 1(iii) Question
Q.No: 2 - Similar to Exercise 3.1 – 4th Question
Q.No: 3 - Similar to Exercise 3.1 – 5th Question
Q.No: 4 - Similar to Exercise 3.2 – 2nd Question
Q.No: 5 - Similar to Exercise 3.2 – 1st Question
Q.No: 6 - Similar to Exercise 3.4 – 1st Question
Q.No: 7 - Similar to Exercise 3.5 – 2nd Question
Q.No: 8 - Similar to Exercise 3.6 -8th Question
Q.No: 9 – Similar to Exercise 3.8 -1st Question
Q.No: 10 - Similar to Exercise 3.10 -1st Question
Q.No: 11 – Similar to Exercise 3.12 – 3rd Question
Q.No: 12 - Similar to Exercise 3.12 – 2nd Question
Q.No: 13, 14 - Similar to Exercise 3.12 – 9th Question
Q.No: 15 - Similar to Exercise 3.14 – 2nd Question
Q.No: 16 - Similar to Exercise 3.16 – 4th Question
Q.No: 17 – For Practice
Q.No: 18 - Similar to Exercise 3.18 -7th Question
Q.No: 19, 20 – Similar to Exercise 3.19 -7th Question

For Practice:

Eg. 3.70: Solve $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ **(2M)**

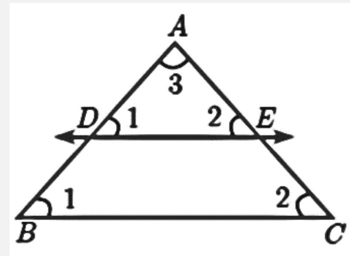
4. Geometry

Theorems

Theorem 1: Basic Proportionality Theorem (BPT) or Thales theorem

MAY-22

Statement: A straight line drawn parallel to a side of triangle intersecting the other two sides, divides the sides in the same ratio.



Proof:

Given: In ΔABC, D is a point on AB and E is a point on AC.

To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

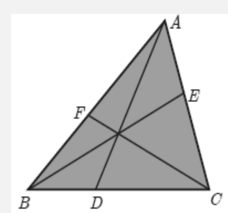
Construction: Draw a line $DE \parallel BC$

No.	Statement	Reason
1.	$\angle ABC = \angle ADE = \angle 1$	Corresponding angles are equal because $DE \parallel BC$
2.	$\angle ACB = \angle AED = \angle 2$	Corresponding angles are equal because $DE \parallel BC$
3.	$\angle DAE = \angle BAC = \angle 3$	Both triangles have a common angle
4.	$\Delta ABC \backsim \Delta ADE$ $\frac{AB}{AD} = \frac{AC}{AE}$ $\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$ $1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$ $\frac{DB}{AD} = \frac{EC}{AE}$ $\frac{AD}{DB} = \frac{AE}{EC}$	By AAA similarity Corresponding sides are proportional Split AB and AC using the points D and E On simplification Cancelling 1 on both sides Taking reciprocals
Hence proved		

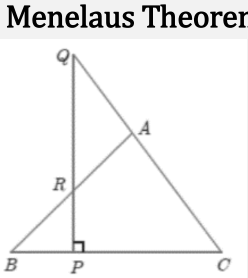
Corollary: If in ΔABC, a straight line DE parallel to BC, intersects AB at D and AC at E, then (i) $\frac{AB}{AD} = \frac{AC}{AE}$ (ii) $\frac{AB}{DB} = \frac{AC}{EC}$

Ceva's Theorem (without proof)

Statement: Let ABC be a triangle and let D, E, F be points on lines $BC,\ CA,\ AB$ respectively. Then the cevians $AD,\ BE,\ CF$ are concurrent if and only if $\frac{BD}{DC} \times \frac{CE}{EA} \times \frac{AF}{FB} = 1$ where the lengths are directed. This also works for the reciprocal of each of the ratios as the reciporcal of 1 is 1.



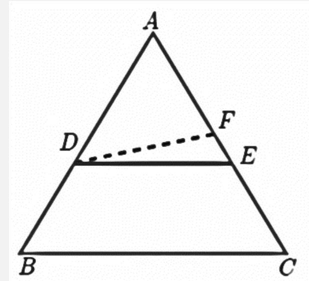
Menelaus Theorem (without proof)



Statement: A necessary and sufficient condition for points $P,\ Q,\ R$ on the respective sides $BC,\ CA,\ AB$ (or their extension) of a triangle ABC to be collinear is that $\frac{BP}{PC} \times \frac{CQ}{QA} \times \frac{AR}{RB} = -1$ where all segments in the formula are directed segments.

Theorem 2: Converse of Basic Proportionality Theorem

Statement: If a straight line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.



Proof:

Given: In $\Delta ABC, \frac{AD}{DB} = \frac{AE}{EC}$

To prove: $DE \parallel BC$

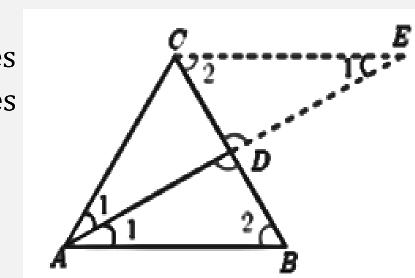
Construction: If DE is not parallel to BC. Draw $BF \parallel DE$

No.	Statement	Reason
1.	$\frac{AD}{DB} = \frac{AE}{EC}$ ……….. (1)	Given
2.	In $\Delta ABC,\ DF \parallel BC$	Construction
3.	$\frac{AD}{DB} = \frac{AF}{FC}$ ………… (2)	Thales theorem
4.	$\frac{AE}{EC} = \frac{AF}{FC}$ $\frac{AE}{EC} + 1 = \frac{AF}{FC} + 1$ $\frac{AE+EC}{EC} = \frac{AF+FC}{FC}$ $\Rightarrow \frac{AC}{EC} = \frac{AC}{FC}$ $EC = FC$ Therefore, $E = F$ Thus $DE \parallel BC$	From (1) and (2) Adding 1 to both sides Cancelling AC on both sides Our assumption that DE is not parallel to BC is wrong. Hence Proved

Theorem 3: Angle Bisector Theorem

Statement: The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the corresponding sides containing the angle.

APR-23, PTA-5, SEP-20, JUL-22



Proof:

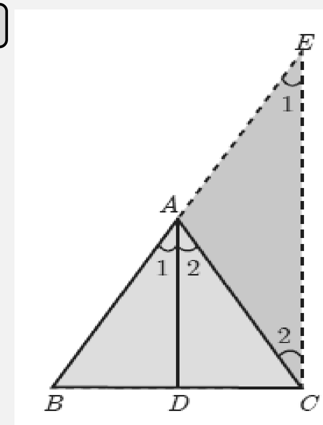
Given : In $\Delta ABC,\ AD$ is the internal bisector

To prove: $\frac{AB}{AC} = \frac{BD}{CD}$

Construction : Draw a line through C parallel to AB. Extend AD to meet line through C at E

No.	Statement	Reason
1.	$\angle AEC = \angle BAE = \angle 1$	Two parallel lines cut by a transversal make alternate angles equal.
2.	ΔACE is isosceles $AC = CE$ …… (1)	In $\Delta ACE,\ \angle CAE = \Delta CEA$
3.	$\Delta ABD \backsim \Delta ECD$ $\frac{AB}{CE} = \frac{BD}{CD}$	By AA similarity
4.	$\frac{AB}{AC} = \frac{BD}{CD}$	From (1) $AC = CE$ Hence proved.

Theorem 4: Converse of Angle Bisector Theorem PTA-3, 4



Statement: If a straight line through one vertex of a triangle divides the opposite side internally in the ratio of the other two sides, then the line bisects the angle internally at the vertex.

Proof:

Given : ABC is a triangle.

AD divides BC in the ratio of the sides containing the angles $\angle A$ to meet BC at D.

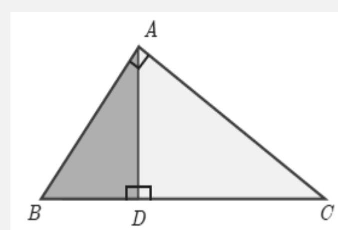
That is $\frac{AB}{AC} = \frac{BD}{DC}$ ……… (1)

To prove : AD bisects $\angle A$ i.e. $\angle 1 = \angle 2$

Construction : Draw $CE \parallel DA$. Extend BA to meet at E.

No.	Statement	Reason
1.	Let $\angle BAD = \angle 1$ and $\angle DAC = \angle 2$	Assumption
2.	$\angle BAD = \angle AEC = \angle 1$	Since $DA \parallel CE$ and AC is transversal, corresponding angles are equal
3.	$\angle DAC = \angle ACE = \angle 2$	Since $DA \parallel CE$ and AC is transversal, Alternate angles are equal
4.	$\frac{BA}{AE} = \frac{BD}{DC}$ ……….. (2)	In ΔBCE by thales theorem
5.	$\frac{AB}{AC} = \frac{BD}{DC}$	From (1)
6.	$\frac{AB}{AC} = \frac{BA}{AE}$	From (1) and (2)
7.	$AC = AE$……….(3)	Cancelling AB
8.	$\angle 1 = \angle 2$	ΔACE is isosceles by (3)
9.	AD bisects $\angle A$	Since, $\angle 1 = \angle BAD = \angle 2 = \angle DAC$. Hence proved

Theorem 5: Pythagoras Theorem



Statement: In a right angle triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

Proof: JUN-23, SEP-21, PTA-4

Given : In ΔABC, $\angle A = 90°$

To prove : $AB^2 + AC^2 = BC^2$

Construction : Draw $AD \perp BC$

No.	Statement	Reason
1.	Compare ΔABC and ΔDBA $\angle B$ is common $\angle BAC = \angle BDA = 90°$ Therefore, $\Delta ABC \sim \Delta DBA$ $\frac{AB}{BD} = \frac{BC}{AB}$ $AB^2 = BC \times BD$ …(1)	Given $\angle BAC = 90°$ and by construction $\angle BDA = 90°$ By AA similarity

2.	Compare ΔABC and ΔDAC $\angle C$ is common $\angle BAC = \angle ADC = 90°$ Therefore, $\Delta ABC \backsim \Delta DAC$ $\frac{BC}{AC} = \frac{AC}{DC}$ $AC^2 = BC \times DC$ ... (2)	Given $\angle BAC = 90°$ and by construction $\angle ADC = 90°$ By AA similarity

Adding (1) and (2) we get

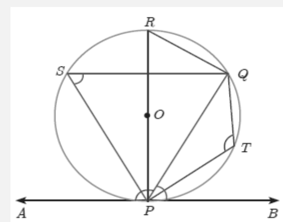
$$AB^2 + AC^2 = (BC \times BD) + (BC \times DC)$$
$$= BC \times (BD + DC)$$
$$= BC \times BC$$
$$AB^2 + AC^2 = BC^2$$

Hence the theorem is proved.

Converse of Pythagoras Theorem

Statement: If the square of the longest side of a triangle is equal to sums of squares of other two sides, then the triangle is a right angle triangle.

Theorem 6: Alternate Segment theorem



Statement: If a line touches a circle and from the point of contact a chord is drawn, the angles between the tangent and the chord are respectively equal to the angles in the corresponding alternate segments.

Proof:

Given : A circle with centre at O, tangent AB touches the circle at P and PQ is a chord. S and T are two points on the circle in the opposite sides of chord PQ.

To prove : (i) $\angle QPB = \angle PSQ$ and (ii) $\angle QPA = \angle PTQ$

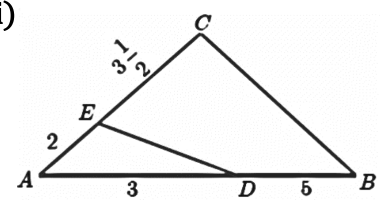
Construction : Draw the diameter POR. Draw QR, QS and PS.

No.	Statement	Reason
1.	$\angle RPB = 90°$ Now, $\angle RPQ + \angle QPB = 90°$ ... (1)	Diameter RP is perpendicular to tangent AB.
2.	In $\Delta RPQ, \angle PQR = 90°$ ... (2)	Angle in a semicircle is 90°.
3.	$\angle QRP + \angle RPQ = 90°$ ... (3)	In a right angled triangle, sum of the two acute angles is 90°.
4.	$\angle RPQ + \angle QPB = \angle QRP + \angle RPQ$ $\angle QPB = \angle QRP$ ... (4)	From (1) and (3).
5.	$\angle QRP = \angle PSQ$ ... (5)	Angles in the same segment are equal.
6.	$\angle QPB = \angle PSQ$ ... (6)	From (4) and (5); Hence (i) is proved.
7.	$\angle QPB + \angle QPA = 180°$ ... (7)	Linear pair of angles.
8.	$\angle PSQ + \angle PTQ = 180°$ ...(8)	Sum of opposite angles of a cyclic quadrilateral is 180°.
9.	$\angle QPB + \angle QPA = \angle PSQ + \angle PTQ$	From (7) and (8).
10.	$\angle QPB + \angle QPA = \angle QPB + \angle PTQ$	$\angle QPB = \angle PSQ$ from (6)
11.	$\angle QPA = \angle PTQ$	Hence (ii) is proved. This completes the proof.

Exercise 4.1

1. Check whether the which triangles are similar and find the value of x. 2M

(i)



$\frac{AD}{DB} = \frac{AE}{EC}$

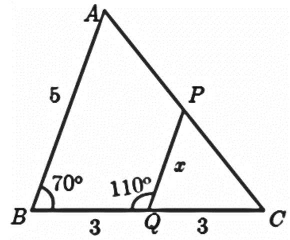
$\frac{3}{5} = \frac{2}{7/2}$

$\frac{3}{5} \neq \frac{4}{7}$

$\Rightarrow ABC \not\sim \Delta AED$

Not similar

(ii)



In $\Delta \boldsymbol{ABC}$ & $\Delta \boldsymbol{PQC}$

$\angle C$ is common

$\angle PQC = 180° - \angle PQB$

$= 180° - 110° = 70°$

$\angle A + \angle B + \cancel{\angle C} = \angle P + \angle Q + \cancel{\angle C}$

$\angle A + \cancel{70^0} = \angle P + \cancel{70^0}$

$\angle A = \angle P$

$\therefore \Delta ABC \sim \Delta POC$

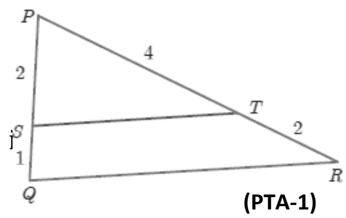
Similar, $\frac{AB}{PQ} = \frac{BC}{QC} \Rightarrow \frac{5}{x} = \frac{6}{3}$

$x = \frac{5}{2} \Rightarrow \boldsymbol{x = 2.5cm}$

Similar Problems

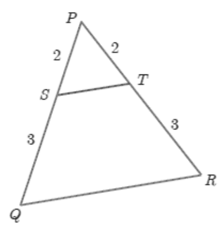
Solve Your Self

Eg. 4.1: Show that $\Delta PST \backsim \Delta PQR$ i)



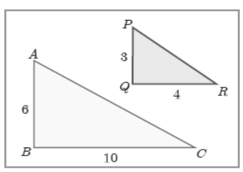
(PTA-1)

ii)

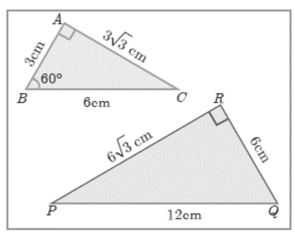


2M

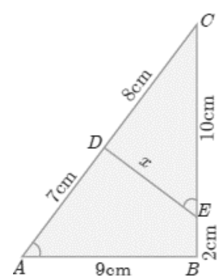
Eg. 4.2: Is $\Delta ABC \sim \Delta PQR$? 2M



Eg. 4.3: Observe Figure and find $\angle P$. 2M



Eg. 4.5: In Figure $\angle A = \angle CED$ prove that $\Delta CAB \sim \Delta CED$. Also find the value of x.



2M

UE-7: A man whose eye-level is 2 m above the ground wishes to find the height of a tree. He places a mirror horizontally on the ground 20 m from the tree and finds that if he stands at a point C which is 4 m from the mirror B, he can see the reflection of the top of the tree. How height is the tree? 5M

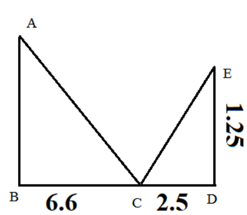
2. A girl looks the reflection of the top of the lamp post on the mirror which is 6.6 m away from the foot of the lamppost. The girl whose height is 1.25 m is standing 2.5 m away from the mirror. Assuming the mirror is placed on the ground facing the sky and the girl, mirror and the lamp post are in a same line, find the height of the lamp post. 2M

$\frac{AB}{DE} = \frac{BC}{CD}$

$\frac{AB}{1.25} = \frac{6.6}{2.5}$

$AB = \frac{6.6 \times 1.25}{2.5}$

$= \boldsymbol{3.3}$ **m**



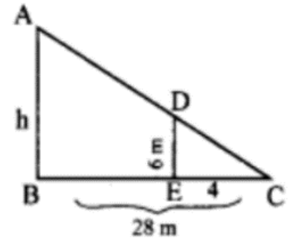
Similar Problems 5M

Solve Your Self

Eg. 4.4: A boy of height 90 cm is walking away from the base of lamp post at a speed of $1.2m/sec$. If the lamp post is $3.6m$ above the ground, find the length of his shadow cast after 4 seconds.

3. **A vertical stick of length 6 m casts a shadow 400 cm long on the ground and at the same time a tower casts a shadow 28 m long. Using similarity, find the height of the tower.** **2M**

In the picture $\Delta ABC, \Delta DEC$ are similar triangles



$$\frac{AB}{DE} = \frac{BC}{EC}$$

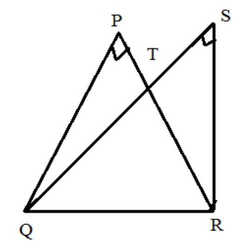
$$\frac{h}{6} = \frac{28}{4}$$

$$h = \frac{28\times 6}{4} = 7 \times 6 = 42$$

Height of a tower $= \mathbf{42\ m}$

4. **Two triangle QPR and QSR, right angled at P and S respectively are drawn on the same base QR and on the same side of QR. If PR and SQ intersect at T, prove that $PT \times TR = ST \times TQ$.** PTA-6 **5M**

In ΔPQR and ΔSQR



$\angle P = \angle S = 90^\circ$ and ΔSQR

$\angle P = \angle S = 90^\circ$

And $\angle PTQ = \angle STR$ (vertically opposite angles)

Thus by AA criterion of similarity we have $\Delta PTQ \sim \Delta STR$

$$\frac{PT}{ST} = \frac{TQ}{TR}$$

$\Rightarrow PT \times TR = TQ \times ST$

5. **In the adjacent figure, ΔABC is right angled at C and $DE \perp AB$. Prove that $\Delta ABC \sim \Delta ADE$ and hence find the lengths of AE and DE.** **5M**

In ΔABC $\angle C = 90^\circ$ and $DE \perp AB$ also in $\Delta AED, \angle E = 90^\circ$

$\angle A$ is common for both ΔABC and ΔAED

$\Rightarrow$ By AA criterion. $\Delta ABC \sim \Delta AED$

$AC = 3 + 2 = 5, \quad BC = 12$

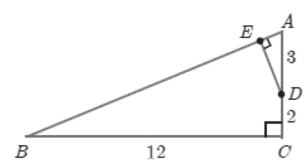
$$AB = \sqrt{AC^2 + BC^2}$$
$$= \sqrt{12^2 + 5^2}$$
$$= \sqrt{169}$$
$$AB = 13$$

$$\frac{AD}{AB} = \frac{ED}{BC} = \frac{AE}{AC} \Rightarrow \frac{3}{13} = \frac{DE}{12} = \frac{AE}{5}$$

$15 = 13AE$

$\frac{15}{13} = AE$

$\Rightarrow \boldsymbol{AE = \frac{15}{13}}$

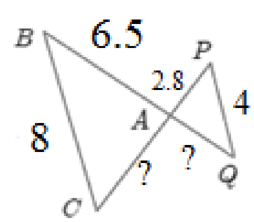


$\frac{DE}{12} = \frac{3}{13}$

$\Rightarrow 13DE = 36$

$\boldsymbol{DE = \frac{36}{13}}$

6. **In the adjacent figure, $\Delta ACB \sim \Delta APQ$. If $BC = 8\ cm$, $PQ = 4\ cm$, $BA = 6.5\ cm$ and $AP = 2.8\ cm$, find CA and AQ.** **2M**



Given $\Delta ABC \sim \Delta APQ$

$$\frac{BC}{PQ} = \frac{AB}{AQ} = \frac{AC}{AP}$$

$$\frac{8}{4} = \frac{6.5}{AQ} = \frac{AC}{2.8}$$

$2 = \frac{6.5}{AQ}$

$AQ = \frac{6.5}{2}$

$\mathbf{AQ = 3.25cm}$

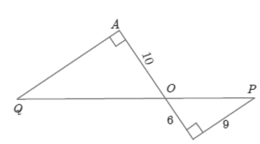
$2 = \frac{AC}{2.8}$

$AC = 2 \times 2.8$

$\mathbf{AC = 5.6cm}$

Similar Problems
Solve Your Self

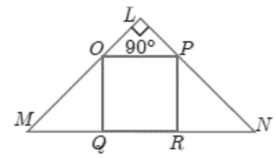
Eg. 4.6: In Figure, QA and PB are perpendicular to AB. If $AO = 10cm$, $BO = 6cm$ and $PB = 9cm$. Find AQ. **2M**



7. In figure $OPRQ$ is a square and $\angle MLN = 90^{\circ}$. Prove that

(i) $\Delta LOP \sim \Delta QMO$ (ii) $\Delta LOP \sim \Delta RPN$

(iii) $\Delta QMO \sim \Delta RPN$ (iv) $QR^2 = MQ \times RN$



2M

(i) In ΔLOP and QMO

we have $\angle OLP = \angle MQO \ (= 90^{\circ})$ and

$\angle LOP = \angle QMO$ (corresponding angles)

$\Delta LOP \sim \Delta QMO$ (By AA criterion of similarity)

(ii) ΔLOP and ΔPRN, we have

$\angle PLO = \angle NRP$ (=90°)

$\angle LPO = \angle PNR$ (Corresponding angles)

$\Delta LOP \sim \Delta RPN$

(iii) In ΔQMO and ΔRPN (From (i) & (ii))

Since $\Delta LOP \sim \Delta QMO$ and

$\Delta LOP \sim \Delta \angle RPN$

$\Delta QMO \sim \Delta RPN$

(iv) We have $\Delta QMO \sim \Delta RPN$ (using iii)

$\frac{MQ}{RP} = \frac{QO}{RN}$ ($\because PRQO$ is a square)

$\frac{MQ}{QO} = \frac{QO}{RN}$ ($\because RP = QO; QO = QR$)

$MQ.RN = QO^2$

$MQ.RN = QR^2$

$QR^2 = MQ \times RN$

8. If $\Delta ABC \sim \Delta DEF$ such that area of ΔABC is $9cm^2$ and the area ΔDEF is $16cm^2$ and $BC = 2.1cm$. find the length of EF. 2M

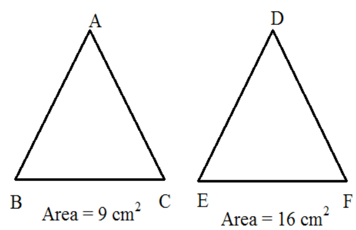
$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{BC^2}{EF^2}$

$\frac{9}{16} = \frac{2.1^2}{EF^2}$

$\frac{3}{4} = \frac{2.1}{EF}$

$EF = \frac{2.1 \times 4}{3}$

$\boldsymbol{EF = 2.8cm}$



Similar Problems 2M

Solve Your Self

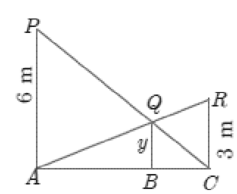
Eg. 4.7: The perimeters of two similar triangles ABC and PQR are respectively $36cm$ and $24cm$. If $PQ = 10cm$, find AB.

Eg. 4.8: If ΔABC is similar to ΔDEF such that $BC = 3cm, EF = 4cm$ & area of $\Delta ABC = 54cm^2$. Find the area of ΔDEF. **(PTA-2)**

9. Two vertical poles of heights $6m$ and $3m$ are erected above a horizontal ground AC. Find the value of y.

PTA-5

5M



In $\Delta PAC, \Delta QBC$ are similar triangles

$\frac{PA}{QB} = \frac{AC}{BC} = \frac{PQ}{QC}$

$\frac{6}{y} = \frac{AC}{BC}$

$y(AC) = 6BC$ .... (1)

ΔACR and ΔABQ are similar triangles

$\frac{CR}{QB} = \frac{AC}{AB}$

$\frac{3}{y} = \frac{AC}{AB}$

$3(AB) = (AC)y$................ (2)

(1) & (2) $\Rightarrow 3AB = 6BC$

$\frac{AB}{BC} = \frac{6}{3} = 2$

$AB = 2BC$

$AC = AB + BC$

$AC = 2BC + BC$ $(AB = 2BC)$

$AC = 3BC$

Substitute $AC = 3BC$ in (1) we get

$(3BC)y = 6BC$

$y = \frac{6BC}{3BC}$

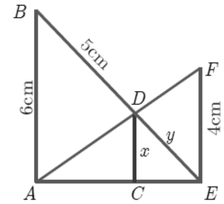
$\boldsymbol{y = 2\ m}$

Similar Problems

Solve Your Self

Eg. 4.9: Two poles of height 'a' metres and 'b' metres are 'p' metres apart. Prove that the height of the point of intersection of the lines joining the top of each pole to the foot of the opposite pole is given by $\frac{ab}{a+b}$ metres. **(APR-23)** 5M

UE-2: In the given figure $AB \parallel CD \parallel EF$. If $AB = 6\ cm,\ CD = x\ cm, EF = 4\ cm,\ BD = 5\ cm$ and $DE = y\ cm$. Find x and y. 2M



Creative Questions

1. **P and Q are points on sides. AB and AC repsectively, of ΔABC. If $AP = 3$cm, $PB = 6$cm, $AQ = 5$ cm, and $QC = 10$cm, show that $BC = 3\ PQ$** PTA-6

$\frac{AP}{AB} = \frac{3}{9} = \frac{1}{3}$, $\frac{AQ}{AC} = \frac{5}{15} = \frac{1}{3}$

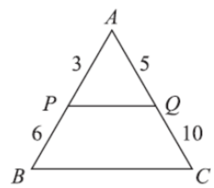
In $\Delta APQ, \Delta ABC$, we have

$\angle APQ = \angle ABC$

[Corresponding angles]

$\angle A = \angle A$

[Common angle]



By SAS criteria $\Delta APQ \sim \Delta ABC$ 2M

$$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC}$$

$$\frac{AP}{AB} = \frac{PQ}{BC} \Rightarrow \frac{PQ}{BC} = \frac{3}{9} = \frac{1}{3}$$

$\therefore\ BC = 3PQ$ Hence Proved.

Exercise 4.2

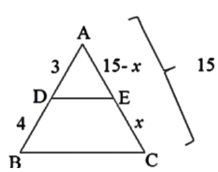
1. **In ΔABC, D and E are points on the sides AB and AC respectively such that $DE||BC$** SEP-21

(i) If $\frac{AD}{DB} = \frac{3}{4}$ and $AC = 15cm$ find AE. 2M

Given in $\Delta ABC, D$ and E are points an the sides AB & AC respectively such that $DE||BC$

$\therefore$ By Thales theorem, $\frac{AD}{DB} = \frac{AE}{EC}$

Let $EC = x$, $AE = 15 - x$



$$\frac{3}{4} = \frac{15-x}{x}$$

$$3x = 60 - 4x$$

$$3x + 4x = 60$$

$$7x = 60$$

$$x = \frac{60}{7}$$

$$x = 8.57$$

$$AE = 15 - 8.57$$

$$\boldsymbol{AE = 6.43cm}$$

Similar Problems **Solve Your Self** 2M

1. In $\Delta ABC, D$ and E are points on the sides AB and AC respectively such that $DE||BC$
 (ii) If $AD = 8x - 7, DB = 5x - 3, AE = 4x - 3$ and $EC = 3x - 1$, find the value of x.

Eg. 4.12: In ΔABC, if $DE \parallel BC,\ AC = x,\ DB = x - 2$, $AE = x + 2$ and $EC = x - 1$ then find the lengths of the sides AB and AC. 5M

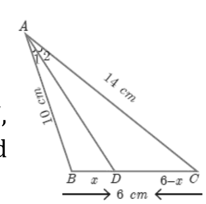
Eg. 4.13: D and E are respectively the points on the sides AB and AC of a ΔABC such that $AB = 5.6\ cm,\ AD = 1.4cm, AC = 7.2cm$ and $AE = 1.8cm$, show that $DE \parallel BC$.

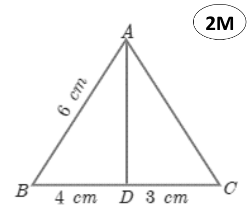
Similar Problems

Solve Your Self 2M

Eg. 4.15: In the Figure AD is the bisector of $\angle A$. If $BD = 4cm, DC = 3cm$ and $AB = 6cm$, find AC. **(PTA-5, MAY-22)**

Eg.4.16: In the Figure, AD is the bisector of $\angle BAC$, if $AB = 10cm,\ AC = 14cm$ and $BC = 6cm$. Find BD and DC. **(APR-23, PTA-3)**





2. ABCD is a trapezium which $AB||DC$ and P, Q are points on AD and BC respectively, such that $PQ||DC$ if $PD = 18\ cm$, $BQ = 35\ cm$ and $QC = 15\ cm$, and find AD. JUL-22

2M

In trapezium $ABCD$, $AB \parallel CD \parallel PQ$

Join AC, meets PQ at R

In ΔACD, $PR \parallel CD$

By BPT, $\frac{AP}{PD} = \frac{AR}{RC}$

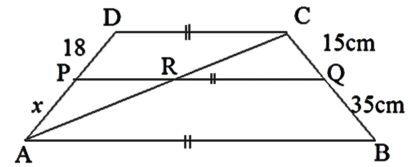
$\frac{x}{18} = \frac{AR}{RC}$ ....(1)

In ΔABC, $RQ \parallel AB$

By BPT, $\frac{BQ}{QC} = \frac{AR}{RC}$

$\frac{35}{15} = \frac{AR}{RC}$

$\frac{7}{3} = \frac{AR}{RC}$ ....(2)



From (1) and (2), $\frac{x}{18} = \frac{7}{3}$

$3x = 126$

$x = \frac{126}{3} = 42$

If $AP = x$

$AP = 42$

$AD = AP + PD = 42 + 18 = \mathbf{60\ cm}$

3. In ΔABC, D and E are points on the sides AB and AC respectively. Show that $DE \parallel BC$ if $AB = 12$cm, $AD = 8$cm, $AE = 12$cm and $AC = 18$cm

2M

Given: In ΔABC, D and E are points on the sides AB & AC respectively

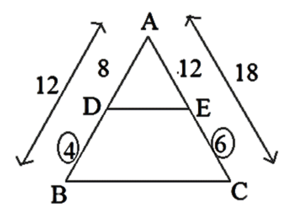
Given $AB = 12cm$ $AD = 8cm$

$AE = 12cm$ $AC = 18\ cm$

By corollary of Thales theorem, $\frac{AB}{AD} = \frac{AC}{AE}$

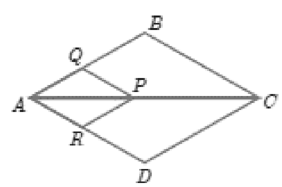
$\Rightarrow \frac{12}{8} = \frac{18}{12} \Rightarrow \frac{3}{2} = \frac{3}{2}$

$\Rightarrow DE||BC$



4. In fig. if $PQ||BC$ and $PR||CD$ prove that

(i) $\frac{AR}{AD} = \frac{AQ}{AB}$ (ii) $\frac{QB}{AQ} = \frac{DR}{AR}$



2M

(i) $\frac{AR}{AD} = \frac{AQ}{AB}$

In ΔABC & ΔADC,

$PQ||BC$ & $PR||CD$

$\therefore$ By Thales theorem

$\frac{AQ}{QB} = \frac{AP}{PC}$ and $\frac{AR}{RD} = \frac{AP}{PC}$

$\Rightarrow \frac{AQ}{QB} = \frac{AR}{RD} \Rightarrow \frac{QB}{AQ} = \frac{RD}{AR}$

$\Rightarrow \frac{QB}{AQ} + 1 = \frac{RD}{AR} + 1$

$\frac{QB+AQ}{AQ} = \frac{RD+AR}{AR}$

$\frac{AB}{AQ} = \frac{AD}{AR} \Rightarrow \frac{AQ}{AB} = \frac{AR}{AD}$

(ii) $\frac{QB}{AQ} = \frac{DR}{AR}$

In ΔABC & ΔAQP $PQ||BC$

$\therefore$ By Thales theorem

$\frac{AP}{PC} = \frac{AQ}{QB}$................ (1)

$PR||CD$

$\therefore$ By Thales theorem

$\frac{AP}{PC} = \frac{AR}{RD}$.............. (2)

(1) and (2) $\Rightarrow \frac{AQ}{QB} = \frac{AR}{RD}$

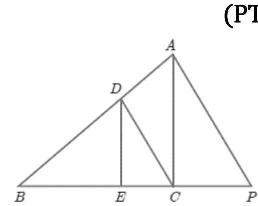
$\Rightarrow \frac{QB}{AQ} = \frac{DR}{RA}$

Similar Problems 2M

Solve Your Self

Eg. 4.14: In the Figure, $DE \parallel AC$ and $DC \parallel AP$. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$.

(PTA-4)



5. **Rhombus PQRB is inscribed in ΔABC such that $\angle B$ is one of its angle. P, Q and R lie on AB, AC and BC respectively. If $AB = 12cm$ and $BC = 6cm$, find the sides PQ, RB of the rhombus.** 5M

In a diagram ABC is a triangle and $PQRB$ is a rhombus inscribed is ΔABC

$PQ||BR \Rightarrow \angle RBP = \angle CRQ$ (Corresponding angle)

$\angle BAC = \angle RQC$ (Corresponding angle)

$\frac{RC}{BC} = \frac{QR}{AB}$.............. (1)

$\therefore \Delta ABC \sim \Delta RQC$ (by AA criterion)

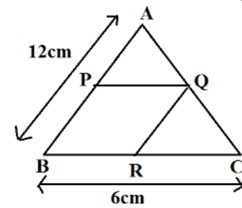
Let the side length of rhombus is x

(ie) $PQ = BP = BR = RQ = x$

Given: $AB = 12cm$ and $BC = 6cm$

$RC = BC - BR = 6 - x$

From (1)



$\frac{6-x}{6} = \frac{x}{12}$

$12(6 - x) = 6x$

$72 - 12x = 6x$

$72 = 6x + 12x$

$72 = 18x$

$x = \frac{72}{18}$

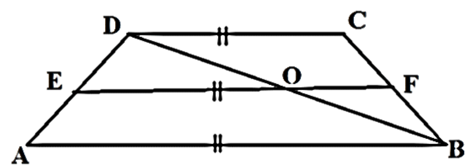
$\Rightarrow x = 4cm$

$PQ, RB = 4$ cm

6. **In trapezium $ABCD, AB||DC, E$ and F are points on non-parallel sides AD and BC respectively, such that $EF||AB$. Show that $\frac{AE}{ED} = \frac{BF}{FC}$** 5M

Given: $ABCD$ is trapezium $AB||DC, E$ and F are points an non – parallel sides AD and BC respectively

Such that $EF||AB$



Join BD and if intersects EF at O

In the $\Delta ABD, EO||AB$

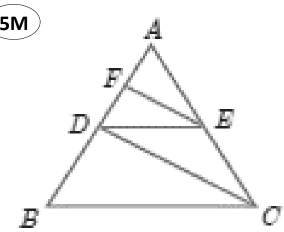
$\frac{AE}{ED} = \frac{BO}{OD}$......... (1) (By Thales theorem)

In the $\Delta BDC, OF||DC$

$\frac{BO}{OD} = \frac{BF}{FC}$............... (2) (By Thales theorem)

(1) & (2) $\Rightarrow \frac{AE}{ED} = \frac{BF}{FC}$

7. **In figure $DE||BC$ and $CD||EF$. Prove that $AD^2 = AB \times AF$** 5M



Given: In figure $DE||BC$ and $EF||CD$ in ΔABC; $DE||BC$.

$\frac{AD}{AB} = \frac{AE}{AC}$............. (1) ($\because$ By corollary of Thales theorem)

In ΔADC; $EF||DC$

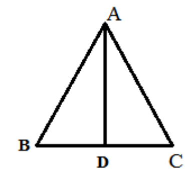
$\frac{AF}{AD} = \frac{AE}{AC}$................ (2) ($\because$ By corollary of Thales theorem)

(1) & (2) $\Rightarrow \frac{AD}{AB} = \frac{AF}{AD}$

$\Rightarrow AD \times AD = AB \times AF$

$AD^2 = AB \times AF$

8. **Check whether AD is bisector of $\angle A$ of ΔABC in each of the following**
(i) $AB = 5cm, AC = 10cm, BD = 1.5cm$ and $CD = 3.5cm$



SEP-20

5M

Given: In the ΔABC,

$\frac{AB}{AC} = \frac{5}{10}$

$\frac{AB}{AC} = \frac{1}{2}$………………… (1)

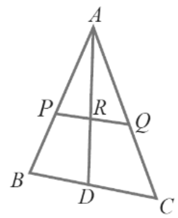
$\frac{BD}{DC} = \frac{1.5}{3.5}$

$\frac{BD}{DC} = \frac{15}{35}$

$\frac{BD}{DC} = \frac{3}{7}$ ……………… (2)

(1) & (2) $\Rightarrow \frac{AB}{AC} \neq \frac{BD}{DC}$

$\therefore AD$ is **not an angle bisector** of $\angle A$



Similar Problems 5M

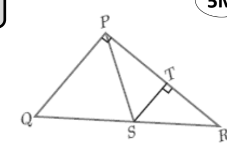
Solve Your Self

8. Check whether AD is bisector of $\angle A$ of ΔABC in each of the following (ii) $AB = 4cm$, $AC = 6cm, BD = 1.6cm$ and $CD = 2.4cm$ **(JUN-23)**

9. **In figure $\angle QPR = 90^{\circ}$, PS is its bisector. If $ST \perp PR$, prove that $ST \times (PQ + PR) = PQ \times PR$.**

PTA-2

5M



Given: In the figure $\angle QPR = 90^{\circ}$,

PS is its bisector and $ST \perp PR$

$\frac{PQ}{PR} = \frac{QS}{SR}$ By Angle bisector theorem

$\frac{PQ}{PR} + 1 = \frac{QS}{SR} + 1$ Add 1 both side

$\frac{PQ+PR}{PR} = \frac{QS+SR}{SR}$

$\frac{PQ+PR}{PR} = \frac{QR}{SR}$………….. (1)

In ΔPQR and ΔSTR

$\angle QPR = 90^0$, $\angle STR = 90^{\circ}$

$\angle PRS = \angle TRS = \angle R$ is common ,

By AA similarity

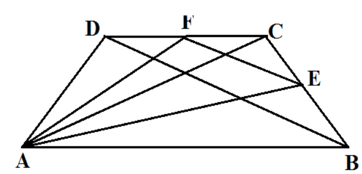
$\therefore \frac{PQ}{ST} = \frac{QR}{SR} = \frac{PR}{TR}$ ……………..(2)

(1) & (2) $\Rightarrow \frac{PQ+PR}{PR} = \frac{PQ}{ST}$

$ST\ (PQ + PR) = PQ \times PR$. Hence proved

10. **$ABCD$ is a quadrilateral in which $AB = AD$, the bisector of $\angle BAC$ and $\angle CAD$ intersect the sides BC and CD at the points E and F respectively. Prove that $EF||BD$.** 5M

Given $ABCD$ is a quadrilateral in which $AB = AD$, the bisector of $\angle BAC$ and $\angle CAD$ intersect the sides BC and CD at the points E and F respectively



Construction: Join: AC & BD

In $\Delta ABC, AE$ is the angle bisector of $\angle BAC$

$\therefore$ By angle bisector theorem

$\frac{AB}{AC} = \frac{BE}{EC}$…………….. (1)

In $\Delta ADC, AF$ is the angle bisector of $\angle DAC$

$\therefore$ By angle bisector theorem

$\frac{AD}{AC} = \frac{DF}{FC}$

But $AD = AB \Rightarrow \frac{AB}{AC} = \frac{DF}{FC}$……………. (2)

(1) & (2)

$\Rightarrow \frac{BE}{EC} = \frac{DF}{FC}$

In ΔBDC

$\Rightarrow \frac{BE}{EC} = \frac{DF}{FC}$

$\Rightarrow EF||BD$ (By corollary of Thales theorem)

Exercise 4.3

1. A man goes 18 m due east and then 24 m due north. Find the distance of his current position from the starting point? (2M) JUL-22

Given, $BC = 18\text{m}, BA = 24\text{m}$

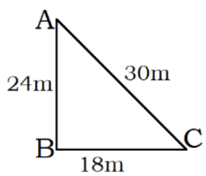
By Pythagoras theorem, $AC^2 = AB^2 + BC^2$

$= 24^2 + 18^2$

$= 576 + 324$

$AC^2 = 900 = 30^2$

$\boldsymbol{AC = 30m}$

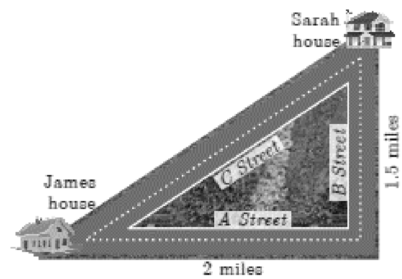


Similar Problems (5M)

Solve Your Self

Eg. 4.20: An insect 8 m away initially from the foot of a lamp post which is 6 m tall. crawls towards it moving through a distance. If its distance from the top of the lamp post is equal to the distance it has moved, how far is the insect away from the foot of the lamp post ?

2. There are two paths one can choose to go from Sarah's house to James house. One way is to take C street, and the other way requires to take B street and then A street. How much shorter is the direct path along C street? (Using figure). (2M)



$SJ = \sqrt{1.5^2 + 2^2}$

$= \sqrt{2.25 + 4}$

$= \sqrt{6.25} = 2.5\text{Miles}$

When one can choose the C street he requires to go 2.5 miles, and the other way requires to take A street and then B street he requires to go (2 + 1.5) 3.5 miles.

$\therefore$ **1 miles** is shorter the direct path along C street.

Similar Problems

Solve Your Self

Eg. 4.22: What length of ladder is needed to reach a height of 7 ft along the wall when the base of the ladder is 4 ft from the wall? Round off your answer to the next tenth place. **(MDL)** (2M)

Eg. 4.23: An Aeroplane after take off from an airport and flies due north at a speed of 1000 km/hr. At the same time, another aeroplane take off from the same airport and flies due west at a speed of 1200 km/hr. How far apart will be the two planes after 1½ hours? **(MAY-22)** (5M)

3. To get from point A to point B you must avoid walking through a pond. You must walk 34 m south and 41 m east. To the nearest meter, how many meters would be saved if it were possible to make a way through the pond? (5M)

Path: I (Through Pond)

$AB = \sqrt{34^2 + 41^2}$

$= \sqrt{1156 + 1681}$

$= \sqrt{2837}$

$= \mathbf{53.26\ m}$

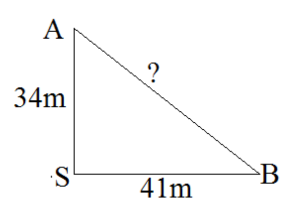
Path: II (South to East)

$AB = AS + SB$

$= 34 + 41 = 75$ m

Required Distance $= 75 - 53.26$

$= \mathbf{21.74\ m}$



Similar Problems (5M)

Solve Your Self

UE-5: Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels at a speed of 20 km/hr and the second train travels at 30 km/hr. After 2 hours, what is the distance between them?

4. **In the rectangle $WXYZ, XY + YZ = 17\ cm$, and $XZ + YW = 26\ cm$. Calculate the length and breadth of the rectangle?** 5M

Given: In rectangle $WXYZ$;

$XY + YZ = 17\text{cm}$ and $XZ + YW = 26$ cm

$l + b = 17 \Rightarrow b = 17 - l$

$d = XZ + YW = 26 \Rightarrow d + d = 26 \Rightarrow 2d = 26 \Rightarrow d = 13$

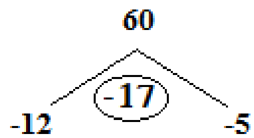
$l^2 + (17 - l)^2 = 13^2$

$l^2 + 289 - 34l + l^2 = 169$

$2l^2 - 34l + 289 - 169 = 0$

$2l^2 - 34l + 120 = 0$

$\div$ *by* 2 , $\Rightarrow l^2 - 17l + 60 = 0$



$(l - 12)(l - 5) = 0$

$l = 12 \quad or \ \ l = 5$

$\boldsymbol{l = 12cm \Rightarrow b = 17 - 12 = 5\ cm}$

5. **The hypotenuse of a right triangle is 6 m more than twice of the shortest side. If the third side is 2 m less than the hypotenuse, find the sides of the triangle.** PTA-3 5M

In $\Delta ABC; \angle B = 90^0$

Let $AB = x \Rightarrow AC = 2x + 6$ *and*

$BC = 2x + 4$

$(2x + 6)^2 = x^2 + (2x + 4)^2$

$4x^2 + 36 + 24x = x^2 + 4x^2 + 16x + 16$

$x^2 + 16x - 24x + 16 - 36 = 0$

$x^2 - 8x - 20 = 0$

$(x - 10)(x + 2) = 0$

$x = 10$ (or) $x = -2$

But $x \neq -2$

If $x = 10$

$\Rightarrow AC = 2x + 6$

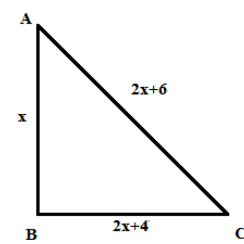
$= 20 + 6 = 26$

$\Rightarrow BC = 2x + 4$

$= 20 + 4 = 24$

$\therefore$ The sides are

$\boldsymbol{AB = 10\ m;\ BC = 24\ m; AC = 26\ m}$



6. **5 m long ladder is placed leaning towards a vertical wall such that it reaches the wall at a point 4 m high. If the foot of the ladder is moved 1.6 m towards the wall, then find the distance by which the top of the ladder would slide upwards on the wall.** 5M

Given, in the ΔABC

$AB = 4\text{m}, AC = 5\text{m}$

$BC = \sqrt{AC^2 - AB^2}$

$= \sqrt{5^2 - 4^2}$

$= \sqrt{25 - 16} = \sqrt{9}$

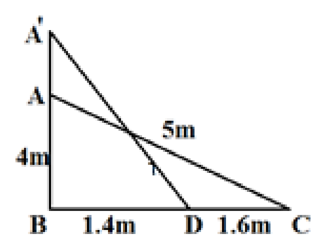
$BC = 3m$

Given $DC = \boldsymbol{1.6m}$

$BD = BC - DC$

$= 3 - 1.6m$

$= 1.4\ m$



$A'B = \sqrt{A'D^2 - BD^2}$

$= \sqrt{5^2 - 1.4^2}$

$= \sqrt{25 - 1.96}$

$= \sqrt{23.04} = 4.8m$

$AA' = A'B - AB = 4.8 - 4 = \boldsymbol{0.8\ m}$

7. The perpendicular PS on the base QR of a ΔPQR intersects QR at S, such that $QS = 3SR$. Prove that $2PQ^2 = 2PR^2 + QR^2$ PTA-6

5M

Given the ΔPQR, the perpendicular on the base QR at S, such that $QS = 3SR$

In $\Delta PQS \Rightarrow PQ^2 = PS^2 + QS^2$

$\Delta PSR \Rightarrow PR^2 = PS^2 + SR^2$

$\Rightarrow PS^2 = PR^2 - SR^2$

$QR = QS + SR$

$= 3SR + SR$

$QR = 4SR$

$\frac{QR}{4} = SR$

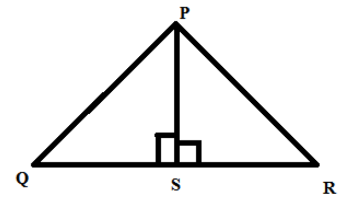
$PQ^2 = PR^2 - SR^2 + (3SR)^2$

$PQ^2 = PR^2 - SR^2 + 9SR^2$

$PQ^2 = PR^2 + 8SR^2$

$PQ^2 = PR^2 + \frac{8QR^2}{16}$

$\Rightarrow \mathbf{2PQ^2 = 2PR^2 + QR^2}$



Similar Problems 5M

Solve Your Self

8. In the adjacent figure, ABC is a right angled triangle with right angle at B and points D, E trisect BC. Prove that $8AE^2 = 3AC^2 + 5AD^2$

Eg. 4.21: P and Q are the mid-points of the sides CA and CB respectively of a ΔABC, right angled at C. Prove that $4(AQ^2 + BP^2) = 5AB^2$. **(MDL)**

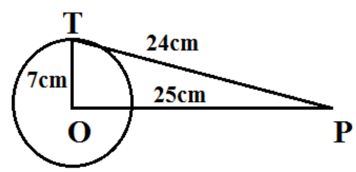
Exercise 4.4

1. The length of the tangent to a circle from a point *P*, which is 25 cm away from the center is 24 cm. What is the radius of the circle? 2M

$OT = \sqrt{OP^2 - PT^2}$

$= \sqrt{25^2 - 24^2} = \sqrt{(25+24)(25-24)}$

$= \sqrt{(49)(1)} = \sqrt{49} = 7cm$



Similar Problems 5M

Solve Your Self

Eg. 4.25: PQ is a chord of length 8 cm to a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the length of the tangent TP.

Eg. 4.24: Find the length of the tangent drawn from a point whose distance from the centre of a circle is 5 cm and radius of the circle is 3 cm.

2. ΔLMN is a right angled triangle with $\angle L = 90^\circ$. A circle is inscribed in it. The lengths of the sides containing the right angle are 6 cm and 8 cm. Find the radius of the circle. 2M

ΔLMN is a right angled triangle with $\angle L = 90^\circ$. A circle inscribed in it.

The lengths of the sides containing the right angle are 6cm and 8cm

$MN = \sqrt{LM^2 + LN^2} = \sqrt{8^2 + 6^2} = \sqrt{64 + 36} = \sqrt{100} = 10cm$

Let radius of the incircle is "r" units

Area of the ΔLMN = Area of $\Delta LON + \Delta LOM + \Delta OMN$

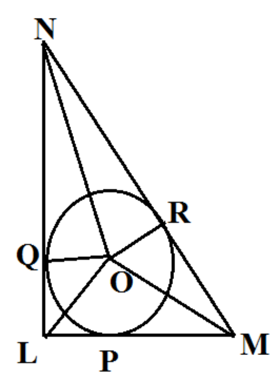
$$\left(\frac{1}{2}LM \times LN\right) = \left(\frac{1}{2} \times LN \times r\right) + \left(\frac{1}{2}r \times LM\right) + \left(\frac{1}{2} \times MN \times r\right)$$

$\times\ by\ 2 \Rightarrow LM \times LN = LN \times r + r \times LM + MNr$

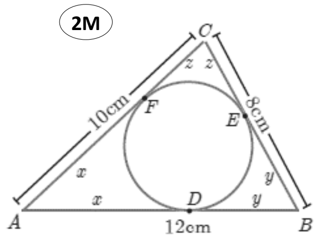
$8 \times 6 = 6r + 8 \times r + 10r$

$48 = 24r$

$r = \frac{48}{24} \Rightarrow \mathbf{r = 2cm}.$



3. A circle is inscribed in ΔABC having sides 8 cm, 10 cm and 12 cm as shown in figure, find AD, BE and CF. 2M



Given a circle is inscribed in ΔABC having sides 8cm, 10cm, and 12cm as shown in figure

Let $AF = AD = x; BD = BE = y; CE = CF = z$

$x + y = 12\ cm$ ………… (1)

$z + y = 8\ cm$ ……………. (2)

$x + z = 10\ cm$…………… (3)

$(1)+(2)+(3) \Rightarrow 2(x + y + z) = 30$

$x + y + z = 15$

$x + y = 12 \Rightarrow 12 + z = 15$

$z = 15 - 12 \Rightarrow\ z = 3cm$

$x + z = 10 \Rightarrow y + 10 = 15$

$y = 15 - 10 \Rightarrow y = 5cm$

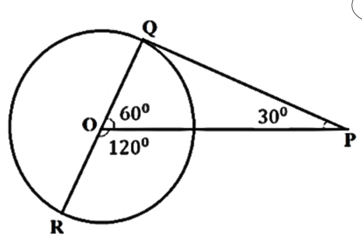
$z + y = 8 \Rightarrow x + 8 = 15$

$x = 15 - 8$

$x = 7cm$

$\therefore \boldsymbol{AD = 7cm; BE = 5cm; CF = 3cm}$

4. PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that $\angle POR = 120^\circ$. Find $\angle OPQ$. 2M



$\angle POQ = 180^\circ - \angle POR$

$= 180^\circ - 120^\circ$

$= 60^\circ$

$\angle OPQ = 180^\circ - (\angle OQP + \angle POQ)$

$= 180^\circ - (90^\circ + 60^\circ)$

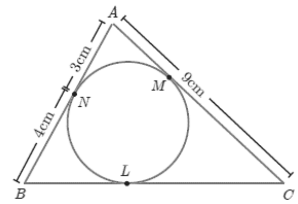
$= 180^\circ - 150^\circ$

$= \boldsymbol{30^\circ}$

Similar Problems 2M

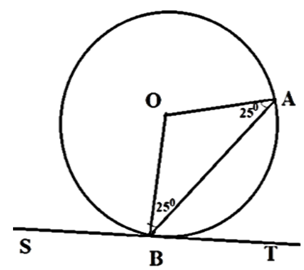
Solve Your Self

Eg. 4.27: In Figure, ΔABC is circumscribing a circle. Find the length of BC.



5. A tangent ST to a circle touches it at B. AB is a chord such that $\angle ABT = 65^\circ$. 2M

Find $\angle AOB$, where "O" is the centre of the circle.



A tangent "ST" to a circle it at B. AB is a chord such that $\angle ABT = 65^\circ$, where O is the centre of the circle

$\angle OBT = 90^\circ$

($\because OB$ radius & $BT -$ Tangents)

$\angle OBA = \angle OBT - \angle ABT$

$= 90^\circ - 65^\circ$

$= 25^\circ$

$\angle OAB = 25^\circ; \angle OBA = 25^\circ$

$\therefore \angle AOB = 180^\circ - (25^\circ + 25^\circ)$

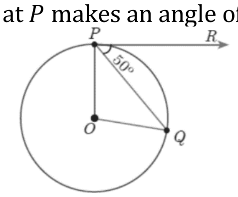
$= 180^\circ - 50^\circ$

$= 130^\circ$

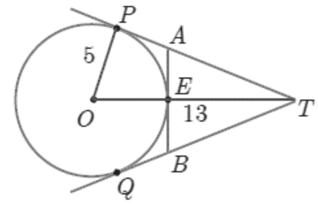
Similar Problems 2M

Solve Your Self

Eg. 4.26: In Figure, O is the centre of a circle. PQ is a chord and the tangent PR at P makes an angle of 50° with PQ. Find $\angle POQ$.



6. **In figure, O is the centre of the circle with radius 5cm. T is a point such that $OT = 13cm$ and OT intersects the circle E, if AB is the tangent to the circle at E, find the length of AB.**



5M

If AB is the tangent to the circle at E.

$OE = 5$ cm, $OT = 13$cm,

$ET = OT - OE$

$= 13 - 5$

$= 8$cm

$OP \perp PT$

$\angle OPT = 90°$

$OT^2 = OP^2 + PT^2$

$13^2 = 5^2 + PT^2$

$13^2 - 5^2 = PT^2$

$169 - 25 = PT^2$

$PT^2 = 144$

$PT = 12$cm

In ΔOPT & ΔAET,

$\angle PTO = \angle ATE$ (common angle)

$\angle TPO = \angle AET = 90°$

By AA similarity

$\Delta OPT \sim \Delta AET$

$\frac{OT}{AT} = \frac{OP}{AE} = \frac{PT}{ET}$

$\frac{13}{AT} = \frac{5}{AE} = \frac{12}{8}$

$\frac{5}{AE} = \frac{12}{8}$

$40 = 12AE$

$AE = \frac{40}{12}$

$AE = \frac{10}{3}$

$AB = 2 \times AE$

$= 2 \times \frac{10}{3}$

$AB = \frac{20}{3}$ cm

8. **Two circles with centres O & O' of radii 3 cm & 4 cm, respectively intersect at two points P & Q, such that OP & $O'P$ are tangents to the two circles. Find the length of the common chord PQ.**

Given: $OP = 3$ cm, $O'P = 4$ cm, $OQ = 3$ cm, $O'Q = 4$ cm

5M

OO' is the $\perp$ bisector of the chord PQ and let R be the point of intersection of PQ and OO'

$OO' = \sqrt{OP^2 + O'P^2}$

$= \sqrt{3^2 + 4^2}$

$= \sqrt{9 + 16} = \sqrt{25} = 5cm$

In $\Delta OPR, OP^2 = PR^2 + OR^2$

$3^2 = PR^2 + OR^2$

$PR^2 + OR^2 = 9$

$PR^2 = 9 - OR^2$ .... (1)

$O'R = OO' - OR = 5 - OR$

In $\Delta O'PR$, $O'P^2 = PR^2 + O'R^2$

$4^2 = PR^2 + (5 - OR)^2$

$16 = PR^2 + (5 - OR)^2$

$PR^2 = 16 - (5 - OR)^2$

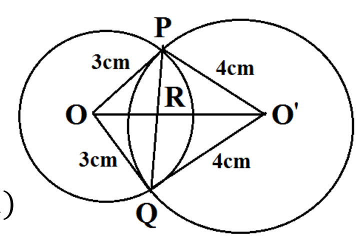
$PR^2 = 16 - (25 - 10(OR) + OR^2)$

$PR^2 = 16 - 25 + 10(OR) - OR^2$

$PR^2 = -9 + 10(OR) - OR^2$ .... (2)

(1) & (2) $\Rightarrow$

$9 - OR^2 = -9 + 10(OR) - OR^2$



$9 + 9 = 10\ (OR)$

$10(OR) = 18$

$OR = \frac{18}{10}$

$OR = 1.8cm$

$PR = \sqrt{9 - (1.8)^2}$

$= \sqrt{9 - 3.24}$

$= \sqrt{5.76}$

$= 2.4cm$

$PQ = 2PR = 2(2.4) = \mathbf{4.8cm}$

7. In two concentric circles, a chord of length 16 cm of larger circle becomes a tangent to the smaller circle whose radius is 6 cm. Find the radius of the larger circle. (2M)

In two concentric circle, a chord of length16cm of larger circle becomes a tangent to the smaller circle whose radius is 6cm

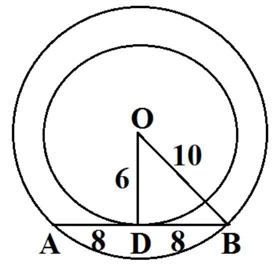
$OD \perp AB \Rightarrow AD = DB = 8cm$

In the right ΔODB, $OB = \sqrt{OD^2 + DB^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100} = \mathbf{10cm}$

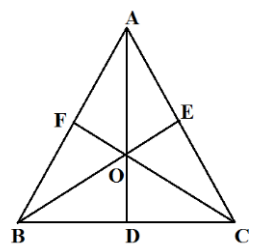


Similar Problems (2M)
Solve Your Self
Eg. 4.28: If radii of two concentric circles are 4 *cm* and 5 *cm* then find the length of the chord of one circle which is a tangent to the other circle.

9. Show that the angle bisectors of a triangle are concurrent. (5M) PTA-4

In the ΔABC, "O" is any point inside the Δ

The angle bisector $\angle AOB, \angle BOC$, and $\angle AOC$ meet the sides AB, BC & CA at D, E & F respectively.



$\therefore$ In $\Delta BOC, OD$ is the bisector of $\angle BOC$

$\therefore \frac{OB}{OC} = \frac{BD}{DC}$ .... (1)

Similarly in the triangle AOC & AOB we get

$\frac{OC}{OA} = \frac{CE}{AE}$ .... (2)

$\frac{OA}{OB} = \frac{AF}{FB}$ .... (3)

$(1) \times (2) \times (3) \Rightarrow \frac{OB}{OC} \times \frac{OC}{OA} \times \frac{OA}{OB} = \frac{BD}{DC} \times \frac{CE}{AE} \times \frac{AF}{EB}$

$\frac{BD}{DC} \times \frac{CE}{AE} \times \frac{AF}{FB} = 1$ .... (4)

If AD, BE & CF are the bisectors of $\angle A, \angle B$ & $\angle C$ then by ABT

$\frac{AB}{AC} = \frac{BD}{DC}; \frac{BC}{CA} = \frac{AF}{FB}; \frac{AB}{BC} = \frac{AE}{EC}$

$\frac{AB}{AC} \times \frac{BC}{CA} \times \frac{AB}{BC} = \frac{BD}{DC} \times \frac{AF}{FB} \times \frac{AE}{EC}$

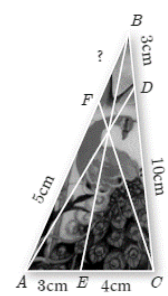
$1 = 1$ (By (4))

$\therefore O$ is the point of concurrent.

The angle bisectors of a triangle concurrent.

10. An artist has created a triangular stained glass window and has one strip of small length left before completing the window. She needs to figure out the length of left out portion based on the lengths of the other sides as shown in the figure. (5M)

In the diagram $\Delta ABC, AD, BE, CF$ are the angle bisectors of $\angle AOB, \angle BOC$ and $\angle AOC$ respectively, and O is concurrent point.



$\frac{OB}{OA} = \frac{BF}{FA}$ .... (1)

$\frac{OC}{OB} = \frac{DC}{DB}$ .... (2)

$\frac{OA}{OC} = \frac{AE}{EC}$............ (3)

$(1) \times (2) \times (3) \Rightarrow$

$\frac{OB}{OA} \times \frac{OC}{OB} \times \frac{OA}{OC} = \frac{BF}{FA} \times \frac{DC}{DB} \times \frac{AE}{EC}$

$1 = \frac{BF}{5} \times \frac{10}{3} \times \frac{3}{4}$

$BF = 2cm$

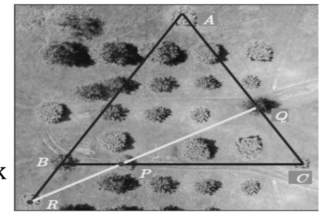
Similar Problems (5M)
Solve Your Self
UE-3: O is any point inside a triangle ABC. The bisector of $\angle AOB$, $\angle BOC$, $\angle COA$ meet the sides AB, BC and CA in point D, E and F respectively.
Show that $AD \times BE \times CF = DB \times EC \times FA$

Eg. 4.32: Show that in a triangle, the medians are concurrent. **(SEP-21)**

Similar Problems
Solve Your Self 5M

Eg. 4.33: In ΔABC, points D, E, F lies on BC, CA, AB respectively. Suppose AB, AC and BC have lengths 13, 14 and 15 respectively. If $\frac{AF}{FB} = \frac{2}{5}$ and $\frac{CE}{EA} = \frac{5}{8}$. Find BD and DC.

Eg. 4.34: In a garden containing several trees, three particular trees P, Q, R are located in the following way, $BP = 2m$, $CQ = 3m$, $RA = 10m$, $PC = 6m$, $QA = 5m$, $RB = 2m$, where A, B, C are points such that P lies on BC, Q lies on AC and R lies on AB. Check whether the trees P, Q, R lie on a same straight line.

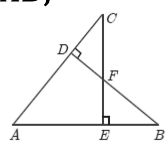


Unit Exercise - 4

Note for Unit Exercise – 4

Q.No: 2 – Similar to Exercise 4.1- 9th Question
Q.No: 3 - Similar to Exercise 4.4 – 9th Question
Q.No: 5 - Similar to Exercise 4.3 – 3rd Question
Q.No: 7 - Similar to Exercise 4.1 – 1st Question

1. In the figure, if $BD \perp AC$ and $CE \perp AB$, prove that (i) $\Delta AEC \backsim \Delta ADB$ (ii) $\frac{CA}{AB} = \frac{CE}{DB}$ 2M



(i) Given ΔAEC and ΔADB are two triangles

Also $\angle AEC = \angle ADB = 90^\circ$

[BD perpendicular AC & CE perpendicular AB]

$\therefore$ $\angle EAC = \angle DAB$

$\angle A$ is common

By AA – criterion of similarity, we know that

$\Delta AEC \backsim \Delta ADB$

(ii) Also we know

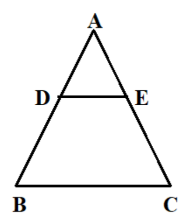
$\Delta AEC \backsim \Delta ADB$ (from the proof above)

$\Rightarrow \frac{CA}{BA} = \frac{EC}{DB}$

$\Rightarrow \frac{CA}{AB} = \frac{CE}{DB}$

Hence it is proved.

4. In the figure, ABC is a triangle in which $AB = AC$. Points D and E are points on the side AB and AC respectively such that $AD = AE$. Show that the points B, C, E and D lie on a same circle. 5M

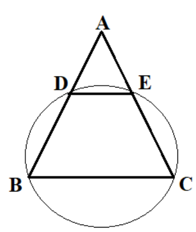


we can prove that

$\angle ABC + \angle CED = 180^\circ$

$\angle ABC + \angle BDE = 180^\circ$

To prove the points B, C, E and D are concyclic



In ΔABC we have

$AD = AE$, $AB = AC$

$AB - AD = AC - AE$

$DB = EC$

By converse of Thales theorem we have,

$AD = AE$, $DB = EC$

$\Rightarrow \frac{AD}{BD} = \frac{AE}{EC}$

$\Rightarrow$ $\angle ABC = \angle ADE$ (corresponding angles)

$\Rightarrow$ $\angle ABC + \angle BDE = \angle ADE + \angle BDE$

Adding $\angle BDE$ on both sides.

we have

$\angle ABC + \angle BDE = 180^\circ$

$\angle ACB + \angle BDE = 180^\circ$

$\because$ $\angle ABC = \angle ACB$ (we have $AB = AC$)

Also we know, $DE \parallel BC$

$\Rightarrow$ $\angle ACB = \angle AED$

$\Rightarrow$ $\angle ACB + \angle CED = \angle AED + \angle CED$

Adding $\angle CED$ on both sides

$\Rightarrow \angle ACB + \angle CED = 180^\circ$

$\angle ABC + \angle CED = 180^\circ$

[reason is $\angle ABC = \angle ACB$)

$\therefore$ we have

$BDEC$ is a quadrilateral such that

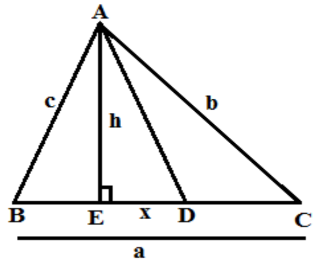
$\Rightarrow$ $\angle ABC + \angle CED = 180^\circ$

$\Rightarrow$ $\angle ACB + \angle BDE = 180^\circ$

$\therefore$ B, C, E and D are con-cyclic points.

6. D is the mid point of side BC and $AE \perp BC$. If $BC = a,\ AC = b,\ AB = c,\ ED = x,$ $AD = p$ and $AE = h$, prove that **5M**

(i) $b^2 = p^2 + ax + \frac{a^2}{4}$ (ii) $c^2 = p^2 - ax + \frac{a^2}{4}$ (iii) $b^2 + c^2 = 2p^2 + \frac{a^2}{2}$



D is the mid point of BC

we know that $\angle AED = 90^\circ$

$\angle ADE < 90^\circ$ and $\angle ADC > 90^\circ$

$\therefore$ $\angle ADE$ is acute angle

$\angle ADC$ is obtuse

(i) In ΔADC, $\angle ADC$ is obtuse angle

$AC^2 = AD^2 + DC^2 + 2\,DC \times DE$

$\Rightarrow AC^2 = AD^2 + (\frac{1}{2}BC)^2 + 2\left(\frac{1}{2}\,BC \cdot DE\right)$

$\Rightarrow AC^2 = AD^2 + \frac{1}{4}BC^2 + (\,BC \cdot DE)$

$\Rightarrow AC^2 = AD^2 + (DE \cdot BC) + \frac{1}{4}BC^2 \ldots (1)$

$\Rightarrow b^2 = p^2 + ax + \frac{1}{4}a^2$

Hence proved.

(ii) In $\Delta ABD, \angle\, ADE$ is an acute angle

$$AB^2 = AD^2 + BD^2 - 2\,BD \cdot DE$$

$$AB^2 = AD^2 + \left[\frac{1}{2}BC\right]^2 - \left[2 \times \frac{1}{2}BC \cdot DE\right]$$

$$AB^2 = AD^2 + \frac{1}{4}BC^2 - BC \cdot DE$$

$$AB^2 = AD^2 - BC \cdot DE + \frac{1}{4}BC^2 \ldots\ldots\ldots\ldots (2)$$

$$\Rightarrow C^2 = p^2 - ax + \frac{1}{4}a^2$$

(iii)From (1) and (2)

$$AB^2 + BC^2 = 2\,AD^2 + \frac{1}{2}BC^2$$

$$C^2 + b^2 = 2P^2 + \frac{a^2}{2}$$

Hence it is proved.

8. **An Emu which is 8 ft tall is standing at the foot of a pillar which is 30 ft high. It walks away from the pillar. The shadow of the Emu falls beyond Emu. What is the relation between the length of the shadow and the distance from the Emu to the pillar?** **5M**

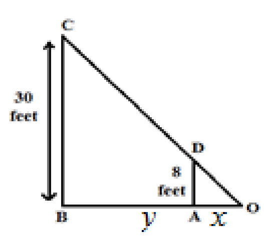
Let OA be the shadow of emu$= x$

$AB = y$

The shadow of pillar is OB

$OB = OA + AB$

$= x + y$



From BPT theorem

$\frac{OA}{OB} = \frac{AD}{BC}$

$\frac{x}{x+y} = \frac{8}{30}$

On reciprocaly we get

$\frac{x+y}{x} = \frac{30}{8}$

$\frac{x}{x} + \frac{y}{x} = \frac{30}{8}$

$1 + \frac{y}{x} = \frac{30}{8}$

$\frac{y}{x} = \frac{30}{8} - 1$

$\frac{y}{x} = \frac{30-8}{8}$

$\frac{y}{x} = \frac{22}{8} = \frac{11}{4}$

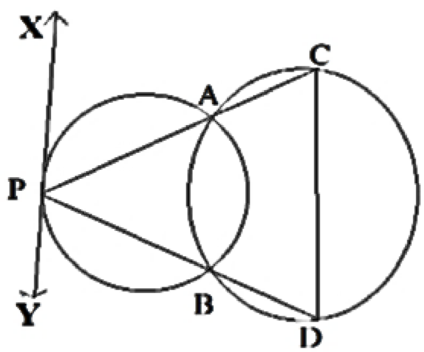
$\frac{y}{x} = \frac{11}{4}$

$x = \frac{4}{11} \times y$

shadow of emu $= \frac{\mathbf{4}}{\mathbf{11}} \times$ **distance**

9. Two circles intersect at A & B. From a point P on one of the circles lines PAC & PBD are drawn intersecting the second circle at C & D. Prove that CD is parallel to the tangent at P.

5M



Let XY be the tangent at a point P

To prove:

CD is $\parallel$ XY

Join AB

$ABCD$ is a cyclic quadrilateral

$\angle BDC + \angle BAC = 180^{\circ}$ .... (1)

$\angle BDC = 180^{\circ} - \angle BAC$ .... (2)

$\angle BDC = \angle PAB$

$\angle PBA = \angle ACD$

Since XY is the tangent to the circle at the point P

$\angle PAB = \angle BPY$

(Alternative segment theorem)

$\therefore$ $\angle PAB = \angle PDC$

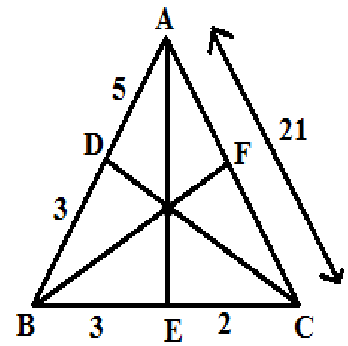
$\angle BPY = \angle PDC$

$\therefore$ $XY \parallel CD$

Thus proved.

10. Let ABC be a triangle and D, E, F are points on the respective sides AB, BC, AC (or their extensions). Let $AD : DB = 5 : 3$, $BE : EC = 3 : 2$ and $AC = 21$. Find the length of the line segment CF.

5M



We know that

$$\frac{AD}{DB} = \frac{5}{3}$$

Also, $\frac{BE}{EC} = \frac{3}{2}$

$$AC = 21$$

$$\frac{CF}{FA} = \frac{CF}{21-CF}$$

$\therefore$ By Ceva's theorem

$$\frac{BE}{EC} \times \frac{CF}{FA} \times \frac{AD}{DB} = 1$$

$$\frac{3}{2} \times \frac{CF}{21-CF} \times \frac{5}{3} = 1$$

$$\frac{CF}{21-CF} \times \frac{5}{2} = 1$$

$$\frac{CF}{21-CF} = \frac{2}{5}$$

$$5CF = 42 - 2CF$$

$$7CF = 42$$

$$CF = \frac{42}{7}$$

$= \mathbf{6}$ **units.**

5. Coordinate Geometry

Exercise 5.1

1. Find the area of the triangle formed by the points.

(i) $(1,-1), (-4,6)$ and $(-3,-5)$ (5M)

Area of the triangle

$$= \frac{1}{2}\begin{bmatrix} 1 & -4 & -3 & 1 \\ -1 & 6 & -5 & 1 \end{bmatrix}$$

$$= \frac{1}{2}[(6+20+3)-(4-18-5)]$$

$$= \frac{1}{2}[29-(-19)]$$

$$= \frac{1}{2}[29+19] = \frac{1}{2}(48)$$

$= \mathbf{24}$ **Sq. units**

> **Similar Problems** (5M)
> **Solve Your Self**
> 1. Find the area of the triangle formed by the points
> (ii) $(-10,-4), (-8,-1)$ and $(-3,-5)$
>
> **Eg. 5.1:** Find the area of the triangle whose vertices are $(-3,5)$, $(5,6)$ and $(5,-2)$

2. Determine whether the sets of points are collinear?

(i) $\left(-\frac{1}{2}, 3\right), (-5,6)$ and $(-8,8)$ (5M)

Area of the triangle

$$= \frac{1}{2}\begin{bmatrix} -\frac{1}{2} & -5 & -8 & -\frac{1}{2} \\ 3 & 6 & 8 & 3 \end{bmatrix}$$

$$= \frac{1}{2}[(-3-40-24)-(-15-48-4)]$$

$$= \frac{1}{2}[(-67)-(-67)]$$

$$= \frac{1}{2}[-67+67]$$

$= \mathbf{0}$

If Area of the triangle is 0, the set of points are collinear.

Hence the set of points are **collinear**.

> **Similar Problems** (5M)
> **Solve Your Self**
> 2. Determine whether the sets of points are collinear? (ii) $(a, b+c), (b, c+a)$ & $(c, a+b)$
> 4. In each of the following, find the values of 'a' for which the given points are collinear.
> (i) $(2,3), (4,a)$ and $(6,-3)$
> (ii) $(a, 2-2a), (-a+1, 2a)$ & $(-4-a, 6-2a)$
>
> **Eg.5.2:** Show that the points $P(-1.5,3)$, $Q(6,-2)$, $R(-3,4)$ are collinear. **(PTA-4, MAY-22)**

3. Vertices of given triangles are taken in order and their areas are provided aside. In each case, find the value of $'p'$. (5M)

(i) Vertices: $(0,0), (p,8), (6,2)$ Area: 20 sq.units

Area of the triangle $= 20$

$$\frac{1}{2}\begin{bmatrix} 0 & p & 6 & 0 \\ 0 & 8 & 2 & 0 \end{bmatrix} = 20$$

$$\frac{1}{2}[(0+2p+0)-(0+48+0)] = 20$$

$$2p-48 = 40$$

$$2p = 40+48$$

$$2p = 88$$

$$\boldsymbol{p = 44}$$

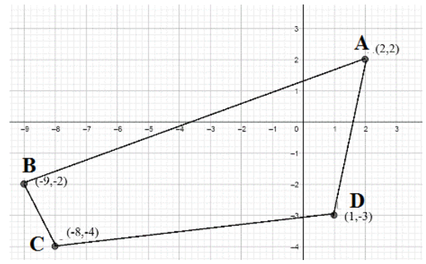
> **Similar Problems** (5M)
> **Solve Your Self**
> 3. Vertices of given triangles are taken in order and their areas are provided aside. In each case, find the value of $'p'$.
> (ii) Vertices: $(p,p), (5,6), (5,-2)$ Area : 32 sq.units
>
> **Eg. 5.3:** If the area of the triangle formed by the vertices $A(-1,2)$, $B(k,-2)$ and $C(7,4)$ (taken in order) is 22 sq. units, find the value of k. **(JUL-22)**

5. Find the area of the quadrilateral whose vertices are at JUN-23 5M

(i) $(-9,-2), (-8,-4), (2,2)$ and $(1,-3)$

By taking the points in anti clockwise we have to find the area of $ABCD$

$A(2,2), B(1,-3), C(-8,-4), D(-9,-2)$



$$\text{Area of } ABCD = \frac{1}{2}\begin{bmatrix} 2 & -9 & -8 & 1 & 2 \\ 2 & -2 & -4 & -3 & 2 \end{bmatrix}$$

$$= \frac{1}{2}[(-4+36+24+2)-(-18+16-4-6)]$$

$$= \frac{1}{2}[(62-4)-(-28+16)]$$

$$= \frac{1}{2}[58-(-12)]$$

$$= \frac{1}{2}[58+12]$$

$$= \frac{1}{2}(70)$$

$$= \mathbf{35 \text{ sq. units}}$$

Similar Problems 5M

Solve Your Self

5. Find the area of the quadrilateral whose vertices are at
(ii) $(-9,0), (-8,6), (-1,-2)$ and $(-6,-3)$

Eg. 5.6: Find the area of the quadrilateral formed by the points $(8,6)$, $(5,11)$, $(-5,12)$ and $(-4,3)$. **(APR-23, JUL-22)**

6. Find the value of k, if the area of a quadrilateral is 28 sq. units, whose vertices are taken in the order $(-4,-2), (-3,k), (3,-2)$ and $(2,3)$ 5M PTA-5, SEP-20

Area of quadrilateral $= 28$ square units

$$\frac{1}{2}\begin{bmatrix} -4 & -3 & 3 & 2 & -4 \\ -2 & k & -2 & 3 & -2 \end{bmatrix} = 28$$

$$[(-4k+6+9-4)-(6+3k-4-12)] = 56$$

$$(-4k+11)-(3k-10) = 56$$

$$-4k+11-3k+10 = 56$$

$$-7k = 56-21$$

$$-7k = 35$$

$$k = \frac{35}{-7}$$

$$\boldsymbol{k = -5}$$

Similar Problems 5M

Solve Your Self

UE-4: If vertices of a quadrilateral are at $A(-5,7), B(-4,k), C(-1,-6)$ and $D(4,5)$ and its area is 72 sq.units. Find the value of k.

7. If the points $A(-3,9), B(a,b)$ and $C(4,-5)$ are collinear and if $a+b=1$, then find a and b. 5M

$$\frac{1}{2}\begin{bmatrix} -3 & a & 4 & -3 \\ 9 & b & -5 & 9 \end{bmatrix} = 0$$

$$[(-3b-5a+36)-(9a+4b+15)] = 0$$

$$-3b-5a+36-9a-4b-15 = 0$$

$$-5a-9a-3b-4b+21 = 0$$

$$-14a-7b+21 = 0$$

$$-2a-b+3 = 0$$

$$2a+b = 3 \text{ ……………. (1)}$$

$$a+b = 1 \text{ (Given)……….. (2)}$$

Subtract $(1)-(2)$

$\Rightarrow 2a+b-a-b = 3-1$

$\boldsymbol{a = 2}$

Sub $a = 2$ in (2)

$2+b = 1 \Rightarrow \quad b = 1-2$

$\boldsymbol{b = -1}$

Similar Problems 5M

Solve Your Self

Eg. 5.4: If the points $P(-1,-4)$, $Q(b,c)$ and $R(5,-1)$ are collinear and if $2b+c=4$, then find the values of b and c. **(SEP-21)**

8. **Let $P(11,7), Q(13.5,4)$ and $R(9.5,4)$ be the mid points of the sides AB, BC and AC respectively of ΔABC. Find the coordinates of the vertices A, B and C. hence find the area of ΔABC and compare this with area of ΔPQR.** (5M)

To find the vertices of the triangle from the midpoints of the sides.

Vertex A: **Formula** $(x_1 + x_3 - x_2,\ y_1 + y_3 - y_2)$, $P(11,7), R(9.5,4)$

$= (11 + 9.5 - 13.5,\ 7 + 4 - 4)$

$= A(7,7)$

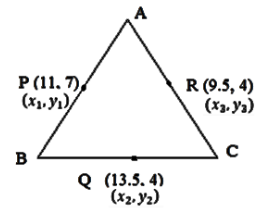
Vertex B: **Formula** $(x_1 + x_2 - x_3,\ y_1 + y_2 - y_3)$,

$= (11 + 13.5 - 9.5,\ 7 + 4 - 4)$

$= B(15,7)$

Vertex C **Formula** $(x_2 + x_3 - x_1,\ y_2 + y_3 - y_1)$,

$= (13.5 + 9.5 - 11, 4 + 4 - 7) = C(12,1)$



Area of triangle ABC

$= \frac{1}{2}\begin{bmatrix} 7 & 15 & 12 & 7 \\ 7 & 7 & 1 & 7 \end{bmatrix}$

$= \frac{1}{2}[(49 + 15 + 84) - (105 + 84 + 7)]$

$= \frac{1}{2}[148 - 196]$

$= -\frac{48}{2}$

$A = -24 =$ **24 sq.units**

Area of triangle PQR

$P(11,7)$ $Q(13.5,4)$ and $R(9.5,4)$

$= \frac{1}{2}\begin{bmatrix} 11 & 13.5 & 9.5 & 11 \\ 7 & 4 & 4 & 7 \end{bmatrix}$

$= \frac{1}{2}[(44 + 54 + 66.5) - (94.5 + 38 + 44)]$

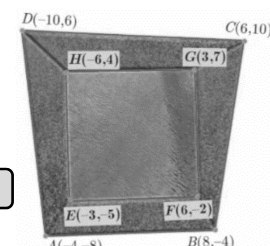
$= \frac{1}{2}[164.5 - 176.5]$

$= \frac{1}{2}(12) =$ **6 sq.units**

Area of triangle $PQR = 6$

Area of $\Delta ABC = 24 = 4(6) = 4 \times$ Area of triangle PQR

9. **In the figure, the quadrilateral swimming pool shown is surrounded by concrete patio. Find the area of the patio.** (5M) PTA-2



To find the area of patio we have to subtract area EFGH from area of ABCD

Area of ABCD $A(-4,-8), B(8,-4), C(6,10), D(-10,6)$

$= \frac{1}{2}\begin{bmatrix} -4 & 8 & 6 & -10 & -4 \\ -8 & -4 & 10 & 6 & -8 \end{bmatrix}$

$= \frac{1}{2}[(16 + 80 + 36 + 80) - (-64 - 24 - 100 - 24)]$

$= \frac{1}{2}[212 - (-212)]$

$= \frac{1}{2}[212 + 212] = \frac{1}{2}[424]$

$=$ **212 Square units.**

Area of EFGH $E(-3,-5), F(6,-2), G(3,7), H(-6,4)$

$= \frac{1}{2}\begin{bmatrix} -3 & 6 & 3 & -6 & -3 \\ -5 & -2 & 7 & 4 & -5 \end{bmatrix}$

$= \frac{1}{2}[(6 + 42 + 12 + 30) - (-30 - 6 - 42 - 12)]$

$= \frac{1}{2}[90 - (-90)]$

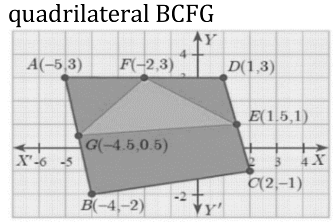
$= \frac{1}{2}[180]$

$=$ **90 Square units.**

Area of the concrete patio $=$ Area of $ABCD -$ Area of $EFGH = 212 - 90 =$ **122 sq.units.**

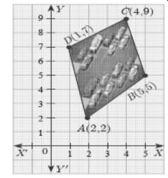
Similar Problems **Solve Your Self** (5M)

10. A triangular shaped glass with vertices at $A(-5,-4), B(1,6)$ and $C(7,-4)$ has to be painted. If one bucket of paint covers 6 square feet, how many buckets of paint will be required to paint the whole glass, if only one coat of paint is applied.
11. In the figure find the area of
    (i) triangle AGF (ii) triangle FED
    (iii) quadrilateral BCFG



Similar Problems 5M

Solve Your Self



Eg. 5.5: The floor of a hall is covered with identical tiles which are in the shapes of triangles. One such triangle has the vertices at $(-3,2)$, $(-1,-1)$ and $(1,2)$. If the floor of the hall is completely covered by 110 tiles, find the area of the floor.

Eg. 5.7: The given diagram shows a plan for constructing a new parking lot at a campus. It is estimated that such construction would cost ₹ 1300 per square feet. What will be the total cost for making the parking lot?

Exercise 5.2

1. What is the slope of a line whose inclination with positive direction of x- axis is 2M

(i) 90°

$m = \tan\theta$

$= \tan 90^\circ = \infty$

$\boldsymbol{m} =$ **Undefined**

(ii) 0°

$m = \tan\theta$

$m = \tan 0^\circ$

$\boldsymbol{m = 0}$

Similar Problems 2M

Solve Your Self

Eg. 5.8:(i) What is the slope of a line whose inclination is 30°?

2. What is the inclination of a line whose slope is 2M PTA-3

(i) 0

$m = 0$

$\tan\theta = 0$

Angle of inclination is $\mathbf{0}^\circ$

(ii) 1

Slope $m = 1$

$\tan\theta = 1 \Rightarrow \quad \theta = 45^\circ$ ($\because \tan 45^\circ = 1$)

Angle of inclination is 45°.

Similar Problems 2M

Solve Your Self

Eg. 5.8: (ii) What is the inclination of a line whose slope is $\sqrt{3}$?

3. Find the slope of a line joining the points

(ii) $(\sin\boldsymbol{\theta}, -\cos\boldsymbol{\theta})$ and $(-\sin\boldsymbol{\theta}, \cos\boldsymbol{\theta})$ 2M PTA-2

Slope $m = \frac{y_2-y_1}{x_2-x_1}$

$(x_1, y_1) = (\sin\theta, -\cos\theta)$
$(x_2, y_2) = (-\sin\theta, \cos\theta)$

$= \frac{\cos\theta-(-\cos\theta)}{-\sin\theta-\sin\theta}$

$= \frac{2\cos\theta}{-2\sin\theta}$

$= \frac{-\cos\theta}{\sin\theta}$

$= -\cot\theta$

$\boldsymbol{m = -\cot\theta}$

Similar Problems **Solve Your Self**

3. Find the slope of a line joining the points 2M
(i) $(5, \sqrt{5})$With the origin **(JUN-23, JUL-22)**

7. The line through the points $(-2, a)$ and $(9,3)$ has slope $-\frac{1}{2}$. find the value of a. 2M

Eg. 5.9: Find the slope of a line joining the given points
(i) $(-6,1)$ and $(-3,2)$ (ii) $\left[-\frac{1}{3}, \frac{1}{2}\right]$ and $\left[\frac{2}{7}, \frac{3}{7}\right]$ 2M
(iii) $(14,10)$ and $(14,-6)$ **(SEP-20)**

Eg. 5.13: Let $A(1,-2), B(6,-2), C(5,1)$ & $D(2,1)$ be four points (i) Find the slope of the line segments (a) AB (b) CD (ii) Find the slope of the line segments (a) BC (b) AD 5M
(iii) What can you deduce from your answer.

4. What is the slope of a line perpendicular to the line joining $\boldsymbol{A(5,1)}$ and $\boldsymbol{P}$ where $\boldsymbol{P}$ is the mid-point of the segment joining $(\mathbf{4},\mathbf{2})$ and $(-\mathbf{6},\mathbf{4})$. 2M

P is the mid-point of the segment joining (4,2) and $(-6,4)$

First let us find the point P.

$(x_1 y_1) = (4,2)$
$(x_2, y_2) = (-6,4)$

Mid-point $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{4+(-6)}{2}, \frac{2+4}{2}\right) = \left(\frac{-2}{2}, \frac{6}{2}\right) = (-1,3)$

Now we have to find the slope or the line which is $\perp r$ to the line joining the points $A(5,1)$ and $P(-1,3)$

Slope of $AP \times$ slope of the required line $\Rightarrow m_1 \times m_2 = -1$

Slope of $AP = \frac{y_2-y_1}{x_2-x_1} = \frac{3-1}{-1-5} = \frac{2}{-6} = \frac{-1}{3}$

$\therefore$ Slope of a required line $= \frac{-1}{-\frac{1}{3}} = 3$

5. Show that the given points are collinear: $(-3,-4),(7,2)$ and $(12,5)$ SEP-21 2M

Let the given points be

$A(-3,-4), B(7,2)$ and $C(12,5)$

Slope of $AB = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{[2-(-4)]}{[7-(-3)]}$

$= \frac{2+4}{7+3}$

$= \frac{6}{10}$

$\boldsymbol{m = \frac{3}{5}}$

Slope of $BC = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{5-2}{12-7}$

$\boldsymbol{m = \frac{3}{5}}$

Slope of AB = Slope of BC

∴ The given points are collinear.

Similar Problems
Solve Your Self 2M

6. If the three points $(3,-1),(a,3)$ and $(1,-3)$ are collinear, find the value of a.

Eg. 5.12: Show that the points $(-2,5)$, $(6,-1)$ and $(2,2)$ are collinear. 5M

8. The line through the points $(-2,6)$ and $(4,8)$ is perpendicular to the line through the points $(8,12)$ and $(x,24)$. Find the value of x. 2M PTA-6

Slope of the line passing through the points $(-2,6)$ and $(4,8)$

Slope $m_1 = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{8-6}{4-(-2)} = \frac{2}{4+2} = \frac{2}{6} = \frac{1}{3}$ ....(1)

Slope of the line passing through the point $(8,12)$ and $(x,24)$

Slope $m_2 = \frac{y_2-y_1}{x_2-x_1} = \frac{24-12}{x-8} = \frac{12}{x-8}$........ (2)

Since these lines are perpendicular to each other

$m_1 \times m_2 = -1 \Rightarrow \quad \frac{1}{3} \times \frac{12}{x-8} = -1$

$\frac{4}{x-8} = -1$

$4 = -(x-8)$

$4 = -x+8$

$x = 8-4$

$\boldsymbol{x = 4}$

9. Show that the given points form a right angled triangle and check whether they satisfies Pythagoras theorem (i) $A(1,-4), B(2,-3)$ and $C(4,-7)$ 5M

Slope of $AB = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{[-3-(-4)]}{2-1}$

$= \frac{-3+4}{1} = 1$

$\boxed{\begin{array}{c} A(1,-4) = (x_1,y_1) \\ B(2,-3) = (x_2,y_2) \end{array}}$

Slope of $BC = \frac{[-7-(-3)]}{4-2}$

$= \frac{-7+3}{2}$

$= -\frac{4}{2}$

$= -2$

$\boxed{\begin{array}{c} B(2,-3) = (x_1,y_1) \\ C(4,-7) = (x_2,y_2) \end{array}}$

Slope of $CA = \frac{[-7-(-4)]}{4-1}$

$= \frac{-7+4}{3}$

$= -\frac{3}{3} \quad = -1$

$\boxed{\begin{array}{c} C(4,-7) = (x_1,y_1) \\ A(1,-4) = (x_2,y_2) \end{array}}$

Slope of $AB \times$ slope of $CA = -1$

$1 \times (-1) = -1$

$-1 = -1$

Hence the given points are vertices of right angled triangle.

In order to check this with Pythagoras theorem let us find the length of AB, BC and CA.

$A(1,-4), B(2,-3)$ and $C(4,-7)$

$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-3+4)^2+(2-1)^2}$

$= \sqrt{1+1} \quad = \sqrt{2}$

$BC = \sqrt{(-7+3)^2+(4-2)^2}$

$= \sqrt{(-4)^2+(2)^2} = \sqrt{16+4} = \sqrt{20}$

$CA = \sqrt{(-7+4)^2+(4-1)^2}$

$= \sqrt{(-3)^2+(3)^2} = \sqrt{9+9} = \sqrt{18}$

$BC^2 = AB^2 + CA^2$

$(\sqrt{20})^2 = (\sqrt{2})^2 + (\sqrt{18})^2$

$20 = 20$

Yes , they satisfies Pythagoras theorem.

Similar Problems

Solve Your Self

9. Show that the given points form a right angled triangle and check whether they satisfies Pythagoras theorem **5M**

(ii) $L(0,5), M(9,12)$ and $N(3,14)$

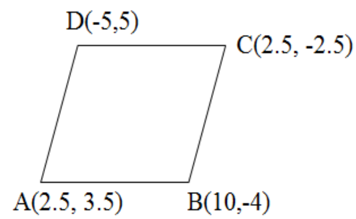
Eg. 5.15: Without using Pythagoras theorem, show that the points $(1,-4)$, $(2,-3)$ and $(4,-7)$ form a right angled triangle. **(PTA-4)** **2M**

10. Show that the given points form a parallelogram: **5M**

$\boldsymbol{A(2.5, 3.5), B(10,-4), C(2.5,-2.5), D(-5,5)}$

In a parallelogram opposite sides will be parallel by proving that slope of opposite sides are equal we may say that opposite sides are parallel.

Slope $m = \frac{y_2-y_1}{x_2-x_1}$



$A(2.5,3.5), B(10,-4), C(2.5-2.5)$ and $D(-5,5)$

Slope of AB: $m_1 = \frac{-4-3.5}{10-2.5} = \frac{-7.5}{7.5} = -1$

Slope of CD: $m_2 = \frac{[5-(-2.5)]}{-5-2.5}$

$= \frac{5+2.5}{-7.5}$

$= \frac{7.5}{-7.5} = -1$

$m_1 = m_2$

AB, CD are parallel

Slope of BC

$m_1 = \frac{(-2.5+4)}{2.5-10}$

$= \frac{1.5}{-7.5}$

$= \frac{-15}{75}$

$= -\frac{1}{5}$

Slope of DA

$m_2 = \frac{5-3.5}{-5-2.5}$

$= \frac{1.5}{-7.5}$

$= \frac{-15}{75}$

$= -\frac{1}{5}$

$\therefore$ Slope of AB =slope of $CD = \mathbf{-1}$

Slope of BC = slope of $DA = \mathbf{-1/5}$

Hence the given points form a parallelogram.

Similar Problems **5M**

Solve Your Self

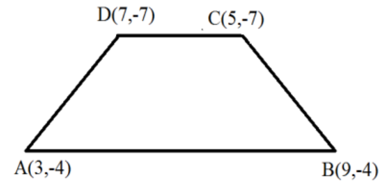
11. If the points $A(2,2), B(-2,-3), C(1,-3)$ and $D(x,y)$ form a parallelogram then find the value of x and y.

UE-1: PQRS is a rectangle formed by joining the points $P(-1,-1), Q(-1,4)$, $R(5,4)$ and $S(5,-1)$. A, B, C and D are the mid-points of PQ, QR, RS and SP respectively. Is the quadrilateral ABCD a square, a rectangle or a rhombus? Justify your answer.

UE-5: Without using distance formula, show that the points $(-2,-1), (4,0), (3,3)$ and $(-3,2)$ are vertices of a parallelogram.

12. Let $A(3,-4)$, $B(9,-4)$, $C(5,-7)$ and $D(7,-7)$. Show that $ABCD$ is a trapezium. 5M

A trapezium will always contain two parallel sides and two non-parallel sides.



Slope of AB: $m = \frac{-4+4}{9-3} = \frac{0}{6} = \mathbf{0}$

Slope of BC: $m = \frac{-7+4}{5-9} = \frac{-3}{-4} = \mathbf{\frac{3}{4}}$

Slope of CD: $m = \frac{-7+7}{7-5} = \frac{0}{2} = \mathbf{0}$

Slope of DA: $m = \frac{-7+4}{7-3} = \mathbf{\frac{-3}{4}}$

The sides AB and CD are parallel. Slope of BC and slope of DA are non parallel.

Hence $ABCD$ is a trapezium.

13. A quadrilateral has vertices at $A(-4,-2)$, $B(5,-1)$, $C(6,5)$ and $D(-7,6)$. 5M MAY-22

Show that the mid-points of its sides form a parallelogram.

Mid-point of the side $AB = \left(\frac{-4+5}{2}, \frac{-2-1}{2}\right)$

$= \left(\frac{1}{2}, \frac{-3}{2}\right) = P$

Mid-point of the side $BC = \left(\frac{5+6}{2}, \frac{-1+5}{2}\right)$

$= \left(\frac{11}{2}, \frac{4}{2}\right)$

$= \left(\frac{11}{2}, 2\right) = Q$

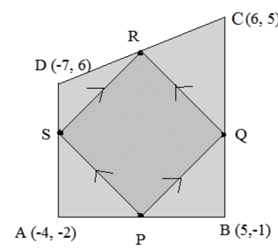
Mid-point of the side $CD = \left(\frac{6-7}{2}, \frac{5+6}{2}\right)$

$= \left(-\frac{1}{2}, \frac{11}{2}\right) = R$

Mid-point of the side $DA = \left(\frac{-7-4}{2}, \frac{6-2}{2}\right)$

$= \left(-\frac{11}{2}, \frac{4}{2}\right) = \left(-\frac{11}{2}, 2\right) = S$

Slope of opposite sides:



Slope of the PQ

$= \frac{2+\frac{3}{2}}{\frac{11}{2}-\frac{1}{2}} = \frac{7/2}{10/2} = \frac{7}{10}$

Slope of RS

$= \frac{2-\frac{11}{2}}{-\frac{1}{2}+\frac{11}{2}} = \frac{7/2}{10/2} = \frac{7}{10}$

Slope of $QR = \frac{\frac{11}{2}-2}{-\frac{1}{2}-\frac{11}{2}} = \frac{\frac{7}{2}}{-\frac{12}{2}} = -\frac{7}{12}$

Slope of $PS = \frac{2+\frac{3}{2}}{-\frac{11}{2}-\frac{1}{2}} = \frac{\frac{7}{2}}{\frac{-11-1}{2}} = \frac{7/2}{-12/2} = -\frac{7}{12}$

$\therefore\ PQ = RS,\quad QR = PS$

Hence, mid-points of its sides form a parallelogram

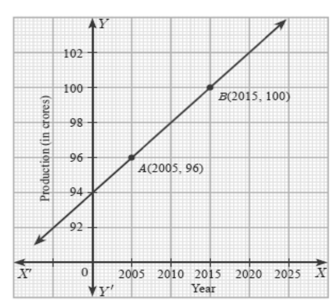
Similar Problems

Solve Your Self

Eg. 5.10: The line r passes through the points $(-2,2)$ and $(5,8)$ and the line s passes through the points $(-8,7)$ and $(-2,0)$. Is the line r perpendicular to s? 2M

Eg. 5.11: The line p passes through the points $(3,-2)$, $(12,4)$ and the line q passes through the points $(6,-2)$ and $(12,2)$. Is p parallel to q? **(MAY-22)** 5M

Eg. 5.14: Consider the graph representing growth of population (in crores).
Find the slope of the line AB and hence estimate the population in the year 2030? 5M



Eg. 5.16: Prove analytically that the line segment joining the mid-points of two sides of a triangle is parallel to the third side and is equal to half of its length. 5M

CQ: $PQRS$ is a rhombus. Its diagonals PR and QS intersect at the point M and satisfy $QS = 2PR$.
If the coordinates of S and M are $(1,1)$ and $(2,-1)$ respectively. Find the coordinates of P. **(PTA-4)** 5M

Exercise 5.3

1. Find the equation of a straight line passing through the mid-point of a line segment joining the points $(1,-5),(4,2)$ and parallel to (i) X axis (ii) Y axis 2M

Midpoint of the line segment

$$=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$$

$$=\left(\frac{1+4}{2},\frac{-5+2}{2}\right)=\left(\frac{5}{2},\frac{-3}{2}\right)$$

(i) X axis

The required line is passing through the point $\left(\frac{5}{2},-\frac{3}{2}\right)$ and parallel to x-axis.

If the line is similarly to x-axis then slope of the required line $y=c$

$$y=-\frac{3}{2}$$

$$2y=-3$$

$$\mathbf{2y+3=0}$$

(ii) Y axis

The required line is passing through the point $\left(\frac{5}{2},-\frac{3}{2}\right)$ and parallel to y-axis.

If the line is parallel to y-axis then slope of the required line $x=b$

$$x=\frac{5}{2}$$

$$2x=5$$

$$\mathbf{2x-5=0}$$

> **Similar Problems** 2M
> **Solve Your Self**
> **Eg. 5.17:** Find the equation of a straight line passing through $(5,7)$ and is (i) parallel to X axis (ii) parallel to Y axis.

2. The equation of a straight line is $2(x-y)+5=0$. Find its slope, inclination and intercept on the Y axis. 2M

$$2(x-y)+5=0$$

$$2x-2y+5=0$$

$$2y=2x+5$$

$$y=x+\frac{5}{2}$$

Slope $m=1$

Angle of inclination: $m=1$

$$\tan\theta=\tan 45$$

$$\theta=45^\circ$$

Intercept of y axis

y Intercept $(c)=\mathbf{\frac{5}{2}}$

3. Find the equation of line whose inclination is 30° and making an intercept -3 on the Y axis. 2M

$$\theta=30^\circ$$

$$m=\tan\theta$$

$$m=\tan 30=\frac{1}{\sqrt{3}}$$

Intercept on the y axis $=-3=c$

Equation of the line:

$$y=mx+c$$

$$=\frac{1}{\sqrt{3}}x+(-3)$$

$$y=\frac{x}{\sqrt{3}}-3$$

$$\sqrt{3}y=x-3\sqrt{3}$$

The required equation is

$$\mathbf{x-\sqrt{3}y-3\sqrt{3}=0}$$

> **Similar Problems** 2M
> **Solve Your Self**
> **Eg. 5.18:** Find the equation of a straight line whose (i) Slope is 5 and y intercept is -9 (ii) Inclination is 45° and y intercept is 11

4. Find the slope and y intercept of $\sqrt{3}x+(1-\sqrt{3})y=3$ 2M

By comparing the given equation with the form $y=mx+c$

$$(1-\sqrt{3})y=-\sqrt{3}x+3$$

$$y=\frac{(-\sqrt{3}x+3)}{1-\sqrt{3}}$$

$$y=\frac{-\sqrt{3}x}{1-\sqrt{3}}+\frac{3}{1-\sqrt{3}}$$

Slope $m=-\frac{\sqrt{3}}{1-\sqrt{3}}$

$$=\frac{\sqrt{3}}{\sqrt{3}-1}\times\frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$=\frac{3+\sqrt{3}}{3-1}$$

$$=\mathbf{\frac{3+\sqrt{3}}{2}}$$

y Intercept $=\frac{3}{1-\sqrt{3}}$

$$=\frac{3}{1-\sqrt{3}}\times\frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$=\frac{3(1+\sqrt{3})}{1-3}$$

$$=\mathbf{\frac{3+3\sqrt{3}}{-2}}$$

> **Similar Problems**
> **Solve Your Self**
> **Eg. 5.19:** Calculate the slope and y intercept of the straight line $8x-7y+6=0$
> **(SEP-21)** 2M

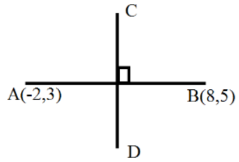
5. Find the value of 'a' if the line through $(-2,3)$ and $(8,5)$ is perpendicular to $y = ax + 2$

Slopes of line joining points $(-2,3)$ and $(8,5)$

2M

$m_1 = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{5-3}{8+2} = \frac{2}{10} = \frac{1}{5}$



$AB \perp CD$

$m_1 \times m_2 = -1$

$\frac{1}{5} \times m_2 = -1$

$m_2 = -1 \times \frac{5}{1} = -5$

Slope of the line $y = ax + 2$

$m_2 = a$

$m_2 = -5$

$\boldsymbol{a = -5}$

Similar Problems
Solve Your Self
Eg. 5.22: Find the equation of a line passing through the point $A(1,4)$ & perpendicular to the line joining points $(2,5)$ and $(4,7)$ 2M

6. The hill in the form of a triangle has its foot at $(19,3)$ the inclination of the hill to the ground is $45°$. Find the equation of the hill joining the foot and top. 2M

Equation of the hill joining the foot and top:

$\theta = 45^{\circ}$; $(x_1, y_1) = (19,3)$

Slope $m = \tan 45^{\circ} = 1$

Equation: $y - y_1 = m(x - x_1)$

$y - 3 = 1(x - 19)$

$y - 3 = x - 19$

$0 = x - y - 19 + 3$

$\therefore x - y - 16 = 0$

The required equation of the Straight line is $\boldsymbol{x - y - 16 = 0}$.

7. Find the equation of the line through the given pair of points (i) $\left(2,\frac{2}{3}\right)$ and $\left(-\frac{1}{2},-2\right)$ 2M

Equation is $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$\frac{y-\frac{2}{3}}{-2-\frac{2}{3}} = \frac{x-2}{-\frac{1}{2}-2}$

$\frac{\frac{3y-2}{3}}{-\frac{8}{3}} = \frac{x-2}{-\frac{5}{2}} \Rightarrow \frac{-(3y-2)}{8} = \frac{-2(x-2)}{5}$

$5(3y-2) = 16(x-2)$

$15y - 10 = 16x - 32$

$16x - 15y - 32 + 10 = 0$

The required equation of the straight line is $\boldsymbol{16x - 15y - 22 = 0}$

Similar Problems
Solve Your Self
Eg. 5.23: Find the equation of a straight line passing through $(5,-3)$ & $(7,-4)$
7 (ii). Find the equation of the line through the given pair of points $(2,3)$ & $(-7,-1)$ 2M

8. A cat is located at the point $(-6,-4)$ is xy- plane. A bottle of milk is kept at $(5,11)$ The cat wish to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk. 5M APR-23

Equation of the path $\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$

$(-6,-4)$ and $(5,11)$

$(x_1, y_1) = (-6,-4)$
$(x_2, y_2) = (5,11)$

$\frac{y+4}{15} = \frac{x+6}{11}$

$11(y+4) = 15(x+6)$

$11y + 44 = 15x + 90$

$0 = 15x - 11y + 90 - 44$

The required equation is $\boldsymbol{15x - 11y + 46 = 0}$

Similar Problems **Solve Your Self**
CQ: A cat is located at the point (6,4) is xy-plane. A bottle of milk is kept at $(-5,-11)$. The cat wish to consume the milk travelling through shortest possible distance. Find the equation of the path it needs to take its milk. **(JUL -22)** 5M
Eg. 5.24: Two buildings of different heights are located at opposite sides of each other. If a heavy rod is attached joining the terrace of the buildings from $(6,10)$ to $(14,12)$, find the equation of the rod joining the buildings? 2M
UE-7: The owner of a milk store finds that, he can sell 980 litres of milk each week at ₹ 14/litre and 1220 litres of milk each week at ₹ 16/litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at ₹ 17/ litre? 2M

10. Find the equation of a straight line which has slope $-\frac{5}{4}$ and passing through to the point $(-1,2)$ MAY-22

Slope $m = -\frac{5}{4}$ 2M

Equation of the line passing through the point $(-1,2) \Rightarrow$ $y - y_1 = m(x - x_1)$

$y - 2 = \left(-\frac{5}{4}\right)(x - (-1))$

$4(y-2) = -5(x+1)$

$4y - 8 = -5x - 5$

$5x + 4y + 5 - 8 = 0$

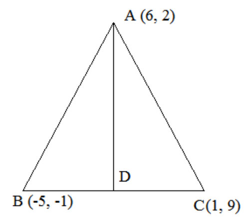
The required equation is $\boldsymbol{5x + 4y - 3 = 0}$

Similar Problems 2M
Solve Your Self
Eg. 5.21: Find the equation of a line passing through the point $(3,-4)$ and having slope $\frac{-5}{7}$
CQ: Find the equation of a line passing through the point $(-4,3)$ and having slope $-\frac{7}{5}$. **(PTA-1)**

9. Find the equation of the median and altitude of triangle ABC through A where the vertices are $A(6,2)$, $B(-5,-1)$ and $C(1,9)$ (5M)

SEP-21, PTA-6

The median drawn passing through the vertex A intersect the side BC at the mid point.



$$D = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$D = \left(\frac{-5+1}{2}, \frac{-1+9}{2}\right)$$

$$= \left(\frac{-4}{2}, \frac{8}{2}\right) = (-2,4)$$

$(x_1, y_1) = B(-5,-1)$
$(x_2, y_2) = C(1,9)$

Equation of the median AD:

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$

$(x_1, y_1) = A(6,2)$
$(x_2, y_2) = D(-2,4)$

$$\frac{y-2}{4-2} = \frac{x-6}{-2-6}$$

$$\frac{y-2}{2} = \frac{x-6}{-8}$$

$$-8(y-2) = 2(x-6)$$

$$-8y+16 = 2x-12$$

$$0 = 2x+8y-12-16$$

$$2x+8y-28 = 0$$

$$\div 2, \quad x+4y-14 = 0$$

If a line passing through the vertex A is altitude, then it will be perpendicular to BC

Slope of BC $\quad m = \frac{y_2-y_1}{x_2-x_1} = \frac{9+1}{1+5} = \frac{10}{6} = \frac{5}{3}$

$$m_1 \times m_2 = -1$$

$$\frac{5}{3} \times m_2 = -1$$

$$m_2 = -1 \times \frac{3}{5}$$

$$= -\frac{3}{5}$$

Equation of altitude passing through A

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{3}{5}(x-6)$$

$$5(y-2) = -3(x-6)$$

$$5y - 10 = -3x + 18$$

$$3x + 5y - 10 - 18 = 0$$

$$\mathbf{3x + 5y - 28 = 0}$$

Similar Problems
Solve Your Self (2M)

CQ: The vertices of a triangle are $A(-1,3)$, $B(1,-1)$ and $C(5,1)$. Find the length of the median through the vertex C. **(MDL)**

11. You are downloading a song, the percent y (in decimal form) of megabytes remaining to get downloaded in x seconds is given by $y = -0.1x + 1$

(i) Find the total MB of the song

(ii) After how many seconds will 75% of the song gets downloaded ?

(iii) After how many seconds the song will be downloaded completely ? (5M)

(i) To find the total MB of the song we have to assign the value of x as 0

$$y = -0.1x + 1$$

If $x = 0 \Rightarrow y = -0.1(0) + 1 = 0 + 1 = \mathbf{1}$

Hence the size of song to be downloaded is 1MB.

(ii) $y = 75\% = 0.75 = 1 - 0.25$

$y = 0.25$ MB to be downloaded

$$y = -0.1x + 1$$

$$0.25 = -0.1x + 1$$

$$0.25 - 1 = -0.1x$$

$$-0.75 = -0.1x$$

$$x = \frac{0.75}{0.1}$$

$$\therefore \mathbf{x = 7.5 \text{ Seconds}}$$

(iii) Now the single of MB is 0

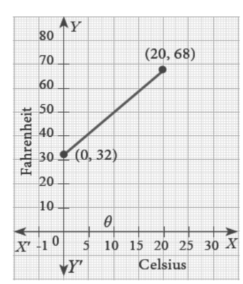
$$0 = -0.1x + 1$$

$$0.1x = 1 \Rightarrow x = \frac{1}{0.1} = 10$$

Hence it will take **10 seconds** to download the song completely.

Similar Problems
Solve Your Self (5M)

Eg. 5.20: The graph relates temperatures y (in Fahrenheit degree) to temperatures x (in Celsius degree) (a) Find the slope and y intercept (b) Write an equation of the line (c) What is the mean temperature of the earth in Fahrenheit degree if its mean temperature is 25° Celsius?



Eg. 5.27: A mobile phone is put to use when the battery power is 100%. The percent of battery power 'y' (in decimal) remaining after using the mobile phone for x hours is assumed as $y = -0.25x + 1$ (i) Find the number of hours elapsed if the battery power is 40%. (ii) How much time does it take so that the battery has no power? (2M)

12. Find the equation of the line whose intercepts the x and y axis given below: (2M)

(i) $(4,-6)$

x Intercept $\Rightarrow a = 4$

y Intercept $\Rightarrow b = -6$

Intercept form

$$\frac{x}{a}+\frac{y}{b}=1$$
$$\frac{x}{4}+\frac{y}{-6}=1$$
$$\frac{x}{4}-\frac{y}{6}=1$$
$$6x-4y=24$$
$$3x-2y=12$$
$$\boldsymbol{3x-2y-12=0}$$

Similar Problems (2M)
Solve Your Self

12. Find the equation of the line whose intercepts the x and y axis given below: (ii) $-5,\frac{3}{4}$

CQ: A straight line AB cuts the co-ordinate axes at A and B. If the mid-point of AB is (2,3), find the equation of AB. **(SEP-20)** (5M)

13. Find the intercept made by following lines on the coordinate axes. (2M)

(i) $3x-2y-6=0$ SEP-21

$3x-2y=6$ Dividing by 6

$$\frac{x}{2}+\frac{y}{-3}=1$$

x Intercept $\Rightarrow$ **2** $\quad$ y Intercept $\Rightarrow$ **-3**

Similar Problems (2M)
Solve Your Self

13. Find the intercept made by following lines on the coordinate axes. (ii) $4x+3y+12=0$

Eg.5.26: Find the intercepts made by the line $4x-9y+36=0$ on the coordinate axes

14. Find the equation of the Straight line

(i) Passing through $(1,-4)$ and has intercept which are in the ratio $2:5$

(ii) Passing through $(-8,4)$ and making equal intercepts on the coordinate axes. (5M)

(i) x Intercept $=2k$, y Intercept $=5k$

Equation of the line when intercepts are given $\frac{x}{a}+\frac{y}{b}=1$

The required line is passing through the point $(1,-4)$

$$\frac{1}{2k}+\left(\frac{-4}{5k}\right)=1 \Rightarrow \frac{5-8}{10k}=1$$
$$-3=10k \Rightarrow k=\frac{-3}{10}$$
$$a=2\left(\frac{-3}{10}\right)=-\frac{3}{5}$$
$$b=5\left(\frac{-3}{10}\right)=-\frac{3}{2}$$

Equation of the line

$$\frac{x}{-\frac{3}{5}}+\frac{y}{-\frac{3}{2}}=1$$
$$-5x-2y=3$$

The required equation is

$$\boldsymbol{5x+2y+3=0}$$

(ii) $a=b$ (Making equal intercept)

equation of line $\frac{x}{a}+\frac{y}{b}=1$

$$-\frac{8}{a}+\frac{4}{a}=1$$
$$(-8+4)=a$$
$$a=-4$$

Equation of the required line is

$$\frac{x}{a}+\frac{y}{b}=1 \Rightarrow \frac{x}{-4}+\frac{y}{-4}=1$$
$$x+y=-4$$
$$\boldsymbol{x+y+4=0}$$

Similar Problems (5M)
Solve Your Self

Eg. 5.25: Find the equation of a line which passes through $(5,7)$ and makes intercepts on the axes equal in magnitude but opposite in sign.

Eg. 5.28: A line makes positive intercepts on coordinate axes whose sum is 7 and it passes through $(-3,8)$. Find its equation.

CQ: Find the equation of the line passing through $(22,-6)$ and having intercept on x-axis exceeds the intercept on y-axis by 5 units. **(MAY-22)**

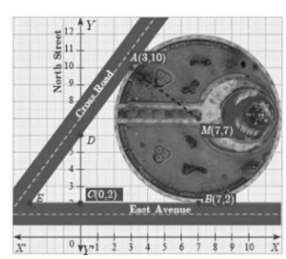
For Practice:

Eg. 5.29: A circular garden is bounded by East Avenue and Cross Road. Cross Road intersects North Street at D and East Avenue at E. AD is tangential to the circular garden at $A(3,10)$. Using the figure.

(a) Find the equation of (i) East Avenue (ii) North Street (iii) Cross Road

(b) Where does the Cross Road intersect ?

(i) North Street $\quad$ (ii) East Avenue ? **(2M)**



Exercise 5.4

1. Find the slope of the following straight line (i) $5y - 3 = 0$ 2M

(i) Slope $m = \frac{-\text{coefficient of } x}{\text{coefficient of } y}$

$= \frac{0}{5} = 0$

$\boldsymbol{m = 0}$

Hence the slope of the given line is 0.

> **Similar Problems** 2M
> **Solve Your Self**
> **Eg. 5.30:** Find the slope of the straight line $6x + 8y + 7 = 0$.
> 1. Find the slope of the following straight line (ii) $7x - \left(\frac{3}{17}\right) = 0$

2. Find the slope of the line which is
(i) parallel to $y = 0.7\,x - 11$ (ii) perpendicular to the line $x = -11$ 2M

(i) If two lines are parallel then the slopes will be equal

Slope of the given line, $m = 0.7$

(ii) Perpendicular to the line $x = -11$

$x + 11 = 0$

Slope of the given line $= -\frac{1}{0}$

$=$ undefined

Slope of the line perpendicular to the given line $= -\frac{1}{undefined} = \frac{1}{1/0} = \boldsymbol{0}$

> **Similar Problems** 2M
> **Solve Your Self**
> **Eg. 5.31:** Find the slope of the line which is
> (i) parallel to $3x - 7y = 11$
> (ii) perpendicular to $2x - 3y + 8 = 0$

3. Check whether the given lines are parallel or perpendicular 2M
(i) $\frac{x}{3} + \frac{y}{4} + \frac{1}{7} = 0$ and $\frac{2x}{3} + \frac{y}{2} + \frac{1}{10} = 0$ (ii) $5x + 23y + 14 = 0$ and $23x - 5y + 9 = 0$

(i) Let us find the slopes of the given lines. $m = \frac{-\text{coefficient of } x}{\text{co efficient of } y}$

Slope of the 1st line $= \left(-\frac{1}{3}\right) \times \frac{4}{1}$

$m_1 = -\frac{4}{3}$

Slope of the 2nd line $= \left(-\frac{2}{3}\right) \times \frac{2}{1}$

$m_2 = -\frac{4}{3}$

Since the slopes are equal the given lines are **parallel**

(ii) Let us find the slope of the given lines

$m = \frac{-\text{coefficient of } x}{\text{co efficient of } y}$

Slope of 1st line $m_1 = \frac{-5}{23}$

Slope of the 2nd line $m_2 = \frac{-23}{-5} = \frac{23}{5}$

$m_1 \times m_2 = -1$

$-\frac{5}{23} \times \frac{23}{5} = -1$

$\Rightarrow -1 = -1$

Hence the given lines are **perpendicular.**

> **Similar Problems** 2M
> **Solve Your Self**
> 4. If the straight lines $12y = -(p + 3)x + 12$, $12x - 7y = 16$ are perpendicular then find p. **(APR-23)**
> **Eg. 5.32:** Show that the straight lines $2x + 3y - 8 = 0$ and $4x + 6y + 18 = 0$ are parallel.
> **Eg. 5.33:** Show that the straight lines $x - 2y + 3 = 0$ and $6x + 3y + 8 = 0$ are perpendicular. **(PTA-5)**
> **CQ:** Show that the straight lines $3x - 5y + 7 = 0$ and $15x + 9y + 4 = 0$ are perpendicular. **(PTA-3)**

5. Find the equation of a straight line passing through the point $P(-5,2)$ and parallel to the line joining the points $Q(3,-2)$ and $R(-5,4)$ (2M)

The required line joining the points QR is similarly to the required line the slopes will be equal.

Slope of $QR = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{[4-(-2)]}{-5-3}$

$= \frac{4+2}{-8} = \frac{6}{-8}$

Slope of $QR = -\frac{3}{4}$

Slope of required line $= -\frac{3}{4}$

Equation of the line passing through the point $P(-5,2)$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = \left(-\frac{3}{4}\right)[x - (-5)]$$

$$4(y-2) = -3(x+5)$$

$$4y - 8 = -3x - 15$$

$$3x + 4y - 8 + 15 = 0$$

$$\therefore 3x + 4y + 7 = 0$$

The required equation of the line is

$\mathbf{3x + 4y + 7 = 0}$

Similar Problems (2M)

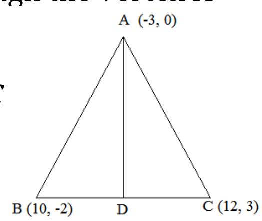
Solve Your Self

6. Find the equation of a line passing through $(6,-2)$ and perpendicular to the line joining the points $(6,7)$ and $(2,-3)$

7. $A(-3,0), B(10,-2)$ and $C(12,3)$ are the vertices of triangle ABC. Find the equation of the altitude through A and B. (5M)

Equation of altitude through A

The altitude passing through the vertex A intersect the side BC at D.



AD is perpendicular to BC

Slope of $BC = \frac{y_2-y_1}{x_2-x_1}$

$= \frac{3-(-2)}{12-10}$

$= \frac{3+2}{2}$

$= \frac{5}{2}$

$(x_1, y_1) \Rightarrow (10,-2)$ $(x_2, y_2) \Rightarrow (12,3)$

Equation of the altitude passing through the vertex A.

$$y - y_1 = m(x - x_1)$$

$A(-3,0)$ and $m = \frac{5}{2}$

$$y - 0 = -\frac{1}{\frac{5}{2}}(x - (-3))$$

$$y = -\frac{2}{5}(x+3)$$

$$5y = -2x - 6$$

$\therefore$ The required equation is $\mathbf{2x + 5y + 6 = 0}$

Equation of altitude through B

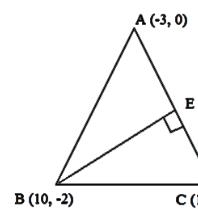
Slope of $AC = \frac{y_2-y_1}{x_2-x_1}$

$(x_1, y_1) \Rightarrow (-3,0)$ $(x_2, y_2) \Rightarrow (12,3)$

$= \frac{3-0}{12-(-3)}$

$= \frac{3}{15}$

$= \frac{1}{5}$



Equation of the altitude passing through the vertex B

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$\Rightarrow B(10,-2)$ and $m = \frac{1}{5}$

$$y - (-2) = -\frac{1}{\frac{1}{5}}(x - 10)$$

$$y + 2 = -5(x - 10)$$

$$y + 2 = -5x + 50$$

$$5x + y + 2 - 50 = 0$$

$\therefore$ The required equation is $\mathbf{5x + y - 48 = 0}$

8. Find the equation of the perpendicular bisector of the line joining the points $A(-4,2)$ and $B(6,-4)$ 5M JUN-23

Perpendicular bisector means the line will pass through the midpoint of the line segment AB and makes an angle 90°

Midpoint $= \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$(x_1, y_1) \Rightarrow (-4,2)$
$(x_2, y_2) \Rightarrow (6,-4)$

$= \left(\frac{-4+6}{2}, \frac{2-4}{2}\right)$

$= \left(\frac{2}{2}, -\frac{2}{2}\right) = (1,-1)$

Slope of AB $\frac{y_2-y_1}{x_2-x_1} = \frac{-4-2}{6+4} = -\frac{6}{10} = -\frac{3}{5}$

Slope of perpendicular line $= \frac{5}{3}$

Equation of perpendicular bisector

$$y - y_1 = -\frac{1}{m}(x - x_1)$$

$$y + 1 = \frac{5}{3}(x-1)$$

$$3(y+1) = 5x - 5 \Rightarrow 3y + 3 = 5x - 5$$

$$0 = 5x - 3y - 5 - 3$$

$\therefore$ The required equation is $\mathbf{5x - 3y - 8 = 0}$

10. Find the equation of a straight line through the intersection of lines $5x-6y=2$, $3x+2y=10$ and perpendicular to the line $4x-7y+13=0$ 5M

$5x - 6y = 2$………… (1)

$3x + 2y = 10$………. (2)

$(1) \Rightarrow 5x - \cancel{6y} = 2$

$3 \times (2) \Rightarrow 9x + \cancel{6y} = 30$

$14x = 32$

$x = \frac{32}{14} = \frac{16}{7}$

Sub $x = \frac{16}{7}$ in (1)

$5\left(\frac{16}{7}\right) - 6y = 2$

$\frac{80}{7} - 6y = 2$

$\frac{80}{7} - 6y = 2$

$6y = \frac{80}{7} - 2$

$y = \frac{66}{6\times 7}$

$y = \frac{11}{7}$

The point of intersection of the given lines is $\left(\frac{16}{7}, \frac{11}{7}\right)$

Similar Problems Solve Your Self

9. Find the equation of a straight line through the intersection of lines $7x+3y=10, 5x-4y=1$ and parallel to the line $13x+5y+12=0$ 5M

UE-10: A person standing at a junction (crossing) of two straight paths represented by the equations $2x-3y+4=0$ and $3x+4y-5=0$ seek to reach the path whose equation is $6x-7y+8=0$ in the least time. Find the equation of the path that he should follow. 5M

Eg. 5.34: Find the equation of a straight line which is parallel to the line $3x-7y=12$ and passing through the point $(6,4)$. **(JUN-23)** 2M

Eg. 5.35: Find the equation of a straight line perpendicular to the line $y=\frac{4}{3}x-7$ and passing through the point $(7,-1)$ 2M

Eg. 5.36: Find the equation of a straight line parallel to Y axis and passing through the point of intersection of the lines $4x+5y=13$ and $x-8y+9=0$ 5M

Eg. 5.37: The line joining the points $A\ (0,5)$ and $B(4,1)$ is a tangent to a circle whose centre C is at the point $(4,4)$ find (i) the equation of the line AB. (ii) the equation of the line through C which is perpendicular to the line AB. (iii) the coordinates of the point of contact of tangent line AB with the circle. 5M

CQ: Find the equation of a straight line parallel to X-axis and passing through the point of intersection of the lines $7x-3y=-12$ & $2y=x+3$ **(APR-23)** 5M

The required line is perpendicular to the line $4x-7y+13=0$

Slope $m = -\frac{a}{b} = -\left(\frac{4}{-7}\right) = \frac{4}{7}$

Slope of the perpendicular line $= -\frac{7}{4}$

Equation $\Rightarrow y - y_1 = -\frac{1}{m}(x - x_1)$

$$y - \frac{11}{7} = -\frac{7}{4}\left(x - \frac{16}{7}\right)$$

$$\frac{7y-11}{7} = -\frac{7}{4}\left(\frac{7x-16}{7}\right)$$

$$4(7y-11) = -7(7x-16)$$

$$28y - 44 = -49x + 112$$

$$49x + 28y - 44 - 112 = 0$$

$\therefore$ The required equation is $\mathbf{49x + 28y - 156 = 0}$

11. Find the equation of a straight line joining the point of intersection of $3x+y+2=0$ and $x-2y-4=0$ to the point of intersection of $7x-3y=-12$ and $2y=x+3$ PTA-3 5M

$3x+y+2=0$………….. (1)

$x-2y-4=0$…………….. (2)

$2\times(1)\Rightarrow 6x+\cancel{2y}+\cancel{4}=0$

$(2)\Rightarrow\ \ x-\cancel{2y}-\cancel{4}=0$

$7x \qquad = 0$

$x=\frac{0}{7}=0$

$x=0$

sub $x=0$ in (1) we get

$3(0)+y+2=0$

$y=-2$

Point of intersection of the first two lines is $(0,-2)$

$7x-3y=-12$………….. (3)

$2y=x+3$

$x-2y=-3$……………. (4)

$2\times(3)\Rightarrow 14x-\cancel{6y}=-24$

$-3\times(4)\Rightarrow\ 3x-\cancel{6y}=-9$

$(-)\quad(+)\qquad(+)$

$11x \qquad =-15$

$x=-\frac{15}{11}$

Sub $x=-\frac{15}{11}$ in (4) we get

$-\frac{15}{11}-2y=-3$

$-2y=-3+\frac{15}{11}$

$-2y=\frac{-33+15}{11}$

$-2y=-\frac{18}{11}$

$y=\frac{9}{11}$

Point of intersection of other set of lines is $\left(\frac{-15}{11},\frac{9}{11}\right)$

To find the equation of the line passing through the points $(0,-2)$ and $\left(\frac{-15}{11},\frac{9}{11}\right)$

$$\frac{y-y_1}{y_2-y_1}=\frac{x-x_1}{x_2-x_1}$$

$$\frac{y+2}{\frac{9}{11}+2}=\frac{x-0}{-\frac{15}{11}-0}$$

$$\frac{y+2}{\frac{31}{11}}=\frac{x-0}{-\frac{15}{11}}$$

$-15(y+2)=31(x-0)$

$-15y-30=31x$

∴ The required equation is $\mathbf{31x+15y+30=0}$

Similar Problems (Solve Your Self) 5M

12. Find the equation of a straight line through the point of intersection of the lines $8x+3y=18, 4x+5y=9$ and bisecting the line segment joining the points $(5,-4)$ and $(-7,6)$

UE-9: Find the equation of a line passing through the point of intersection of the lines 4x + 7y −3 = 0 and 2x −3y +1 = 0 that has equal intercepts on the axes.

Unit Exercise -5

Note for Unit Exercise – 5

Q.No: 1 – Similar to Exercise 5.2- 10th Question	Q.No: 4 - Similar to Exercise 5.1 – 6th Question
Q.No: 5 - Similar to Exercise 5.2 – 10th Question	Q.No: 7 - Similar to Exercise 5.3 – 8th Question
Q.No: 9 - Similar to Exercise 5.4 – 11th Question	Q.No: 10 - Similar to Exercise 5.4 – 10th Question

2. The area of a triangle is 5 sq. Units. Two of its vertices are $(2,1)$ and $(3,-2)$. The third vertex is (x,y) where $y=x+3$. Find the coordinates of the third vertex. 5M PTA-1

Given, area of triangle ABC is 5 sq. Units and $A(2,1), B(3,-2), C(x,y)$ where $y=x+3$

$$\text{Area of }\Delta=\frac{1}{2}\begin{bmatrix}2 & 3 & x & 2\\ 1 & -2 & y & 1\end{bmatrix}=5$$

$(-4+3y+x)-(3-2x+2y)=10$

$x+3y-4-3+2x-2y=10$

$3x+y=17$ …… (1)

Given $y=x+3$ sub in (1)

$3x+x+3=17$

$4x=14\Rightarrow x=\frac{14}{4}\Rightarrow x=\frac{7}{2}$

Substitute, $x=\frac{7}{2}$ in $y=x+3\Rightarrow y=\frac{13}{2}$

∴ Third vertex is $\left(\frac{7}{2},\frac{13}{2}\right)$

3. Find the area of a triangle formed by the lines $3x + y - 2 = 0, 5x + 2y - 3 = 0$ & $2x - y - 3 = 0$.

5M

Given lines are

$3x + y - 2 = 0$ .... (1)

$5x + 2y - 3 = 0$ .... (2)

$2x - y - 3 = 0$ .... (3)

Solving (1) & (2)

$(1) \times 2 \Rightarrow 6x + \cancel{2y} = 4$

$(2) \Rightarrow 5x + \cancel{2y} = 3$

$\quad (-) \quad (-) \quad (-)$

$x = 1$

Substitute $x = 1$ in (1)

$3(1) + y - 2 = 0$

$y = -1$

$A(1, -1)$

Solving (1) & (3)

$3x + y = 2$

$2x - y = 3$

$5x = 5 \Rightarrow x = 1$

Substitute $x = 1$ in (1)

$3(1) + y - 2 = 0 \Rightarrow y = -1$

B is $(1, -1)$

Solving (2) & (3)

$(2) \Rightarrow 5x + \cancel{2y} = 3$

$(3) \times 2 \Rightarrow 4x - \cancel{2y} = 6$

$9x = 9$

$x = 1$

Substitute $x = 1$ in (3)

$2 - y - 3 = 0$

$-y = 1 \Rightarrow \therefore y = -1$

C is $(1, -1)$

$A(1, -1), B(1, -1), C(1, -1)$

All the three points are same.

Area of a triangle $=$ **0 sq.units**

6. Find the equations of the lines, whose sum and product of intercepts are 1 & –6 respectively.

2M

x intercept $= a$, y intercept $= b$

Given, sum of intercepts $= 1$

$\Rightarrow a + b = 1$

$\therefore b = 1 - a$

Given, product of intercepts $= -6$

$ab = -6$

$a(1 - a) = -6$

$a - a^2 = -6$

$a^2 - a - 6 = 0$

$(a - 3)(a + 2) = 0$

$\therefore a = 3, -2$

- 6

-3 2

If $a = 3, b = -2 \Rightarrow \frac{x}{3} + \frac{y}{-2} = 1$

$\frac{x}{3} - \frac{y}{2} = 1$

$2x - 3y - 6 = 0$

If $a = -2, b = 3 \Rightarrow \frac{x}{-2} + \frac{y}{3} = 1$

$\frac{-3x + 2y}{6} = 1$

$-3x + 2y = 6$

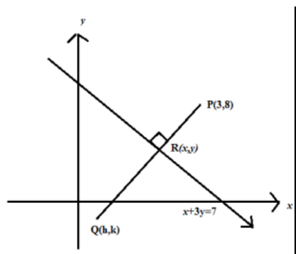
$\mathbf{3x - 2y + 6 = 0}$

8. Find the image of the point $(3, 8)$ with respect to the line $x + 3y = 7$ assuming the line to be plane mirror.

2M

$Q(h, k)$ be the image of the point (3,8) with respect to the line $x + 3y = 7$

$\therefore$ R is the midpoint and PQ is a perpendicular bisector of $x + 3y = 7$



$(x, y) = \left(\frac{h+3}{2}, \frac{k+8}{2}\right) \Rightarrow x = \frac{h+3}{2}, \ y = \frac{k+8}{2}$

Since $R(x, y)$ is a point on $x + 3y = 7$

$\left(\frac{h+3}{2}\right) + 3\left(\frac{k+8}{2}\right) = 7$

$h + 3 + 3k + 24 = 14$

$h + 3k = -13$ .... (2)

Also, slope of $PQ \times$ slope of $(x + 3y = 7) = -1$

$\frac{k-8}{h-3} \times \frac{-1}{3} = -1 \Rightarrow \frac{k-8}{h-3} = 3$

$k - 8 = 3h - 9$

$3h - k = 1$ .... (3)

Solving (2) & (3)

$(2) \Rightarrow h + \cancel{3k} = -13$

$(3) \times 3 \Rightarrow 9h - \cancel{3k} = 3$

$10h = -10$

$h = -1$

Substitute $h = -1$ in (2)

$-1 + 3k = -13$

$3k = -12$

$k = -4$

$\therefore Q$ is $(\mathbf{-1, -4})$

6. Trigonometry

Exercise 6.1

1. Prove the following identities. (2M)

(i) $\cot\theta + \tan\theta = \sec\theta \operatorname{cosec}\theta$

$$\cot\theta + \tan\theta = \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}$$
$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta.\cos\theta}$$
$$= \frac{1}{\cos\theta.\sin\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$
$$= \frac{1}{\cos\theta} \times \frac{1}{\sin\theta}$$
$$= \sec\theta\,.\operatorname{cosec}\theta$$
$$\therefore \cot\theta + \tan\theta = \sec\theta \operatorname{cosec}\theta$$

(ii) $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

$$\tan^4\theta + \tan^2\theta = (\tan^2\theta)^2 + \tan^2\theta$$
$$= \tan^2\theta(\tan^2\theta + 1)$$
$$= (\sec^2\theta - 1)\sec^2\theta$$
$$= \sec^4\theta - \sec^2\theta$$
$$\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$$

Similar Problems
Solve Your Self (2M)

Eg.6.1: Prove that $\tan^2\theta - \sin^2\theta = \tan^2\theta\sin^2\theta$ (JUN-23)

Eg. 6.12: Prove that $\tan^2 A - \tan^2 B = \frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}$ (5M)

2. Prove the following identities (2M)

(i) $\frac{1-\tan^2\theta}{\cot^2\theta - 1} = \tan^2\theta$

$$\frac{1-\tan^2\theta}{\cot^2\theta-1} = \frac{1-\tan^2\theta}{\frac{1}{\tan^2\theta} - 1} \quad \left[\because \cot^2\theta = \frac{1}{\tan^2\theta}\right]$$
$$= \frac{1-\tan^2\theta}{\frac{1-\tan^2\theta}{\tan^2\theta}}$$
$$= \cancel{1-\tan^2\theta} \times \frac{\tan^2\theta}{\cancel{1-\tan^2\theta}}$$
$$= \tan^2\theta$$

Similar Problems Solve Your Self (2M)

UE-1.(ii) Prove that $\frac{\tan^2\theta - 1}{\tan^2\theta + 1} = 1 - 2\cos^2\theta$

Eg. 6.3: Prove that $1 + \frac{\cot^2\theta}{1 + cosec\,\theta} = cosec\,\theta$

Eg. 6.15: Show that $\left[\frac{1+\tan^2 A}{1+\cot^2 A}\right] = \left[\frac{1-\tan A}{1-\cot A}\right]^2$ (5M)

(ii) $\frac{\cos\theta}{1+\sin\theta} = \sec\theta - \tan\theta$

[∵Multiply Numerator & Denominator by the conjugate of 1+sin θ]

$$\frac{\cos\theta}{1+\sin\theta} = \frac{\cos\theta}{1+\sin\theta} \times \frac{1-\sin\theta}{1-\sin\theta}$$
$$= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta} \quad [\because (a+b)(a-b) = a^2 - b^2]$$
$$= \frac{\cancel{\cos\theta}(1-\sin\theta)}{\cos^{\cancel{2}}\theta} \quad [\because 1 - \sin^2\theta = \cos^2\theta]$$
$$= \frac{1-\sin\theta}{\cos\theta}$$
$$= \frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = \sec\theta - \tan\theta$$

Similar Problems
Solve Your Self (2M)

Eg. 6.2: Prove that $\frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$

Eg. 6.4: Prove that $\sec\theta - \cos\theta = \tan\theta\sin\theta$

3. Prove the following identities (i) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sec\theta + \tan\theta$ SEP-20 (2M)

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta} \times \frac{1+\sin\theta}{1+\sin\theta}}$$
$$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$$
$$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$$
$$= \frac{1+\sin\theta}{\cos\theta}$$
$$= \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$$

Similar Problems
Solve Your Self (2M)

3. Prove (ii) $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$ (JUN-23)

Eg. 6.5: Prove that $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \operatorname{cosec}\theta + \cot\theta$

5. **Prove the following identities.** 5M

(i) $\mathbf{\sec^4\theta\,(1-\sin^4\theta) - 2\tan^2\theta = 1}$

$$\sec^4\theta\,(1-\sin^4\theta) - 2\tan^2\theta$$
$$= \sec^4\theta - \sin^4\theta.\sec^4\theta - 2\tan^2\theta$$
$$= \sec^4\theta - \frac{\sin^4\theta}{\cos^4\theta} - 2\tan^2\theta \left(\because \sec^4\theta = \frac{1}{\cos^4\theta}\right)$$
$$= \sec^4\theta - \tan^4\theta - 2\tan^2\theta$$
$$= [(\sec^2\theta)^2 - (\tan^2\theta)^2] - 2\tan^2\theta$$
$$= (\sec^2\theta + \tan^2\theta)(\sec^2\theta - \tan^2\theta) - 2\tan^2\theta \quad (\because a^2-b^2 = (a+b)(a-b))$$
$$= (\sec^2\theta + \tan^2\theta)\times 1 - 2\tan^2\theta \ (\because \sec^2\theta - \tan^2\theta = 1)$$
$$= \sec^2\theta + \tan^2\theta - 2\tan^2\theta$$
$$= \sec^2\theta - \tan^2\theta$$
$$= \mathbf{1}$$

Similar Problems
Solve Your Self
4. Prove the following identities. 5M
(i) $\sec^6\theta = \tan^6\theta + 3\tan^2\theta.\sec^2\theta + 1$
(ii) $(\sin\theta + \sec\theta)^2 + (\cos\theta + \mathrm{cosec}\,\theta)^2 = 1 + (\sec\theta + \mathrm{cosec}\,\theta)^2$
Eg. 6.7: Prove that $\sin^2 A\cos^2 B + \cos^2 A\sin^2 B + \cos^2 A\cos^2 B + \sin^2 A\sin^2 B = 1$ 2M
Eg. 6.9: Prove that $(\mathrm{cosec}\,\theta - \sin\theta)(\sec\theta - \cos\theta)(\tan\theta + \cot\theta) = 1$ 2M

(ii) $\mathbf{\dfrac{\cot\theta-\cos\theta}{\cot\theta+\cos\theta} = \dfrac{\mathrm{cosec}\,\theta-1}{\mathrm{cosec}\,\theta+1}}$

$$\frac{\cot\theta-\cos\theta}{\cot\theta+\cos\theta} = \frac{\frac{\cos\theta}{\sin\theta} - \cos\theta}{\frac{\cos\theta}{\sin\theta} + \cos\theta}$$
$$= \frac{\frac{\cos\theta - \sin\theta.\cos\theta}{\sin\theta}}{\frac{\cos\theta+\sin\theta.\cos\theta}{\sin\theta}}$$
$$= \frac{\cancel{\cos\theta}(1-\sin\theta)}{\cancel{\cos\theta}(1+\sin\theta)}$$
$$= \frac{1-\sin\theta}{1+\sin\theta}$$

$$= \frac{1 - \frac{1}{\mathrm{cosec}\,\theta}}{1 + \frac{1}{\mathrm{cosec}\,\theta}}$$
$$= \frac{\frac{\mathrm{cosec}\,\theta - 1}{\cancel{\mathrm{cosec}\,\theta}}}{\frac{\mathrm{cosec}\,\theta + 1}{\cancel{\mathrm{cosec}\,\theta}}}$$
$$= \mathbf{\frac{\mathrm{cosec}\,\theta - 1}{\mathrm{cosec}\,\theta + 1}}$$

Similar Problems
Solve Your Self 2M
Eg. 6.6: Prove that $\frac{\sec\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \cot\theta$ **(APR-23)**

6. **Prove the following identities.** 2M

(i) $\mathbf{\dfrac{\sin A - \sin B}{\cos A + \cos B} + \dfrac{\cos A - \cos B}{\sin A + \sin B} = 0}$

$$\frac{\sin A - \sin B}{\cos A + \cos B} + \frac{\cos A - \cos B}{\sin A + \sin B} = \frac{\sin^2 A - \sin^2 B + \cos^2 A - \cos^2 B}{(\cos A + \cos B)(\sin A + \sin B)}$$
$$= \frac{(\sin^2 A + \cos^2 A) - (\sin^2 B + \cos^2 B)}{(\cos A + \cos B)(\sin A + \sin B)} \quad (\because \sin^2\theta + \cos^2\theta = 1)$$
$$= \frac{1-1}{(\cos A + \cos B)(\sin A + \sin B)}$$
$$= \mathbf{0}$$

Similar Problems
Solve Your Self
Eg. 6.10: Prove that $\frac{\sin A}{1+\cos A} + \frac{\sin A}{1-\cos A} = 2cosec\ A$ 2M
Eg. 6.14: Prove that $\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{cosec\ A + \cot A - 1} = 1$ 5M

(ii) $\mathbf{\dfrac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \dfrac{\sin^3 A - \cos^3 A}{\sin A - \cos A} = 2}$

$$\frac{\sin^3 A + \cos^3 A}{\sin A + \cos A} + \frac{\sin^3 A - \cos^3 A}{\sin A - \cos A}$$
$$= \frac{\cancel{(\sin A + \cos A)}(\sin^2 A + \cos^2 A - \sin A.\cos A)}{\cancel{\sin A + \cos A}} + \frac{\cancel{(\sin A - \cos A)}(\sin^2 A + \cos^2 A + \sin A.\cos A)}{\cancel{\sin A - \cos A}}$$
$$= 1 - \cancel{\sin A.\cos A} + 1 + \cancel{\sin A.\cos A}$$
$$= 1 + 1$$
$$= \mathbf{2}$$

$$\because a^3 + b^3 = (a+b)(a^2+b^2-ab)$$
$$a^3 - b^3 = (a-b)(a^2+b^2+ab)$$

Similar Problems
Solve Your Self 5M
Eg. 6.16: Prove that $\frac{(1+\cot A + \tan A)(\sin A - \cos A)}{\sec^3 A - cosec^3 A} = \sin^2 A\cos^2 A$
Eg. 6.13: Prove that $\left(\frac{\cos^3 A - \sin^3 A}{\cos A - \sin A}\right) - \left(\frac{\cos^3 A + \sin^3 A}{\cos A + \sin A}\right) = 2\sin A\cos A$ **(PTA-6)**

7. (i) If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$ 5M

Given: $\sin\theta + \cos\theta = \sqrt{3}$

Squaring on both sides, we have $(\sin\theta + \cos\theta)^2 = 3$

$$\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta = 3$$

$$1 + 2\sin\theta.\cos\theta = 3$$

$$2\sin\theta.\cos\theta = 3 - 1$$

$$2\sin\theta.\cos\theta = 2$$

$$\sin\theta.\cos\theta = 1 \text{ .... } (1)$$

> **Similar Problems** 2M
> **Solve Your Self**
> **Eg. 6.8:** If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, then prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

$$\tan\theta + \cot\theta = \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta.\cos\theta} = \frac{1}{1} \qquad \text{[by (1)]}$$

$$\boldsymbol{\tan\theta + \cot\theta = 1}$$

(ii) If $\sqrt{3}\sin\theta - \cos\theta = 0$, then show that $\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}$ PTA-3 5M

Given: $\sqrt{3}\sin\theta - \cos\theta = 0$

$$\sqrt{3}\sin\theta = \cos\theta$$

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}}$$

$$\therefore \theta = 30^0 \qquad \left(\because \tan 30^\circ = \frac{1}{\sqrt{3}}\right)$$

LHS:

$$\tan 3\theta = \tan 3(30^\circ) = \tan 90^\circ = \infty \text{ .... } (1)$$

RHS:

$$\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta} = \frac{3\times\tan 30^\circ - \tan^3 30^\circ}{1 - 3\tan^2 30^\circ}$$

$$= \frac{3\times\frac{1}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)^3}{1 - 3\times\left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{3}{\sqrt{3}} - \left(\frac{1}{\sqrt{3}}\right)^3}{1 - 3\times\frac{1}{3}}$$

$$= \frac{\sqrt{3} - \left(\frac{1}{\sqrt{3}}\right)^3}{0} = \infty \text{ .... } (2)$$

$(1) = (2)$

$$\therefore \boldsymbol{\tan 3\theta = \frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}}$$

8. (i) If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, then prove that $(m^2+n^2)\cos^2\beta = n^2$ (5M)

(ii) If $\cot\theta + \tan\theta = x$ and $\sec\theta - \cos\theta = y$, then prove that $(x^2y)^{\frac{2}{3}} - (xy^2)^{\frac{2}{3}} = 1$

(i) Given: $m = \frac{\cos\alpha}{\cos\beta}$, $n = \frac{\cos\alpha}{\sin\beta}$

$$(m^2+n^2)\cos^2\beta = \left[\left(\frac{\cos\alpha}{\cos\beta}\right)^2 + \left(\frac{\cos\alpha}{\sin\beta}\right)^2\right]\cos^2\beta$$

$$= \left(\frac{\cos^2\alpha}{\cos^2\beta} + \frac{\cos^2\alpha}{\sin^2\beta}\right)(\cos^2\beta)$$

$$= \frac{\cos^2\alpha\sin^2\beta + \cos^2\alpha\cos^2\beta}{\cos^2\beta.\sin^2\beta} \times \cos^2\beta$$

$$= \frac{\cos^2\alpha(\sin^2\beta + \cos^2\beta)}{\sin^2\beta}$$

$$= \frac{\cos^2\alpha(1)}{\sin^2\beta} \qquad [\because \sin^2\beta + \cos^2\beta = 1]$$

$$= \frac{\cos^2\alpha}{\sin^2\beta} = \left(\frac{\cos\alpha}{\sin\beta}\right)^2 = \boldsymbol{n^2} \qquad \left[\text{Given } \frac{\cos\alpha}{\sin\beta} = n\right]$$

(ii) Given: $\cot\theta + \tan\theta = x$, $\sec\theta - \cos\theta = y$

$$(x^2y)^{\frac{2}{3}} = [(\cot\theta + \tan\theta)^2(\sec\theta - \cos\theta)]^{\frac{2}{3}}$$

$$= \left[\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)^2(\sec\theta - \cos\theta)\right]^{\frac{2}{3}}$$

$$= \left[\left[\frac{\cos^2\theta + \sin^2\theta}{\sin\theta.\cos\theta}\right]^2(\sec\theta - \cos\theta)\right]^{\frac{2}{3}}$$

$$= \left[\left(\frac{1}{\sin\theta.\cos\theta}\right)^2\left(\frac{1}{\cos\theta} - \cos\theta\right)\right]^{\frac{2}{3}}$$

$$= \left[\frac{1}{\sin^2\theta.\cos^2\theta} \times \frac{1-\cos^2\theta}{\cos\theta}\right]^{\frac{2}{3}}$$

$$= \left[\frac{1}{\sin^2\theta.\cos^2\theta} \times \frac{\sin^2\theta}{\cos\theta}\right]^{\frac{2}{3}}$$

$$= \left[\frac{1}{\cos^3\theta}\right]^{\frac{2}{3}}$$

$$(x^2y)^{\frac{2}{3}} = \frac{1}{\cos^2\theta} \quad \ldots\ldots\ldots\ldots (1)$$

$$(xy^2)^{\frac{2}{3}} = [(\cot\theta + \tan\theta)(\sec\theta - \cos\theta)^2]^{\frac{2}{3}}$$

$$= \left[(\cot\theta + \tan\theta)\left(\frac{1}{\cos\theta} - \cos\theta\right)^2\right]^{\frac{2}{3}}$$

$$= \left[\left[\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right]\left[\frac{1-\cos^2\theta}{\cos\theta}\right]^2\right]^{\frac{2}{3}}$$

$$= \left[\left(\frac{\cos^2\theta + \sin^2\theta}{\sin\theta.\cos\theta}\right)\left(\frac{\sin^2\theta}{\cos\theta}\right)^2\right]^{\frac{2}{3}}$$

$$= \left[\frac{1}{\cancel{\sin\theta}.\cos\theta} \times \frac{\sin^{\cancel{4}}\theta}{\cos^2\theta}\right]^{\frac{2}{3}}$$

$$= \left[\frac{\sin^3\theta}{\cos^3\theta}\right]^{\frac{2}{3}}$$

$$(xy^2)^{\frac{2}{3}} = \frac{\sin^2\theta}{\cos^2\theta} \quad \ldots\ldots\ldots\ldots (2)$$

(1) − (2)

$$(\boldsymbol{x^2y})^{\frac{2}{3}} - (\boldsymbol{xy^2})^{\frac{2}{3}} = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\sin^2\theta}{\cos^2\theta} = \frac{\cancel{\cos^2\theta}}{\cancel{\cos^2\theta}} = \mathbf{1}$$

9. (i) If $\sin\theta + \cos\theta = p$ and $\sec\theta + \operatorname{cosec}\theta = q$, then prove that $q(p^2-1) = 2p$ 5M

(ii) If $\sin\theta(1+\sin^2\theta) = \cos^2\theta$, then prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$

(i) Given: $p = \sin\theta + \cos\theta,\ q = \sec\theta + \operatorname{cosec}\theta$

LHS: $q(p^2-1) = (\sec\theta + \operatorname{cosec}\theta)((\sin\theta+\cos\theta)^2 - 1)$

$= (\sec\theta + \operatorname{cosec}\theta)((\sin^2\theta + \cos^2\theta) + 2\sin\theta.\cos\theta - 1)$

$= (\sec\theta + \operatorname{cosec}\theta)(\cancel{1} + 2\sin\theta\,.\cos\theta - \cancel{1})$

$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)(2\sin\theta.\cos\theta)$

$= \frac{\sin\theta+\cos\theta}{\cancel{\sin\theta.\cos\theta}} \times 2\cancel{\sin\theta.\cos\theta}$

$= 2(\sin\theta+\cos\theta) = 2p$ (since $p = \sin\theta+\cos\theta$)

$\therefore \boldsymbol{q(p^2-1) = 2p}$. Hence proved

(ii) Given: $\sin\theta(1+\sin^2\theta) = \cos^2\theta$

$\sin\theta(1+1-\cos^2\theta) = \cos^2\theta$

Squaring on both sides, $\sin^2\theta(2-\cos^2\theta)^2 = \cos^4\theta$

$(1-\cos^2\theta)(4+\cos^4\theta - 4\cos^2\theta) = \cos^4\theta$ ($\because (a-b)^2 = a^2+b^2-2ab$)

$4+\cos^4\theta - 4\cos^2\theta - 4\cos^2\theta - \cos^6\theta + 4\cos^4\theta = \cos^4\theta$

$4 - 4\cos^2\theta - 4\cos^2\theta - \cos^6\theta + 4\cos^4\theta = \cos^4\theta - \cos^4\theta$

$4 - 8\cos^2\theta - \cos^6\theta + 4\cos^4\theta = 0$

$4 = \cos^6\theta - 4\cos^4\theta + 8\cos^2\theta$

$\boldsymbol{\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4}$ Hence proved.

Similar Problems 5M

Solve Your Self

Eg. 6.17:If $\frac{\cos^2\theta}{\sin\theta} = p$ and $\frac{\sin^2\theta}{\cos\theta} = q$, then prove that $p^2q^2(p^2+q^2+3) = 1$

UE-3: If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, then prove that $x^2+y^2 = 1$

UE-4: If $a\cos\theta - b\sin\theta = c$, then prove that, $a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2-c^2}$.

10. If $\frac{\cos\theta}{1+\sin\theta} = \frac{1}{a}$, then prove that $\frac{a^2-1}{a^2+1} = \sin\theta$ 5M

LHS: $\frac{a^2-1}{a^2+1} = \frac{1-\frac{1}{a^2}}{1+\frac{1}{a^2}}$

($\div$ by a^2 on both numerator and denominator)

$= \frac{1-\left(\frac{1}{a}\right)^2}{1+\left(\frac{1}{a}\right)^2}$

$= \frac{1-\left(\frac{\cos\theta}{1+\sin\theta}\right)^2}{1+\left(\frac{\cos\theta}{1+\sin\theta}\right)^2}$

$= \frac{1-\frac{\cos^2\theta}{(1+\sin\theta)^2}}{1+\frac{\cos^2\theta}{(1+\sin\theta)^2}}$

$= \frac{\frac{(1+\sin\theta)^2-\cos^2\theta}{\cancel{(1+\sin\theta)^2}}}{\frac{(1+\sin\theta)^2+\cos^2\theta}{\cancel{(1+\sin\theta)^2}}}$

$= \frac{1+\sin^2\theta+2\sin\theta-\cos^2\theta}{1+\sin^2\theta+2\sin\theta+\cos^2\theta}$

$= \frac{\sin^2\theta+\sin^2\theta+2\sin\theta}{1+1+2\sin\theta}$

$= \frac{2\sin^2\theta+2\sin\theta}{2+2\sin\theta}$

$= \frac{\cancel{2}\sin\theta(\cancel{\sin\theta+1})}{\cancel{2}(\cancel{1+\sin\theta})}$

$\therefore \boldsymbol{\frac{a^2-1}{a^2+1} = \sin\theta}$ RHS

Similar Problems 5M

Solve Your Self

Eg. 6.11: If $cosec\,\theta + \cot\theta = P$, then prove that $\cos\theta = \frac{P^2-1}{P^2+1}$

Creative Questions

Prove the following identities.

1. $\dfrac{\cot A-\cos A}{\cot A+\cos A}=\dfrac{cosec\ A-1}{cosec\ A+1}$ PTA-4 2M

$$\frac{\cot A-\cos A}{\cot A+\cos A}=\frac{\frac{\cos A}{\sin A}-\cos A}{\frac{\cos A}{\sin A}+\cos A}$$

$$=\frac{\frac{\cos A-\cos A\sin A}{\sin A}}{\frac{\cos A+\cos A\sin A}{\sin A}}$$

$$=\frac{\cos A(1-\sin A)}{\cos A(1+\sin A)}$$

$$=\frac{1-\frac{1}{\text{cosec}\,A}}{1+\frac{1}{\text{cosec}\,A}}=\frac{\frac{\text{cosec}\,A-1}{\text{cosec}\,A}}{\frac{\text{cosec}\,A+1}{\text{cosec}\,A}}$$

$$=\frac{\text{cosec}\,A-1}{\text{cosec}\,A+1}$$

2. $\sqrt{\dfrac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}}=\dfrac{1-\sin\theta}{\cos\theta}$ PTA-5 2M

$$\sqrt{\frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}}=\sqrt{\frac{\sec\theta-\tan\theta}{\sec\theta+\tan\theta}\times\frac{\sec\theta-\tan\theta}{\sec\theta-\tan\theta}}$$

$$=\sqrt{\frac{(\sec\theta-\tan\theta)^2}{\sec^2\theta-\tan^2\theta}}$$

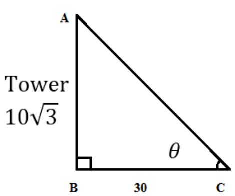
$$=\sqrt{\frac{(\sec\theta-\tan\theta)^2}{1}}$$

$$=\sec\theta-\tan\theta$$

$$=\frac{1}{\cos\theta}-\frac{\sin\theta}{\cos\theta}=\frac{1-\sin\theta}{\cos\theta}$$

Exercise 6.2

1. Find the angle of elevation of the top of a tower from a point on the ground, which is 30m away from the foot of a tower of height $10\sqrt{3}m$. 2M PTA-2, SEP-21, JUL-22



In right angle ΔABC

$AB=$ Tower

$=10\sqrt{3}m$

$BC=30m$

$$tan\theta=\frac{AB}{BC}$$

$$=\frac{10\sqrt{3}}{30}$$

$$=\frac{\sqrt{3}}{3}$$

$$=\frac{\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$$

$$\tan\theta=\frac{1}{\sqrt{3}}\Rightarrow$$

$$\tan 30^{\circ}=\frac{1}{\sqrt{3}}$$

$$\therefore\theta=30^{\circ}$$

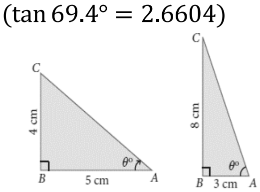
$\therefore$ The angle of elevation

$$\boldsymbol{\theta=30^{\circ}}$$

Similar Problems 2M

Solve Your Self

Eg. 6.18: Calculate the size of $\angle BAC$ in the given triangles. ($\tan 38.7^{\circ}=0.8011$, (**MDL**) ($\tan 69.4^{\circ}=2.6604$)



Eg. 6.19: A tower stands vertically on the ground. From a point on the ground, which is 48 m away from the foot of the tower, the angle of elevation of the top the tower is 30°. Find the height of the tower. **(PTA-1)** 2M

Eg. 6.20: A kite is flying at a height of 75 m above the ground. The string attached to the kite is temporarily tied to a point on the ground. The inclination of the string with the ground is 60°. Find the length of the string, assuming that there is no slack in the string. 2M

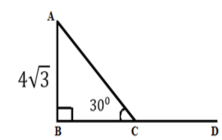
2. A road is flanked on either side by continuous rows of houses of height $4\sqrt{3}m$ with no space in between them. A pedestrian is standing on the median of the road facing a row house. The angle of elevation from the pedestrian to the top of the house is 30°. Find the width of the road. 2M

$AB=$ Height of the house $=4\sqrt{3}m$

$C=$ Centre point of the road

$BD=$ Width of the road

$\therefore BC=CD$



Angle of elevation $\theta=30^{\circ}$

To find the width of the road

In ΔABC, $\tan\theta=\dfrac{\text{Opposite Side}}{\text{Adjacent Side}}$

$$\tan 30^{\circ}=\frac{AB}{BC}=\frac{1}{\sqrt{3}}\Rightarrow\frac{4\sqrt{3}}{BC}=\frac{1}{\sqrt{3}}$$

$$4\sqrt{3}\times\sqrt{3}=BC$$

$$4\times 3=BC$$

$$BC=12\ m$$

$\therefore$ The width of the road $BD=BC+CD$

$=12+12$

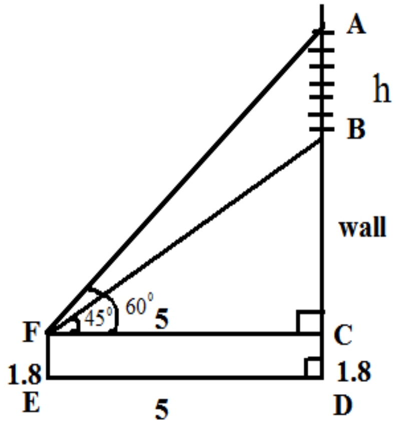
$$\boldsymbol{BD=24\ m}$$

Similar Problems 5M

Solve Your Self

Eg. 6.25: Two trees are standing on flat ground. The angle of elevation of the top of both the trees from a point X on the ground is 40°. If the horizontal distance between X and the smaller tree is 8 m and the distance of the top of the two trees is 20 m, calculate (i) the distance between the point X and the top of the smaller tree. (ii) the horizontal distance between the two trees. ($\cos 40^{\circ}=0.7660$)

3. To a man standing outside his house, the angles of elevation of the top and bottom of a window are 60° and 45° respectively. If the height of the man is 180 cm and if he is 5 m away from the wall, what is the height of the window? $(\sqrt{3} = 1.732)$. 5M JUL-22



Let AB = Window = h

EF = Man = $180cm$

$= 1.8m = CD$

$CF = 5m$

To find the height of the window

In right angle ΔBCF

$$\tan 45^\circ = \frac{BC}{5}$$

$$1 = \frac{BC}{5}$$

$$\therefore BC = 5\text{m}$$

In right angle ΔACF

$$\tan 60^\circ = \frac{AC}{5}$$

$$\sqrt{3} = \frac{AC}{5}$$

$$AC = 5\sqrt{3}$$

$$BC + AB = 5\sqrt{3}$$

$$5 + h = 5\sqrt{3}$$

$$h = 5\sqrt{3} - 5$$

$$= (5 \times 1.732) - 5$$

$$= 8.660 - 5$$

$$h = 3.66\text{m}$$

Height of the window $\boldsymbol{h = 3.66}$**m**

Similar Problems 5M

Solve Your Self

4. A statue 1.6m tall stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 40°. Find the height of the pedestal $(\tan 40^\circ = 0.8391, \sqrt{3} = 1.732)$

5. A flag pole of height 'h' metres is on the top of the hemispherical dome of radius 'r' metres. A man is standing 7m away from the dome. Seeing the top of the pole at an angle 45° and moving 5m away from the dome and seeing the bottom of the pole at an angle 30°. Find (i) the height of the pole (ii) radius of the dome. $(\sqrt{3} = 1.732)$

UE-5:A bird is sitting on the top of a 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such away that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Determine the speed at which the bird flies. $(\sqrt{3} = 1.732)$

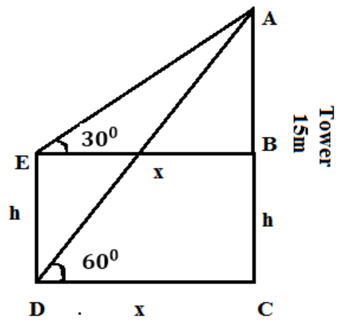
UE-6: An aeroplane is flying parallel to the Earth's surface at a speed of 175 m/sec and at a height of 600 m. The angle of elevation of the aeroplane from a point on the Earth's surface is 37°. After what period of time does the angle of elevation increase to 53°? $(\tan 53^\circ = 1.3270,\ \tan 37^\circ = 0.7536)$

Eg. 6.21: Two ships are sailing in the sea on either sides of a lighthouse. The angle of elevation of the top of the lighthouse as observed from the ships are 30° and 45° respectively. If the lighthouse is $200\ m$ high, find the distance between the two ships. $(\sqrt{3} = 1.732)$ **(JUN-23, PTA-5, SEP-21)**

Eg. 6.22: From a point on the ground, the angles of elevation of the bottom and top of a tower fixed at the top of a $30\ m$ high building are 45° and 60° respectively. Find the height of the tower. $(\sqrt{3} = 1.732)$ **(MAY-22)**

Eg. 6.23: A TV tower stands vertically on a bank of canal. The tower is watched from a point on the other bank directly opposite to it. The angle of elevation of the top of the tower is 58°. From another point $20\ m$ away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°. Find the height of the tower and the width of the canal. $(\tan 58° = 1.6003)$

6. The top of a 15m high tower makes an angle of elevation of 60° with the bottom of an electronic pole and angle of elevation of 30° with the top of the pole. What is the height of the electric pole? 5M



$AC =$ Tower $= 15m$

$DE =$ Electric Pole $= h$

$\therefore BC = h \therefore AB = 15 - h$

Let $CD = BE = x$

In right angle $\Delta\, ACD, \theta = 60^\circ$

$\tan 60^\circ = \frac{AC}{CD} = \sqrt{3}$

$\frac{15}{x} = \sqrt{3}$

$x = \frac{15}{\sqrt{3}}$

$x = \frac{15}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{15\sqrt{3}}{3}$

$x = 5\sqrt{3}m.$

In right angle $\Delta\, ABE$

$\theta = 30^\circ$

$tan\, 30^\circ = \frac{AB}{BE} = \frac{1}{\sqrt{3}}$

$\frac{15-h}{x} = \frac{1}{\sqrt{3}}$ $(\because \text{sub } x = 5\sqrt{3})$

$\frac{15-h}{5\cancel{\sqrt{3}}} = \frac{1}{\cancel{\sqrt{3}}}$

$15 - h = 5$

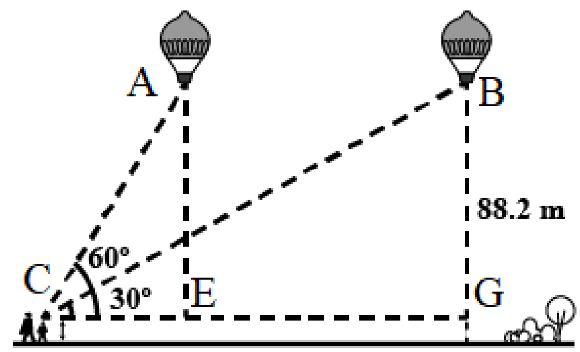
$h = 15 - 5$

$h = 10m$

Height of the electronic pole $=$ **$10m$.**

Creative Questions

1. A 1.2m tall girls spots a balloon moving with the wind in a horizontal line at a height of 88.2m from the ground. The angle of elevation of the balloon from the eyes of the girl at an instant is 60°. After some time the angle of elevation reduces to 30°. find the distance travelled by the balloon during the interval. 5M PTA-2



In ΔACE, $\frac{AE}{CE} = \tan 60°$

$\frac{88.2-1.2}{CE} = \sqrt{3}$

$CE = 29\sqrt{3}$

In ΔBCG, $\frac{BG}{CG} = \tan 30°$

$\frac{88.2-1.2}{CG} = \frac{1}{\sqrt{3}}$

$CG = 87\sqrt{3}m$

Distance travelled by the balloon

$= EG$

$= GC - EC$

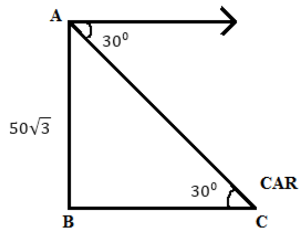
$= 87\sqrt{3} - 29\sqrt{3}$

$= 58\sqrt{3}m$

Exercise 6.3

1. From the top of a rock $50\sqrt{3}m$ high, the angle of depression of a car on the ground is observed to be 30°. Find the distance of the car from the rock. PTA-6, MAY-22

2M



AB = Height of the rock $= 50\sqrt{3}$

Angle of depression $= 30^\circ$

In right angle $\Delta\ ABC$,

$\tan 30^\circ = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$

$\frac{50\sqrt{3}}{BC} = \frac{1}{\sqrt{3}}$

$BC = 50\sqrt{3} \times \sqrt{3} = 50 \times 3$

$= 150m$

The distance of the car from rock

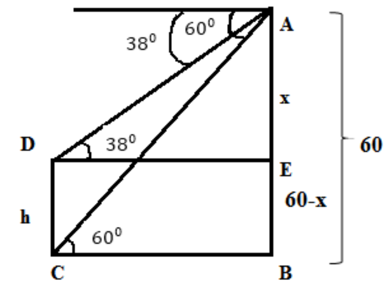
$= \mathbf{150m}$

Similar Problems
Solve Your Self 2M

Eg. 6.26: A player sitting on the top of a tower of height $20\ m$ observes the angle of depression of a ball lying on the ground as 60°. Find the distance between the foot of the tower and the ball. $(\sqrt{3} = 1.732)$ **(PTA-3)**

3. From the top of the tower 60 m high, the angles of depression of the top & bottom of a vertical lamp post are observed to be 38° and 60° respectively. Find the height of the lamp post. $(\tan 38^\circ = 0.7813, \sqrt{3} = 1.732)$

5M SEP-20



AB = Tower $= 60m$

CD = lamp post $= h$

$AE = x$

$CD = BE = 60 - x = h$

In right angle $\Delta\ AEC$

$\tan 38^\circ = \frac{AE}{DE} = 0.7813$

$DE = \frac{x}{0.7813}$......... (1)

In right angle $\Delta\ ABC$

$\theta = 60^\circ$

$\tan 60^\circ = \frac{AB}{BC} = \sqrt{3}$

$\frac{60}{BC} = \sqrt{3}$

$BC = \frac{60}{\sqrt{3}}$

$BC = \frac{60}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{60\sqrt{3}}{3}$

$BC = 20\sqrt{3}$

$BC = DE$

$\therefore DE = 20\sqrt{3}$ .... (2)

From (1) & (2)

$DE \Rightarrow \frac{x}{0.7813} = 20\sqrt{3}$

$x = 20\sqrt{3} \times 0.7813$

$x = 20 \times 1.732 \times 0.7813$

$x = 27.064m$

Height of the lamp post

$h = 60 - x$

$= 60 - 27.064$

$\mathbf{h = 32.93\ m}$

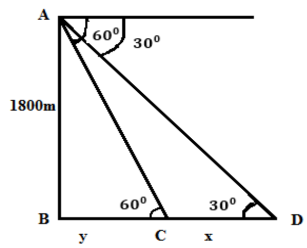
Similar Problems
Solve Your Self 2M

2. The horizontal distance between two buildings is 70 m. The angle of depression of the top of the first building when seen from the top of the second building is 45°. If the height of the second building is 120 m, find the height of the first building.

Eg. 6.27: The horizontal distance between two buildings is 140 m. The angle of depression of the top of the first building when seen from the top of the second building is 30°. If the height of the first building is 60 m, find the height of the second building. $(\sqrt{3} = 1.732)$ 5M

Eg. 6.28: From the top of a tower 50 m high, the angles of depression of the top and bottom of a tree are observed to be 30° and 45° respectively. Find the height of the tree. $(\sqrt{3} = 1.732)$ 5M

4. **An aeroplane at an altitude of 1800m finds that two boats are sailing towards it in the same direction. The angles of depression of the boats as observed from the aeroplane are 60° and 30° respectively. Find the distance between the two boats. $(\sqrt{3} = 1.732)$** 5M



AB = Height of the aeroplane from earth $= 1800m$

C = Boat 1

D = Boat 2

CD = Distance between the two boats

In right angle ΔABC

$\theta = 60^\circ$

$\tan 60^0 = \frac{AB}{BC}$

$\sqrt{3} = \frac{1800}{y}$

$\frac{1800}{y} = \sqrt{3}$

$y = \frac{1800}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{1800\sqrt{3}}{3}$

$y = 600\sqrt{3}m$.......... (1)

In right angle ΔABD, $\theta = 30^\circ$

$\tan 30^\circ = \frac{AB}{BD} = \frac{1}{\sqrt{3}}$

$\frac{1800}{x+y} = \frac{1}{\sqrt{3}}$

$x + y = 1800\sqrt{3}$.......(2)

Substitute (1) in (2)

$x + 600\sqrt{3} = 1800\sqrt{3}$

$x = 1800\sqrt{3} - 600\sqrt{3}$

$x = 1200\sqrt{3}$

$x = 1200 \times 1.732$

$x = 2078.4m$

Distance between the two boats

$= \mathbf{2078.4m}$

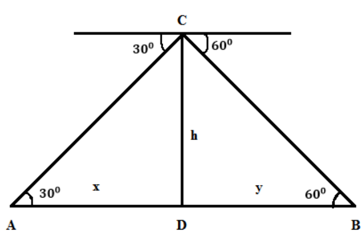
Similar Problems 5M

Solve Your Self

Eg. 6.29: As observed from the top of a 60 m high light house from the sea level, the angles of depression of two ships are 28° and 45°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. $(\tan 28^\circ = 0.5317)$ **(PTA-1)**

Eg. 6.30: A man is watching a boat speeding away from the top of a tower. The boat makes an angle of depression of 60° with the man's eye when at a distance of 200 m from the tower. After 10 seconds, the angle of depression becomes 45°. What is the approximate speed of the boat (in km/hr), assuming that it is sailing in still water? $(\sqrt{3} = 1.732)$

5. **From the top of a lighthouse, the angle of depression of two ships on the opposite sides of it are observed to be 30° and 60°. If the height of the lighthouse is h meters and the line joining the ships passes through the foot of the lighthouse, show that the distance between the ships is $\frac{4h}{\sqrt{3}}m$.** 5M



$A \to$ Ship 1, $B \to$ Ship 2

CD = Height of the Light house $= h$ m

Let $AD = x, BD = y$

In right angle ΔADC

$\theta = 30^\circ$

$\tan 30^\circ = \frac{CD}{AD} = \frac{1}{\sqrt{3}}$

$\frac{h}{x} = \frac{1}{\sqrt{3}}$

$x = \sqrt{3}h$ .... (1)

In right angle ΔBDC, $\theta = 60^\circ$ APR-23

$\tan 60^\circ = \frac{CD}{BD} = \sqrt{3}$

$\frac{h}{y} = \sqrt{3}$

$y = \frac{h}{\sqrt{3}}$.................. (2)

The distance between the two ships (1) + (2)

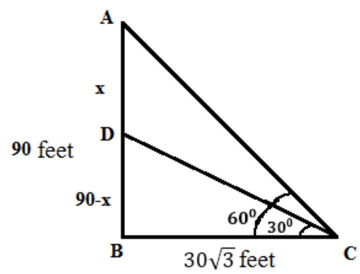
$x + y = \sqrt{3}h + \frac{h}{\sqrt{3}} = \frac{3h+h}{\sqrt{3}}$

$\boldsymbol{AB} = \frac{\mathbf{4h}}{\sqrt{3}}$ m Hence proved.

Similar Problems **Solve Your Self** 5M

UE-8: Two ships are sailing in the sea on either side of the lighthouse. The angles of depression of two ships as observed from the top of the lighthouse are 60° and 45° respectively. If the distance between the ships is $200\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)$ metres, find the height of the lighthouse.

6. A lift in a building of height 90 feet with transparent glass walls is descending from the top of the building. At the top of the building, the angle of depression to a fountain in the garden is 60°. Two minutes later, the angle of depression reduces to 30°. If the fountain is $30\sqrt{3}$ feet from the entrance of the lift, find the speed of the lift which is descending. (5M)



AB = Lift = 90 feet

C = Fountain

$BC = 30\sqrt{3}$ feet

$AD = x$

(The distance travelled by lift in two minutes)

In right angle $\Delta\, DBC$,

$\theta = 30^\circ$

$\tan 30^\circ = \frac{DB}{BC} = \frac{1}{\sqrt{3}}$

$\frac{90-x}{30\sqrt{3}} = \frac{1}{\sqrt{3}}$

$90 - x = \frac{30\sqrt{3}}{\sqrt{3}}$

$90 - x = 30$

$x = 60$ feet

Speed $= \frac{Distance}{Time}$

Distance = 60 feet

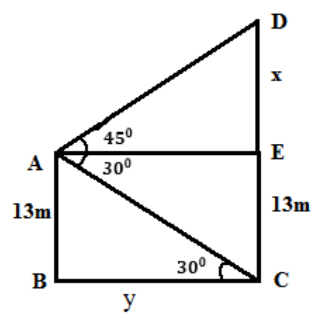
$\therefore$ Time = 2 minutes

Speed $= \frac{60 \text{ feet}}{2 \text{ minutes}} = 30$

Speed = **30 feet/minutes**

Exercise 6.4

1. From the top of a tree of height 13m the angle of elevation and depression of the top & bottom of another tree are 45° & 30° respectively. Find the height of the second tree. ($\sqrt{3} = 1.732$) (5M)



AB = Tree 1 $= 13m$

CD = Tree 2 $= 13 + x$

$DE = x$

$AE = BC = y$

In right angle $\Delta\, AED$

$\theta = 45^\circ$

$\tan 45^\circ = \frac{DE}{AE} = 1$

$\frac{x}{y} = 1$

$x = y$

In right angle $\Delta\, ABC$

$\theta = 30^\circ$

$\tan 30^\circ = \frac{AB}{BC} = \frac{1}{\sqrt{3}}$

$\frac{13}{y} = \frac{1}{\sqrt{3}}$

$y = 13\sqrt{3}$

$= 13 \times 1.732$

$y = 22.516$

$x = y$

$\therefore x = 22.516$

Height of the second tree

$= x + 13$

$= 22.516 + 13$

$= 35.516m$

$\boldsymbol{h = 35.52m}$

Similar Problems (5M)

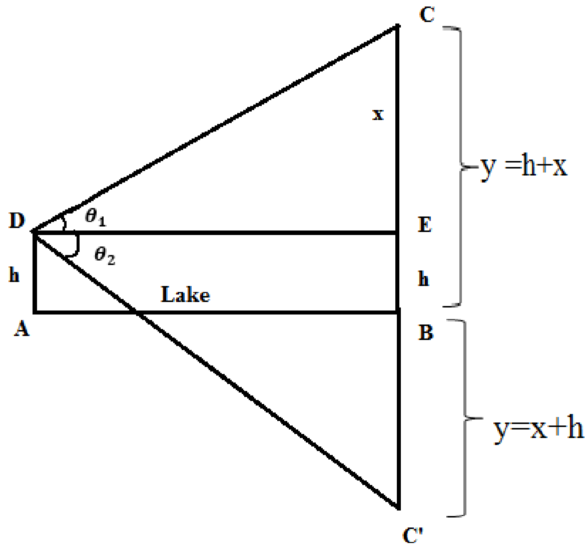
Solve Your Self

2. A man is standing on the deck of a ship, which is 40m above water level. He observes the angle of elevation of the top of a hill as 60° and the angle of depression of the base of the hill as 30°. calculate the distance of the hill form the ship and the height of the hill. ($\sqrt{3} = 1.732$)

Eg. 6.31: From the top of a 12 m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 30°. Determine the height of the tower.

Eg. 6.32: A pole 5 m high is fixed on the top of a tower. The angle of elevation of the top of the pole observed from a point 'A' on the ground is 60° and the angle of depression to the point 'A' from the top of the tower is 45°. Find the height of the tower. ($\sqrt{3} = 1.732$)

3. If the angle of elevation of a cloud from a point 'h' metres above a lake is θ_1 and the angle of depression of its reflection in the lake is θ_2. prove that the height that the cloud is located from the ground is $\frac{h(\tan\theta_1+\tan\theta_2)}{\tan\theta_2-\tan\theta_1}$. 5M



AB →Surface of the lake,

C → Cloud

C' → Reflection of the cloud in the lake

Given

Let $AD = h$

$DE = AB$

The height that the cloud is located from the ground $= h + x$

In right angle ΔDEC

$\tan\theta_1 = \frac{x}{DE}$

$DE = \frac{x}{\tan\theta_1}$……………….. (1)

In right angle $\Delta DEC'$

$\tan\theta_2 = \frac{EC'}{DE} = \frac{h+y}{DE}$

$\tan\theta_2 = \frac{h+x+h}{DE}$ $[\because y = x + h]$

$DE = \frac{2h+x}{\tan\theta_2}$……………… (2)

From (1) & (2)

$\frac{x}{\tan\theta_1} = \frac{2h+x}{\tan\theta_2}$

$x\tan\theta_2 = 2h\tan\theta_1 + x\tan\theta_1$

$x\tan\theta_2 - x\tan\theta_1 = 2h\tan\theta_1$

$x[\tan\theta_2 - \tan\theta_1] = 2h\tan\theta_1$

$x = \frac{2h\tan\theta_1}{\tan\theta_2-\tan\theta_1}$

The height that the cloud is located from the ground $= x + h$

$= \frac{2h\tan\theta_1}{\tan\theta_2-\tan\theta_1} + h$

$= \frac{2h\tan\theta_1+h\tan\theta_2-h\tan\theta_1}{\tan\theta_2-\tan\theta_1}$

$= \frac{h\tan\theta_1+h\tan\theta_2}{\tan\theta_2-\tan\theta_1}$

$= \frac{h[\tan\theta_1+\tan\theta_2]}{\tan\theta_2-\tan\theta_1}$

Hence proved.

Similar Problems

Solve Your Self 5M

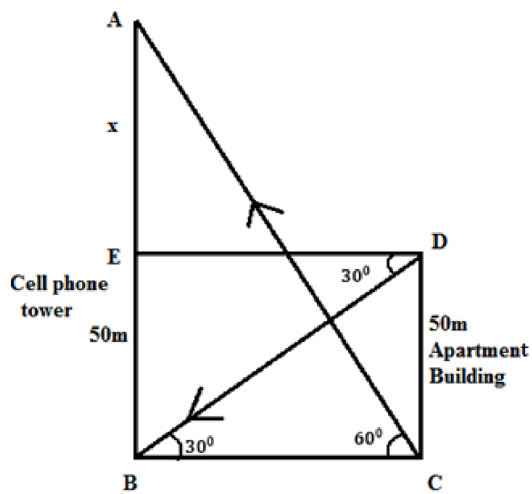
Eg. 6.33: From a window (h metres high above the ground) of a house in a street, the angles of elevation and depression of the top and the foot of another house on the opposite side of the street are θ_1 and θ_2 respectively. Show that the height of the opposite house is $h\left(1+\frac{\cot\theta_2}{\cot\theta_1}\right)$.

4. The angle of elevation of the top of a cell phone tower from the foot of a high apartment is 60° and the angle of depression of the foot of the tower from the top of the apartment is 30°. If the height of the apartment is 50 m, find the height of the cell phone tower. According to radiations control norms, the minimum height of a cell phone tower should be 120 m. State if the height of the above mentioned cell phone tower meets the radiation norms. 5M

$AB \rightarrow$ Cell phone tower $= AE + EB = (x+50)m$

$CD \rightarrow$ Apartment building $= 50m$

In right angle $\Delta\, BCD$



$\theta = 30^{\circ}$

$\tan 30^{\circ} = \frac{CD}{BC} = \frac{1}{\sqrt{3}}$

$\frac{50}{BC} = \frac{1}{\sqrt{3}}$

$BC = 50\sqrt{3}$ ……………… (1)

In right angle $\Delta\, ABC$

$\tan 60^{\circ} = \frac{AB}{BC} = \sqrt{3}$

$\frac{x+50}{BC} = \sqrt{3}$

$BC = \frac{x+50}{\sqrt{3}}$ ……………… (2)

From (1) & (2)

$50\sqrt{3} = \frac{x+50}{\sqrt{3}}$

$x + 50 = 50\sqrt{3} \times \sqrt{3}$

$x + 50 = 50 \times 3$

$x + 50 = 150$

$x = 150 - 50$

$x = 100m$

$\therefore$ Height of the cell phone tower

$x + 50 = 100 + 50$

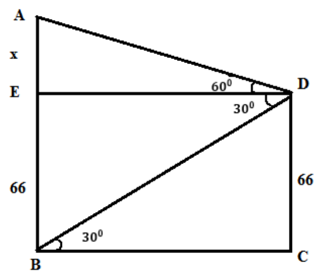
$= \mathbf{150m}$

According to radiation control norms, the minimum height of a cell phone tower should be 120m

Here, the height of the cell phone tower is **150m**

Yes, the cell phone tower meets the radiation norms.

5. **The angle of elevation and depression of the top and bottom of a lamp post from the top of a 66 m high apartment are 60° and 30° respectively. Find**
 (i) The height of the lamp post.
 (ii) The difference between height of the lamp post and the apartment.
 (iii) The distance between the lamp post and the apartment. $(\sqrt{3} = 1.732)$ 5M



$CD = \text{Apartment} = 66m$

$AB = \text{Lamp post}$

$AB = x + 66$

$BC = DE$

In right angle $\Delta\ BCD$

$\theta = 30^{\circ}$

$tan\ 30^{\circ} = \frac{CD}{BC} \Rightarrow \frac{1}{\sqrt{3}} = \frac{CD}{BC}$

$\frac{66}{BC} = \frac{1}{\sqrt{3}} \Rightarrow BC = 66\sqrt{3}$ …………………… (1)

In right angle $\Delta\ AED$

$\theta = 60^{\circ}$

$\tan 60^{\circ} = \frac{AE}{ED} = \sqrt{3}$

$\frac{x}{BC} = \sqrt{3}$

$x = \sqrt{3} \times BC = \sqrt{3} \times 66\sqrt{3} = 66 \times 3 = 198m$

(i) The height of the lamp post $= x + 66$

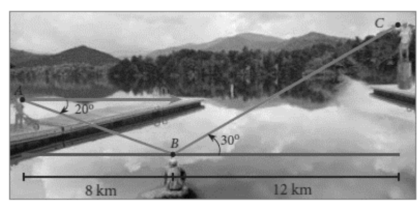
$= 198 + 66 = \mathbf{264m}$

(ii) The difference between height of the lamp post and the apartment $= 264 - 66 = \mathbf{198m}$

(iii) The distance between the lamp post and the apartment $= BC = 66\sqrt{3} = 66 \times 1.732$

$= \mathbf{114.31m}$

6. **Three villagers A, B and C can see each other using telescope across a valley. The horizontal distance between A and B is 8 km and the horizontal distance between B and C is 12 km. The angle of depression of B from A is 20° and the angle of elevation of C from B is 30°. Calculate: (i) The vertical height between A and B. (ii) The vertical height between B and C. $(\tan 20^{\circ} = 0.3640, \sqrt{3} = 1.732)$**



5M

In right angle $\Delta\ ADB$

$\theta = 20^{\circ}$

$\tan 20^{\circ} = \frac{AD}{BD} = 0.3640$

$\frac{x}{8} = 0.3640$

$x = 0.3640 \times 8$

$x = 2.9120$

In right angle $\Delta\ BE,\ \theta = 30^{\circ}$

$\tan 30^{\circ} = \frac{EC}{BE} = \frac{1}{\sqrt{3}}$

$\frac{y}{12} = \frac{1}{\sqrt{3}}$

$y = \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{12\sqrt{3}}{3}$

$= 4\sqrt{3} = 4 \times 1.732$

$y = 6.928$

Answers:

(i) The vertical height between A and B

$x = 2.912 = \mathbf{2.91}$ **km**

(ii) The vertical height between B and C

$y = 6.928 = \mathbf{6.93}$ **km**

Unit Exercise 6

Note for Unit Exercise – 6

Q.No: 1(ii) – Similar to Exercise 6.1- 2(i) Question
Q.No: 3,4 - Similar to Exercise 6.1 – 9th Question
Q.No: 5,6 - Similar to Exercise 6.2 – 3rd Question
Q.No: 8 - Similar to Exercise 6.3 – 5th Question

1. Prove that

(i) $\cot^2 A\left[\frac{\sec A-1}{1+\sin A}\right]+\sec^2 A\left[\frac{\sin A-1}{1+\sec A}\right]=0$ (2M)

$$\text{LHS}=\cot^2 A\left[\frac{\sec A-1}{1+\sin A}\right]+\sec^2 A\left[\frac{\sin A-1}{1+\sec A}\right]$$

$$=\frac{\cot^2 A[(sec\,A-1)(sec\,A+1)]+sec^2A[(sin\,A-1)(sin\,A+1)]}{(1+sin\,A)(1+sec\,A)}$$

$$=\frac{\cot^2 A(sec^2A-1)+sec^2A(sin^2-1)}{(1+sin\,A)(1+sec\,A)}$$ $[\because (a-b)(a+b)=a^2-b^2]$

$$=\frac{\cot^2 A.tan^2A+sec^2A(sin^2A-1)}{(1+sin\,A)(1+sec\,A)}$$ $\boxed{\cot^2 A\ tan^2A=\frac{1}{tan^2A}tan^2A=1}$

$$=\frac{1+\sec^2 A(sin^2A-1)}{(1+sin\,A)(1+sec\,A)}$$

$$=\frac{1-\sec^2 A(1-\sin^2 A)}{(1+\sin A)(1+\sec A)}$$

$$=\frac{1-\sec^2 A.\cos^2 A}{(1+\sin A)(+\sec A)}=\frac{1-1}{(1+\sin A)(1+\sec A)}=0=\text{RHS}$$

2. Prove that $\left[\frac{1+sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right]^2=\frac{1-\cos\theta}{1+\cos\theta}$ (5M)

$$LHS=\left[\frac{1+sin\theta-cos\theta}{1+sin\theta+cos\,\theta}\right]^2=\frac{(1+\sin\theta)^2+\cos^2\theta-2(1+sin\,\theta)(cos\,\theta)}{(1+sin\,\theta)^2+cos^2\theta+2(1+sin\theta)(cos\,\theta)}$$

$(a-b)^2=a^2+b^2-2ab$
$(a+b)^2=a^2+b^2+2ab$
Here,
$a=1+\sin\theta\,,b=\cos\theta$

$$=\frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta-2\cos\theta-2sin\theta.\cos\theta}{1+2sin\theta+sin^2\theta+\cos^2\theta+2cos\,\theta+2sin\,\theta.cos\theta}$$

$$=\frac{1+2\sin\theta+1-2\cos\theta-2\sin\theta.\cos\theta}{1+2\sin\theta+1+2\cos\theta+2\sin\theta.\cos\theta}$$

$$=\frac{2+2\sin\theta-2\cos\theta-2\sin\theta.\cos\theta}{2+2\sin\theta+2\cos\theta+2\sin\theta\,.\cos\theta}$$ (Divide Numerator and Denominator by 2)

$$=\frac{1+\sin\theta-\cos\theta-\sin\theta.\cos\theta}{1+\sin\theta+\cos\theta+\sin\theta.\cos\theta}$$

$$=\frac{(1+\sin\theta)-\cos\theta(1+\sin\theta)}{(1+\sin\theta)+\cos\theta(1+\sin\theta)}$$

$$=\frac{\cancel{1+sin\theta}[1-\cos\theta]}{\cancel{1+sin\theta}[1+\cos\theta]}$$

$$=\frac{1-\cos\theta}{1+\cos\theta}=RHS$$ Hence proved.

Similar Problems Solve Your Self (5M)

Eg. 6.24: An aeroplane sets off from G on a bearing of 24° towards H, a point 250 km away. At H it changes course and heads towards J deviates further by 55° and a distance of 180 km away.

(i) How far is H to the North of G? (ii) How far is H to the East of G?
(iii) How far is J to the North of H? (iv) How far is J to the East of H?

$[\sin 24°=0.4067\quad \sin 11°=0.1908\quad \cos 24°=0.9135\quad \cos 11°=0.9816]$

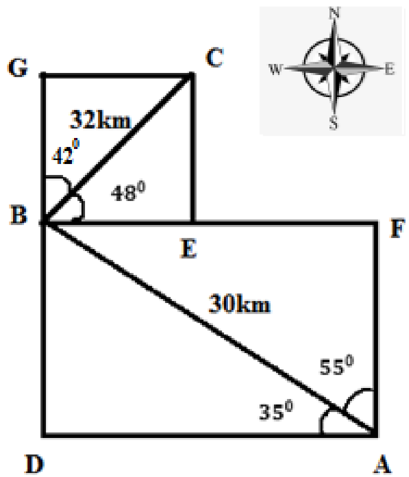
7. **A bird is flying from A towards B at an angle of 35°, a point 30 km away from A. At B it changes its course of flight and heads towards C on a bearing of 48^0 and distance 32 km away.**

(i) How far is B to the North of A? **(ii) How far is B to the West of ?**

(iii) How far is C to the North of B? **(iv) How far is C to the East of ?**

5M

$(\mathbf{\sin 55^\circ = 0.8192, \cos 55^\circ = 0.5736, \sin 42^\circ = 0.6691, \cos 42^\circ = 0.7431})$



(i) Distance of B to the North of $A = BF$

In right angle $\Delta\ BFA$

$\theta = 55^0 \quad (90 - 35^0)$

$\sin 55^0 = \frac{BF}{AB} = 0.8192$

$\frac{BF}{30} = 0.8192$

$BF = 30 \times 0.8192 \quad = 24.576$

$\boldsymbol{BF = 24.58km}$ (approx.)

(ii) Distance of B to the west of A = AF

In right angle ΔBFA

$\theta = 55^\circ \quad (90 - 35^\circ)$

$\cos 55^\circ = \frac{AF}{AB} = 0.5736$

$\frac{AF}{30} = 0.5736$

$AF = 0.5736 \times 30 \quad = 17.208$

$AF = \mathbf{17.21km}$ (approx.)

(iii) Distance of C to the North of B = GC

In right angle ΔBGC

$\theta = 42^\circ \quad (90^\circ - 48^\circ)$

$\sin 42^\circ = \frac{GC}{BC} \quad = 0.6691$

$\frac{GC}{32} = 0.6691$

$GC = 0.6691 \times 32$

$\boldsymbol{GC = 21.41km}$ (approx.)

(iv) Distance of C to the East of B =EC

In right angle $\Delta\ BEC$

$\sin 48^\circ = \frac{EC}{BC}$

$\cos 42^\circ = \frac{EC}{32} = 0.7431$

$EC = 32 \times 0.7431 \ = 23.779$

$\boldsymbol{EC = 23.78km}$ (approx.)

9. **A building and a statue are in opposite side of a street from each other 35 m apart. From a point on the roof of building the angle of elevation of the top of statue is 24° and the angle of depression of top of the statue is 34°. Find the height of the statue.**

$(\mathbf{\tan 24^\circ = 0.4452, \tan 34^\circ = 0.6745})$

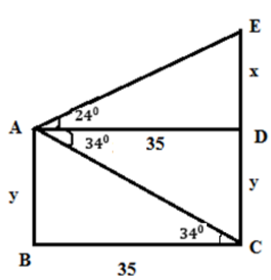
PTA-4

5M

$AB = \text{Building} = y$

$CE = \text{State} = x + y$

$BC = AD = 35m$



In right angle $\Delta\ ADE$

$\tan 24^0 = \frac{ED}{AD} = 0.4452$

$\frac{x}{35} = 0.4452$

$x = 35 \times 0.4452$

$x = 15.582$

In right angle ΔABC,

$\tan 34^\circ = \frac{AB}{BC} = 0.6745$

$\frac{y}{35} = 0.6745$

$y = 0.6745 \times 35 = 23.6075$

Height of the statues

$CE = x + y$

$= 15.582 + 23.608$

$= 39.190$

$CE = 39.19m$

Height of the statue $= \mathbf{39.19}\boldsymbol{m}$

7. Mensuration

Exercise 7.1

1. The radius and height of a cylinder in the ratio 5: 7 and its curved surface area is 5500 sq. cm Find its radius and height. APR-23,JUL-22

2M

$\frac{\text{Radius}}{\text{Height}} = \frac{r}{h} = \frac{5}{7}$

$r = \frac{5h}{7}$ ……………..(1)

CSA of the cylinder $= 2\pi rh = 5500$

$2 \times \frac{22}{7} \times \frac{5h}{7} \times h = 5500$

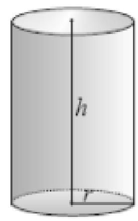
$h^2 = \frac{5500\times7\times7}{2\times22\times5}$

$= 5\times5\times7\times7$

$h = 35\text{cm}$

Substitute h=35 in (1), $r = \frac{5(35)}{7} \Rightarrow r = 25\text{cm}.$

$\boldsymbol{r = 25}$ **cm,** $\boldsymbol{h = 35}$**cm**



r:h = 5:7

Similar Problems 5M
Solve Your Self

2. A solid iron cylinder had total surface area of 1848 sq.m. Its curved surface area is five-sixth of its total surface area. Find the radius and height of the iron cylinder.

Eg. 7.1: A cylindrical drum has a height of 20 cm and base radius of 14 cm. Find its curved surface area and the total surface area. **(JUL-22)** 5M

Eg. 7.2: The curved surface area of a right circular cylinder of height 14 cm is 88 cm^2. Find the diameter of the cylinder. **(JUN-23)** 2M

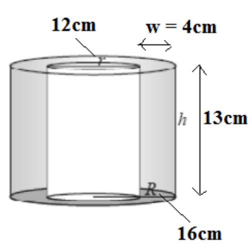
3. The external radius and the length of a hollow wooden log are 16 cm and 13 cm respectively. If its thickness is 4 cm then find its T.S.A. 2M

Given: External Radius (R) =16cm.

Internal Radius (r) $= 16 - 4 =$12cm

[∵width=4cm]

Length (Height) = 13cm.



TSA of hollow cylinder

$= 2\pi(R + r)(R - r + h)$

$= 2 \times \frac{22}{7} \times (16 + 12)(16 - 12 + 13)$

$= 2 \times \frac{22}{7} \times 28 \times 17$

$=$ **2992 sq.cm**

Similar Problems
Solve Your Self

Eg. 7.4: If one litre of paint covers 10 m^2, how many litres of paint is required to paint the internal and external surface areas of a cylindrical tunnel whose thickness is 2 m, internal radius is 6 m and height is 25 m. 2M

4. A right angled triangle $\boldsymbol{PQR}$ **where** $\angle\boldsymbol{Q} = \boldsymbol{90}^\circ$ **is rotated about** $\boldsymbol{QR}$ **and** $\boldsymbol{PQ}$**. If** $\boldsymbol{QR = 16\ cm}$ **and** $\boldsymbol{PR = 20\ cm}$**, compare the curved surface areas of the right circular cones so formed by the triangle.** 2M

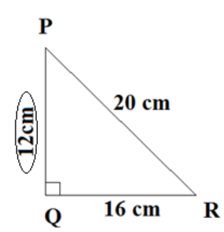
According to Pythagoras theorem,

$PQ = \sqrt{20^2 - 16^2}$

$= \sqrt{400 - 256}$

$= \sqrt{144}$

$PQ = 12\text{cm}.$



CSA of cone $= \pi rl$

(1) CSA (Rotated about QR)

$= \pi \times 12 \times 20 = 240\pi cm^2$ ………..(1)

(2) CSA (Rotated about PQ)

$= \pi \times 16 \times 20 = 320\pi cm^2$…………(2)

$(2) > (1) \Rightarrow$

CSA (Rotated about PQ) > CSA (Rotated about QR)

CSA of the cone when rotated about **'PQ' is larger than 'QR'.**

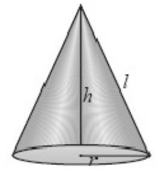
5. **4 persons live in a conical tent whose slant height is 19m. If each person require $22\ m^2$ of the floor area, then find the height of the tent.** 2M

Required base area of cone $= \pi r^2 = 22 \times 4$

$$\frac{22}{7} \times r^2 = 88$$

$$r^2 = 88 \times \frac{7}{22} = 28$$

$$r = \sqrt{28} = \sqrt{4 \times 7} = 2\sqrt{7}\ \text{m.}$$



Height of the tent $h = \sqrt{l^2 - r^2} = \sqrt{19^2 - (2\sqrt{7})^2}$

$$= \sqrt{361 - 28}$$

$$= \sqrt{333}$$

$$\boldsymbol{h \cong 18.25}\textbf{m.}$$

> **Similar Problems Solve Your Self** 5M
>
> **Eg. 7.5:** The radius of a conical tent is $7\ m$ and the height is $24\ m$. Calculate the length of the canvas used to make the tent if the width of the rectangular canvas is $4\ m$? **(APR-23, PTA-3)**
>
> **Eg. 7.6:** If the total surface area of a cone of radius $7\ cm$ is $704\ cm^2$, then find its slant height. **(JUL-22)** 2M

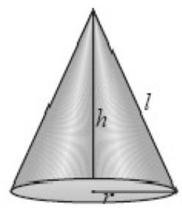
6. **A girl wishes to prepare birthday caps in the form of right circular cones for her birthday party, using a sheet of paper whose area is $5720 cm^2$, how many caps can be made with radius 5cm and height 12cm.** 2M

$r = 5$ cm, $h = 12$ cm

Slant Height $l = \sqrt{h^2 + r^2}$

$$= \sqrt{12^2 + 5^2} = \sqrt{144 + 25}$$

$$= \sqrt{169} = 13\ \text{cm}$$



Number of caps $= \dfrac{\text{Area of paper sheet}}{\text{area of one cap (CSA of cone } \pi r l)}$

$$= \frac{5720 \times 7}{22 \times 5 \times 13}$$

$$= 28$$

Number of caps can be made=28

7. **The ratio of the radii of two right circular cones of same height is 1:3. Find the ratio of their curved surface area when the height of each cone is 3 times the radius of the smaller cone.** PTA-2 2M

Smaller cone:	**Large cone:**
$r_1 \to r$	$r_2 \to 3r$
$h_1 \to 3r$	$h_2 \to 3r$
$l_1 = \sqrt{(3r)^2 + r^2} = \sqrt{10r^2} = r\sqrt{10}$	$l_2 = \sqrt{(3r)^2 + (3r)^2} = \sqrt{18r^2} = \sqrt{9 \times 2}(r) = 3r\sqrt{2}$

CSA of small cone : CSA of large cone

$$\pi r_1 l_1 : \pi r_2 l_2$$

$$r \times r\sqrt{10} : 3r \times 3r\sqrt{2}$$

$$\sqrt{5}\,\sqrt{2} : 9\sqrt{2}$$

$$\sqrt{5} : 9$$

Ratio of the CSA is $\sqrt{5} : 9$

8. The radius of a sphere increases by 25%. Find the percentage increase in its surface area. (2M)

Surface area of sphere $= 4\pi r^2$

Original = SA (when r =100 units)

$= 4\pi(100)^2$

$= 10000 \times 4\pi$ Sq.units.

New = SA (when r =125 units)

i.e. increased by25% $= 4\pi(125)^2 = 15625 \times 4\pi$ sq.units.

Percentage change $= \left[\frac{\text{new SA}}{\text{Original SA}} - 1\right] \times 100 = \left[\frac{15625\times\cancel{4\pi}}{10000\times\cancel{4\pi}} - 1\right] \times 100 = 56.25\%$

Similar Problems

Solve Your Self (2M)

Eg. 7.8: Find the diameter of a sphere whose surface area is $154\ m^2$. **(SEP-20)**

Eg. 7.9: The radius of a spherical balloon increases from 12 cm to 16 cm as air being pumped into it. Find the ratio of the surface area of the balloons in the two cases. **(MAY-22)** (5M)

9. The internal and external diameters of a hollow hemispherical vessel are 20cm and 28cm respectively. Find the cost to paint the vessel all over at ₹ 0.14 per cm^2 (2M)

$r = 10$ cm, $R = 14$ cm Where R- External radius, r- Internal radius

TSA of hemisphere $= \pi(3R^2 + r^2)$

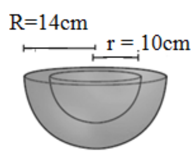
$= \frac{22}{7}(3(14)^2 + 10^2)$

$= \frac{22}{7} \times (588 + 100)$

$= \frac{22}{7} \times 688$

TSA =2162.16 cm^2

Required cost =2162.16×0.14 = ₹302.72



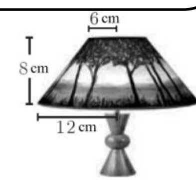
Similar Problems

Solve Your Self (2M)

Eg. 7.10: If the base area of a hemispherical solid is 1386 sq. metres, then find its total surface area?

Eg. 7.11: The internal and external radii of a hollow hemispherical shell are 3 m and 5 m respectively. Find the T.S.A. and C.S.A. of the shell.

10. The frustum shaped outer portion of the table lamp has to be painted including the top part. Find the total cost of painting the lamp if the cost of painting 1 sq.cm is ₹ 2 (5M)



$r = 6$ cm, $R = 12$ cm

Required Area = CSA of Frustum + top part circle area

$= \pi(R + r)l + \pi r^2$

$= \pi(12 + 6)10 + \pi(6)^2$

$= 180\pi + 36\pi$

$= 216\pi$

$= 216 \times \frac{22}{7}$

$= \frac{4752}{7}$

$\cong 678.86\ cm^2$

$l = \sqrt{h^2 + (R - r)^2}$

$= \sqrt{8^2 + (12 - 6)^2}$

$= \sqrt{64 + 36}$

$= \sqrt{100}$

$= 10$cm

Required Cost $= 678.86 \times 2$ = ₹ 1357.72

Similar Problems

Solve Your Self

Eg. 7.13: The slant height of a frustum of a cone is 5 cm and the radii of its ends are 4 cm and 1 cm. Find its curved surface area. (2M)

Eg. 7.14: An industrial metallic bucket is in the shape of the frustum of a right circular cone whose top and bottom diameters are 10 m and 4 m and whose height is 4 m. Find the curved and total surface area of the bucket. (5M)

UE-7: The slant height of a frustum of a cone is 4 m and the perimeter of circular ends are 18 m and 16 m. Find the cost of painting its curved surface area at ₹ 100 per sq. m. (5M)

For Practice:

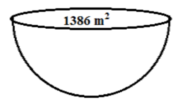
Eg. 7.3: A garden roller whose length is 3 m long and whose diameter is 2.8 m is rolled to level a garden. How much area will it cover in 8 revolutions? **(2M)**

Eg. 7.7: From a solid cylinder whose height is 2.4 cm and diameter 1.4 cm, a conical cavity of the same height and base is hollowed out. Find the total surface area of the remaining solid. **(5M)**

Eg. 7.12: A sphere, a cylinder and a cone are of the same height which is equal to its radius, where as cone and cylinder are of same height. Find the ratio of their curved surface areas. **(2M)**

Creative Questions

1. **If the base area of a hemispherical solid is 1386 sq. metres, then find its**
   (i) Curved surface area
   (ii) Total surface area (2M)



Given that,

Base Area $= \pi r^2 = 1386 m^2$

(i) CSA of hemisphere $= 2\pi r^2$

$= 2 \times 1386$

$= 2772\ m^2$

(ii) TSA of hemisphere $= 3\pi r^2$ SEP-20

$= 3 \times 1386$

$= 4158\ m^2$

2. **If the slant height of the frustum cone is 10cm and perimeters of its circular base are 18cm and 28cm respectively. What is the curved surface area of the frustum?** PTA-6

Slant height of the frustum $l = 10\ cm$ (5M)

Circular base (top) $= 2\pi R = 28\ cm \quad R = \frac{14}{\pi}$ cm

Circular base (bottom) $= 2\pi r = 18\ cm$

$r = \frac{9}{\pi}\ cm$

Curved surface area of the frustum $= \pi(R + r)l$

$= \pi\left(\frac{14}{\pi} + \frac{9}{\pi}\right) \times 10$

$= \pi \times \frac{23}{\pi} \times 10$

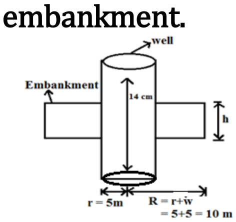
$= 230\ \text{cm}^2$

Exercise 7.2

1. **A 14 m deep well with inner diameter 10 m is dug and the earth taken out is evenly spread all around the well to form an embankment of width 5 m. Find the height of the embankment.**

Volume of embankment = Volume of well (2M)

Volume of hollow cylinder = Volume of cylinder



$\pi(R^2 - r^2)h = \pi r^2 h$

$\pi \times (10^2 - 5^2)h = \pi \times 5 \times 5 \times 14$

$h = \frac{5\times5\times14}{75}$

$= \frac{14}{3}$

$= 4.666.....$

Height of the embankment $= \mathbf{4.67\ m}$

> **Similar Problems** (2M)
> **Solve Your Self**
> **Eg. 7.15:** Find the volume of a cylinder whose height is 2 m and whose base area is 250 m^2 **(SEP-21)**
> **Eg. 7.17:** Find the volume of the iron used to make a hollow cylinder of height 9 cm and whose internal and external radii are 21 cm and 28 cm respectively.

2. **A cylindrical glass with diameter 20 cm has water to a height of 9 cm. A small cylindrical metal of radius 5 cm and height 4 cm is immersed it completely. Calculate the raise of the water in the glass?** SEP-20 (2M)

Volume of water raised in cylindrical glass

= Volume of cylindrical metal immersed

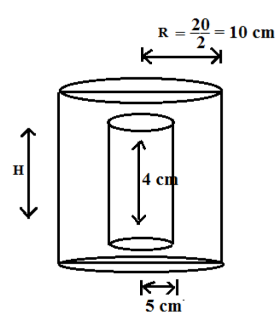
$\pi R^2 H = \pi r^2 h$

$\pi \times 10 \times 10 \times h = \pi \times 5 \times 5 \times 4$

$h = \frac{5\times5\times4}{10\times10}$

$= 1$

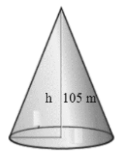
The raise of the water in the glass $= \mathbf{1\ cm}$



> **Similar Problems** (2M)
> **Solve Your Self**
> **Eg. 7.16:** The volume of a cylindrical water tank is 1.078×10^6 litres. If the diameter of the tank is 7 m, find its height.

3. If the circumference of a conical wooden piece is 484 cm then find its volume when its height is 105 cm. (2M) JUL-22

Base circumference of the cone $(2\pi r) = 484$



$$2 \times \frac{22}{7} \times r = 484$$

$$r = \frac{\cancel{484} \times 7}{\cancel{2} \times \cancel{22}}$$

$$r = 77 \text{ cm}$$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 77 \times 77 \times 105 = 652190\ cm^3$$

Volume of the conical wooden piece $= \mathbf{652190\ cm^3}$

Similar Problems **Solve Your Self** (2M)

Eg. 7.19: The volume of a solid right circular cone is 11088 cm^3. If its height is 24 cm then find the radius of the cone. **(JUN-23,PTA-1)**

UE-9: The volume of a cone is $1005\frac{5}{7}$ cu. cm. The area of its base is $201\frac{1}{7}$ sq. cm. Find the slant height of the cone.

4. A conical container is fully filled with petrol. The radius is 10 m and the height is 15 m. If the container can release the petrol through its bottom at the rate of 25 cu.meter per minute, in how many minutes the container will be emptied. Round off your answer to the nearest minute. (2M)

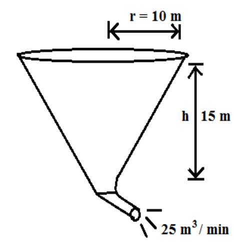
$$\text{Minutes} = \frac{\text{volume of conical container } \left(\frac{1}{3}\pi r^2 h\right)}{\text{volume of petrol released per minute}}$$

$$= \frac{\frac{1}{3} \times \frac{22}{7} \times 10 \times 10 \times 15}{25}$$

$$= \frac{1 \times 22 \times 10 \times 10 \times 15}{3 \times 7 \times 25}$$

$$= \frac{440}{7}$$

$$= 62.8 \cong \mathbf{63}$$



The container will be emptied in 63 minutes (approx.)

5. A right angled triangle whose sides are 6 cm, 8 cm & 10 cm is revolved about the sides containing the right angle in two ways. Find the difference in volumes of the two solids so formed. (2M)

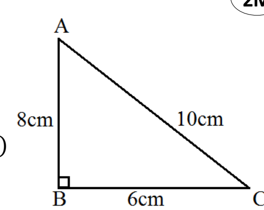
Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of cone (revolved about 6 cm) $= \frac{1}{3} \times \pi \times 8 \times 8 \times 6$ $(r = 8, h = 6)$

$= 128\pi\ cm^3$ ……….. (1)

Volume of cone (revolved about 8 cm) $= \frac{1}{3} \times \pi \times 6 \times 6 \times 8$ $(r = 6, h = 8)$

$= 96\pi$ ……….. (2)



Difference in volumes (1) – (2) $= 128\pi - 96\pi$

$$= 32\pi \quad = 32 \times \frac{22}{7}$$

$$= \frac{704}{7}$$

$$\cong 100.571$$

Difference in the volumes of two solids $= \mathbf{100.58\ cm^3}$

6. The volumes of two cones of same base radius are 3600 cm^3 and 5040 cm^3. Find the ratio of heights. (2M) PTA - 4, MAY-22

Volume of cone $= \frac{1}{3}\pi r^2 h$

Volume of cone 1 : Volume of cone 2 $= 3600 : 5040$

$$\frac{1}{3}\pi r^2 \times h_1 : \frac{1}{3}\pi r^2 \times h_2 = 180 : 252$$

$$h_1 : h_2 = 45 : 63$$

$$h_1 : h_2 = 5 : 7$$

Ratio of the height **5 : 7**

Similar Problems (2M)
Solve Your Self

7. If the ratio of radii of two spheres is $4 : 7$, find the ratio of their volumes. **(APR-23)**

Eg. 7.20: The ratio of the volumes of two cones is $2 : 3$. Find the ratio of their radii if the height of second cone is double the height of the first.

8. A solid sphere and a solid hemisphere have equal total surface area. Prove that the ratio of their volume is $3\sqrt{3} : 4$. 2M PTA-6

TSA of sphere = TSA of hemisphere

$$\cancel{4}\pi r_1^2 = \cancel{3}\pi r_2^2 \Rightarrow \frac{r_1^2}{r_2^2} = \frac{3}{4} \Rightarrow \frac{r_1}{r_2} = \frac{\sqrt{3}}{2}$$

$$\frac{\text{volume of sphere}}{\text{volume of hemisphere}} = \frac{\frac{4}{3}\pi r_1^3}{\frac{2}{3}\pi r_2^3}$$

$$= 2\left(\frac{r_1}{r_2}\right)^3$$

$$= 2\left(\frac{\sqrt{3}}{2}\right)^3$$

$$= \frac{2\times 3\sqrt{3}}{8}$$

$$= \frac{3\sqrt{3}}{4}$$

$\therefore$ Ratio of the volume $\mathbf{3\sqrt{3} : 4}$

Similar Problems 5M
Solve Your Self
Eg. 7.21: The volume of a solid hemisphere is $29106\ cm^3$. Another hemisphere whose volume is two-third of the above is carved out. Find the radius of the new hemisphere.

Similar Problems (Solve Your Self)
Eg. 7.22: Calculate the mass of a hollow brass sphere if the inner diameter is 14 cm and thickness is 1 mm, and whose density is 17.3 g/cm^3. **(MDL)**

9. The outer and the inner surface areas of a spherical copper shell are 576π cm² and 324π cm² respectively. Find the volume of the material required to make the shell. 5M

Surface area of sphere $= 4\pi r^2$

Outer surface Area of sphere $= 4\pi R^2 = 576\pi$

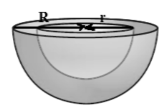
$$\Rightarrow R^2 = \frac{576\pi}{4\pi} \Rightarrow R = 12\ cm$$

Inner surface Area of sphere $= 4\pi r^2 = 324\pi$

$$\Rightarrow r^2 = \frac{324\pi}{4\pi} \Rightarrow r = 9\ cm$$

Volume of hollow sphere $= \frac{4}{3}\pi(R^3 - r^3)$

$$= \frac{4}{3}\times\frac{22}{7}\times(12^3 - 9^3)$$



$$= \frac{4}{3}\times\frac{22}{7}\times(1728 - 729)$$

$$= \frac{4}{3}\times\frac{22}{7}\times 999$$

$$= \frac{29304}{7} = 4186.285$$

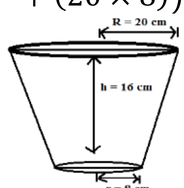
Volume of the material required $\cong \mathbf{4186.29\ cm^3}$

10. A container open at the top is in the form of a frustum of a cone of height $16\ cm$ with radii of its lower and upper ends are $8\ cm$ and $20\ cm$ respectively. Find the cost of milk which can completely fill a container at the rate of ₹ 40 per litre. MAY-22 5M

Volume of frustum $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$

$$= \frac{1}{3}\times\frac{22}{7}\times 16\big(20^2 + 8^2 + (20\times 8)\big)$$

$$= \frac{1}{3}\times\frac{22}{7}\times 16\times 624$$



$$= \frac{73216}{7}$$

$$= 10459.4\ cm^3$$

Volume of frustum = 10.4594 litres

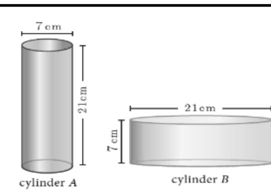
Required cost = 10.4594 × 40

= ₹ 418.376

$\boxed{\therefore 1000\ cm^3 = 1 litre}$

Cost of the milk which can completely fill the container $\cong$ ₹ **418.38**

Similar Problems (Solve Your Self)
Eg. 7.23: If the radii of the circular ends of a frustum which is 45 cm high are 28 cm and 7 cm, find the volume of the frustum. **(PTA-5, SEP-21)** 5M
Eg. 7.18: For the cylinders A and B (i) find out the cylinder whose volume is greater.
(ii) verify whether the cylinder with greater volume has greater total surface area.
(iii) find the ratios of the volumes of the cylinders A and B. 5M



Creative Questions

1. The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes. SEP-21 5M

Let h_1 and h_2 be the heights and r_1 and r_2 be the radii of the two cones respectively.

Ratio of their heights $= h_1 : h_2 = 1 : 2 \Rightarrow \frac{h_1}{h_2} = \frac{1}{2}$

Ratio of perimeters $\Rightarrow \cancel{2\pi}r_1 : \cancel{2\pi}r_2 = 3 : 4 \Rightarrow \frac{r_1}{r_2} = \frac{3}{4}$

Volume of first cone : Volume of Second cone $= \frac{1}{3}\pi r_1^2 h_1 : \frac{1}{3}\pi r_2^2 h_2$

$$= \frac{r_1^2 h_1}{r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2\left(\frac{h_1}{h_2}\right) = \left(\frac{3}{4}\right)^2\left(\frac{1}{2}\right) = \frac{9}{16}\times\frac{1}{2} = \frac{9}{32}$$

2. Find the volume of the iron used to make a hollow cylinder of height 9 cm and whose internal and external radii are 3 cm and 5 cm respectively. SEP-20

Let r, R and h be the internal radius, external radius and height of the hollow cylinder respectively.

Given $r = 3cm, R = 5\ cm, h = 9\ cm$ 2M

Volume of hollow cylinder $= \pi(R^2 - r^2)h$ cu. units

$$= \frac{22}{7}(5^2 - 3^2) \times 9$$

$$= \frac{22}{7}(25 - 9) \times 9$$

$$= \frac{22}{7} \times 16 \times 9 = 452.57\ cm^2$$

3. A well of diameter 3m is dug 14m deep. The earth, taken out of it has been spread evenly all around it in the shape of a circular ring of width 4m to form an embankment. Find the height of the embankment. PTA-5 5M

Well	Embankment
$D = 3$m	$r = \frac{3}{2} = 1.5$m
$r = \frac{3}{2} = 1.5$m	$R = \frac{3}{2} + 4 = \frac{11}{2} = 5.5$m
$h = 14$ m	

Volume of embankment = Volume of well

Volume of hollow cylinder = Volume of cylinder

$$\pi(R^2 - r^2)h = \pi r^2 h$$

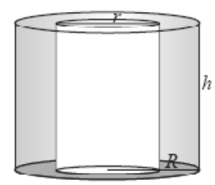
$$\pi \times (5.5^2 - 1.5^2)h = \pi \times 1.5 \times 1.5 \times 14$$

$$(5.5 + 1.5)(5.5 - 1.5)h = 1.5 \times 1.5 \times 14$$

$$7 \times 4 \times h = 31.50$$

$$28 \times h = 31.50$$

$$h = \frac{31.50}{28} = 1.125$$



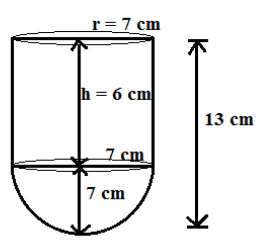
$\therefore$ Height of the embankment $= 1.125\ m$

Exercise 7.3

1. A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter is 14 cm and the height of the vessel is 13 cm. Find the capacity of the vessel. 5M

Capacity of the vessel (V) = Volume of cylinder + Volume of hemisphere

$$= \pi r^2 h + \frac{2}{3}\pi r^3$$



$$= \pi r^2 \left[h + \frac{2r}{3}\right]$$

$$= \frac{22}{7} \times 7 \times 7 \left[6 + \frac{2(7)}{3}\right]$$

$$= 22 \times 7 \times \frac{32}{3}$$

$$V = \frac{4928}{3} = 1642.66\ldots\ldots$$

Capacity of the vessel$\cong$ $\mathbf{1642.67\ cm^3}$

Similar Problems Solve Your Self 5M

Eg. 7.24: A toy is in the shape of a cylinder surmounted by a hemisphere. The height of the toy is 25 cm. Find the total surface area of the toy if its common diameter is 12 cm. **(SEP-21)**

Eg. 7.26: Arul has to make arrangements for the accommodation of 150 persons for his family function. For this purpose, he plans to build a tent which is in the shape of cylinder surmounted by a cone. Each person occupies 4 sq.m of the space on ground and 40 cu. meter of air to breathe. What should be the height of the conical part of the tent if the height of cylindrical part is 8 m? **(APR-23)**

2. **Nathan, an engineering student was asked to make a model shaped like a cylinder with two cones attached at its two ends. The diameter of the model is 3 cm and its length is 12 cm. If each cone has a height of 2 cm, find the volume of the model that Nathan made.** MAY-22

5M

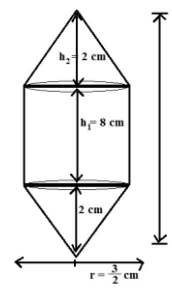
Volume of the model = Volume of cylinder + Volume of cone × 2

$$= \pi r^2 h_1 + \frac{1}{3}\pi r^2 h_2 \times 2$$

$$= \pi r^2 \left[h_1 + \frac{2}{3}h_2\right]$$

$$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \left[8 + \frac{2}{3}(2)\right]$$

$$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{28}{3}$$



Volume of the model $= \mathbf{66\ cm^3}$

3. **From a solid cylinder whose height is 2.4 cm & the diameter 1.4 cm, a cone of the same height and same diameter is carved out. Find the volume of the remaining solid to the nearest cm^3.**

5M

Volume of remaining solid = Volume of cylinder – Volume of cone

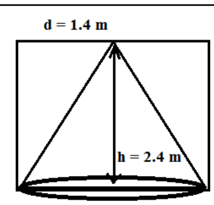
$$= \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$= \frac{2}{3}\pi r^2 h$$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{10} \times \frac{7}{10} \times \frac{24}{10}$$

$$= \frac{2464}{1000}$$

$$= 2.464\ cm^3$$



diameter = 1.4cm

radius r= 0.7 = $\frac{7}{10}$ cm

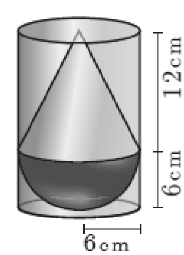
height h = $\frac{24}{10}$ cm

Volume of the remaining solid $= \mathbf{2.46 cm^3}$

4. **A solid consisting of a right circular cone of height 12 cm and radius 6 cm standing on a hemisphere of radius 6 cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of the water displaced out of the cylinder, if the radius of the cylinder is 6 cm and height is 18 cm.** 5M

Volume of water displaced = Volume of solid placed

(Volume of cone + Volume of hemisphere)



$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{1}{3}\pi r^2 (h + 2r)$$

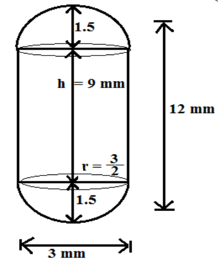
$$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times (12 + 2(6))$$

$$= \frac{44 \times 6}{7} \times 24$$

$$= \frac{6336}{7}$$

Volume of the water displaced $= \mathbf{905.14\ cm^3}$

5. A capsule is in the shape of a cylinder with two hemisphere stuck to each of its ends. If the length of the entire capsule is 12 mm and the diameter of the capsule is 3 mm, how much medicine it can hold? (5M)



Volume of capsule = Volume of cylinder + (2 ×Volume of hemisphere)

$$= \pi r^2 h + \left(2 \times \frac{2}{3}\pi r^3\right)$$

$$= \pi r^2 \left(h + \frac{4r}{3}\right)$$

$$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \left[9 + \frac{4}{3}\left(\frac{3}{2}\right)\right]$$

$$= \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times 11$$

$$= \frac{2178}{28} = \frac{1089}{14}$$

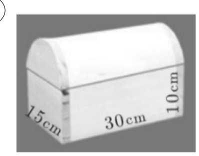
$h = 12 - 3$
$= 9$
$r = \frac{3}{2}$

Volume of capsule $= \frac{1089}{14} \cong 77.785$

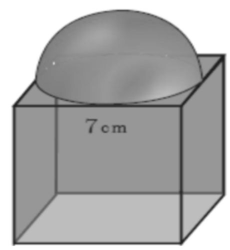
A capsule can hold $\mathbf{77.78\ mm^3}$ medicine.

Similar Problems (Solve Your Self) (5M)

Eg. 7.25: A jewel box is in the shape of a cuboid of dimensions $30\ cm \times 15\ cm \times 10\ cm$ surmounted by a half part of a cylinder as shown in the figure. Find the volume of the box.



6. As shown in figure a cubical block of side 7 cm is surmounted by a hemisphere. Find the surface area of the solid. (5M)



Surface Area of the solid

= TSA of cube – Area of base of hemisphere + CSA of hemisphere

$$= 6a^2 - \pi r^2 + 2\pi r^2$$

$$= 6a^2 + \pi r^2$$

$$= 6a^2 + \pi\left(\frac{a^2}{4}\right)$$

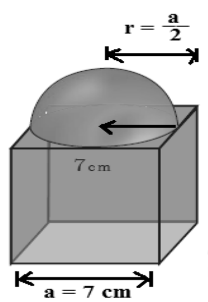
$$= a^2\left(6 + \frac{\pi}{4}\right)$$

$$= \frac{a^2}{4}(24 + \pi)$$

$$= \frac{a^2}{4}\left(24 + \frac{22}{7}\right)$$

$$= \frac{a^2}{4}\left(\frac{190}{7}\right)$$

$$= \frac{95a^2}{14} = \frac{95(7)^2}{14} = \frac{95 \times 49}{14} = 332.5\ \text{cm}^2$$

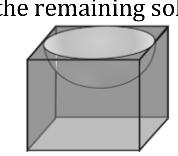


cube + hemisphere

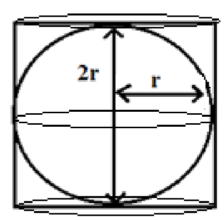
Surface Area of the solid = $\mathbf{332.5\ \ cm^2}$.

Similar Problems
Solve Your Self (5M)

Eg. 7.28: A hemispherical section is cut out from one face of a cubical block such that the diameter l of the hemisphere is equal to side length of the cube. Determine the surface area of the remaining solid.



7. A right circular cylinder just enclose a sphere of radius r units. Calculate (i) the surface area of the sphere (ii) the curved surface area of the cylinder (iii) the ratio of the areas obtained in (i) and (ii). (5M)



(i) Surface Area of sphere $= 4\pi r^2$ sq.units

(ii) CSA of cylinder $= 2\pi rh = 2\pi r(2r) = \mathbf{4\pi r^2}$ **sq.units**

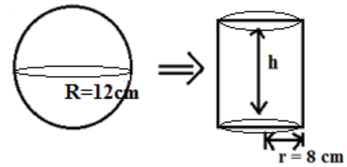
(iii) Ratio of (i) and (ii) $= \frac{4\pi r^2}{4\pi r^2} = \frac{1}{1} = \mathbf{1:1}$

Exercise 7.4

1. An aluminium sphere of radius 12 cm is melted to make a cylinder of radius 8 cm. Find the height of the cylinder. 5M

Radius of sphere R $= 12\ cm$

Radius of cylinder r $= 8\ cm$



Volume of cylinder $(\pi r^2 h)$ = Volume of sphere $\left(\frac{4}{3}\pi r^3\right)$

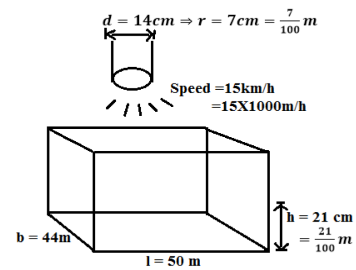
$$\pi \times 8 \times 8 \times h = \frac{4}{3} \times \pi \times 12 \times 12 \times 12$$

$$h = \frac{4 \times \pi \times 12 \times 12 \times 12}{3 \times \pi \times 8 \times 8}$$

$$h = 36\ cm$$

Height of the cylinder $= \mathbf{36\ cm}$

2. Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide. Find the time in which the level of water in the tanks will rise by 21 cm. 5M



Given

Rectangular Tank:	**Cylindrical pipe:**
$l = 50\ m$	diameter $= 14\ cm$
$b = 44m$	$\therefore$ Radius $= 7\ cm = \frac{7}{100}m$
$h = 21\ cm = \frac{21}{100}m$	speed $= 15km/h = 15 \times 1000\ m/h$

$$\text{Required time} = \frac{\text{Volume of Rectangular Tank (lbh)}}{\text{water discharged from pipe per hour } (\pi r^2 \times \text{speed})}$$

$$= \frac{50 \times 44 \times 21 \times 7 \times 100 \times 100}{100 \times 22 \times 7 \times 7 \times 15 \times 1000}$$

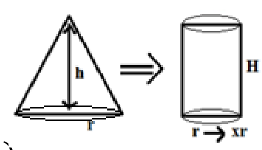
$$= 2 \text{ hours}$$

Time required to rise water level by 21 cm = **2 hours**

> **Similar Problems** (Solve Your Self)
>
> **UE-2:** A hemi-spherical tank of radius 1.75 m is full of water. It is connected with a pipe which empties the tank at the rate of 7 litre per second. How much time will it take to empty the tank completely? 5M

3. A conical flask is full of water. The flask has base radius r units & height h units, the water is poured into a cylindrical flask of base radius xr units. Find the height of water in the cylindrical flask. 5M

Conical flask	**Cylindrical flask**
Radius $\rightarrow r$	Radius $\rightarrow xr$
Height $\rightarrow h$	Height $\rightarrow H$



Volume of cylindrical flask $(\pi r^2 h)$

$$= \text{Volume of conical flask } \left(\frac{1}{3}\pi \times r^2 \times h\right)$$

$$\pi \times xr \times xr \times H = \frac{1}{3}\pi \times r^2 \times h$$

$$H = \frac{1 \times \pi \times r^2 \times h}{3 \times x^2 \times r^2 \times \pi}$$

$$H = \frac{h}{3x^2}$$

Height of water in the cylindrical flask $= \boldsymbol{\frac{h}{3x^2}}$ **units**

> **Similar Problems** (Solve Your Self)
>
> **Eg. 7.30:** A cone of height 24 cm is made up of modeling clay. A child reshapes it in the form of a cylinder of same radius as cone. Find the height of the cylinder. **(PTA-3)** 2M
>
> **Eg. 7.31:** A right circular cylindrical container of base radius 6 cm and height 15 cm is full of ice cream. The ice cream is to be filled in cones of height 9 cm and base radius 3 cm, having a hemispherical cap. Find the number of cones needed to empty the container. **(PTA-5,6)** 5M

4. **A solid right circular cone of diameter $14\ cm$ and height $8\ cm$ is melted to form a hollow sphere. If the external diameter of the sphere is $10\ cm$, find the internal diameter.** (5M)

Given

Cone:

diameter $= 14\ cm$

$\therefore$ Radius $(r_1) = 7\ cm$

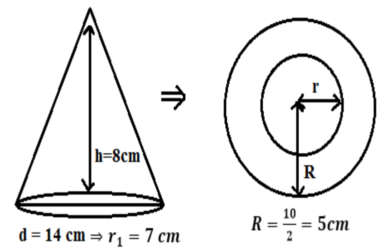
Height $(h) = 8\ cm$

Hollow sphere:

External diameter $= 10\ cm$

$\therefore$ Radius $R = 5\ cm$

Let, Internal Radius $= r$



Volume of the hollow sphere = Volume of cone

$$\frac{4}{3}\pi(R^3 - r^3) = \frac{1}{3}\pi r_1^2 h$$

$$\frac{4}{3} \times \pi \times (5^3 - r^3) = \frac{1}{3} \times \pi \times 7 \times 7 \times 8$$

$$125 - r^3 = 98$$

$$-r^3 = 98 - 125$$

$$-r^3 = -27$$

$$\therefore\ r^3 = 27$$

$$r^3 = 3^3$$

$$r = 3\ cm \ = \text{Internal Radius}$$

Internal diameter of the sphere $= \mathbf{6\ cm}$

5. **Seenu's house has an overhead tank in the shape of a cylinder. This is filled by pumping water from a sump (underground tank) which is in the shape of a cuboid. The sump has dimensions $2\ m \times 1.5m \times 1m$. The overhead tank has its radius of $60\ cm$ and height $105\ cm$. Find the volume of the water left in the sump after the overhead tank has been completely filled with water from the sump which has been full, initially.** (5M)

Given

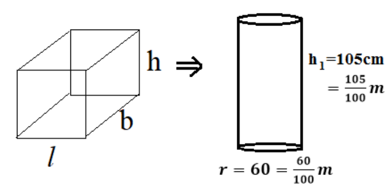
Cuboidal sump

$l \times b \times h = 2m \times 1.5m \times 1m$

Cylindrical tank

Radius $= 60\ cm = \frac{60}{100} m$

Height $= 105\ cm = \frac{105}{100} m$



Volume of water left in the sump = (Volume of cuboidal sump (lbh)) − (Volume of cylinderical overhead tank $(\pi r^2 h)$)

$$= [2 \times 1.5 \times 1] - \left[\frac{22}{7} \times \frac{60}{100} \times \frac{60}{100} \times \frac{105}{100}\right]$$

$$= 3 - \frac{11 \times 6 \times 6 \times 3}{1000}$$

$$= 3 - 1.188$$

$$= 1.812$$

Volume of water left in the sump

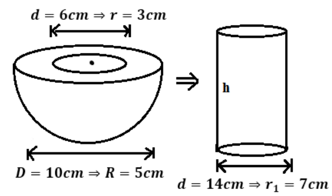
$= 1.812\ m^3$ $\quad (\because 1m^3 = 1000000\ cm^3)$

$= \mathbf{1812000\ cm^3}$ **(or) 1812 litre**

6. **The internal and external diameter of a hollow hemispherical shell are $6\ cm$ and $10\ cm$ respectively. If it is melted and recast into a solid cylinder of diameter $14\ cm$, then find the height of the cylinder.** 5M

Given

Hollow hemisphere	**Cylinder**
Internal diameter $= 6\ cm$	Diameter $= 14\ cm$
$\therefore$ Radius $(r) = 3\ cm$	$\therefore$ Radius $(r) = 7cm$
External diameter $= 10\ cm$	Let Height $\rightarrow h$.
$\therefore$ Radius $(R) = 5\ cm$	



Volume of cylinder $(\pi r^2 h)$ = Volume of hollow hemisphere $\left[\frac{2}{3}\pi(R^3 - r^3)\right]$

$$\pi \times 7 \times 7 \times h = \frac{2}{3} \times \pi \times [5^3 - 3^3]$$

$$\pi \times 7 \times 7 \times h = \frac{2}{3} \times \pi \times [125 - 27]$$

$$\pi \times 7 \times 7 \times h = \frac{2}{3} \times \pi \times 98$$

$$h = \frac{2 \times \pi \times 98}{3 \times \pi \times 7 \times 7} = \frac{4}{3}$$

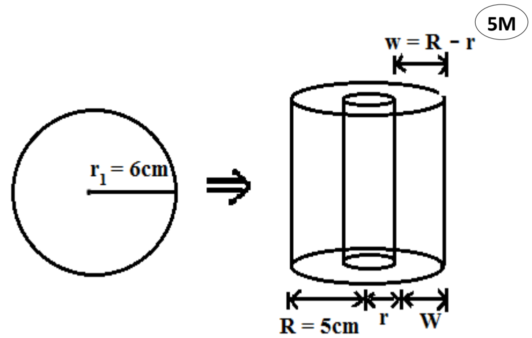
$$h = 1.33\ cm.$$

Height of the cylinder $= \mathbf{1.33\ cm}$

7. **A solid sphere of radius $6\ cm$ is melted into a hollow cylinder of uniform thickness. If the external radius of the base of the cylinder is $5\ cm$ and its height is $32\ cm$, then find the thickness of the cylinder.** 5M

Given

Sphere	**Cylinder (Hollow)**
Radius $= 6\ cm$	External Radius $(R) = 5\ cm$
	Internal Radius $(r) \rightarrow r$
	Height $(h) = 32\ cm$



Volume of hollow cylinder = Volume of sphere

$$\pi h(R^2 - r^2) = \frac{4}{3}\pi r^3$$

$$\pi \times 32 \times (5^2 - r^2) = \frac{4}{3} \times \pi \times 6 \times 6 \times 6$$

$$25 - r^2 = \frac{4 \times \pi \times 6 \times 6 \times 6}{3 \times \pi \times 32} = 9$$

$$-r^2 = 9 - 25 = -16$$

$$r = 4cm$$

Width $= R - r$

$= 5 - 4 = 1\ cm$

Thickness of the cylinder $= \mathbf{1\ cm}$

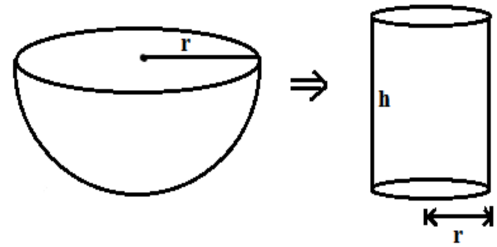
8. **A hemispherical bowl is filled to the brim with juice. The juice is poured into a cylindrical vessel whose radius is 50% more than its height. If the diameter is same for both the bowl and the cylinder then find the percentage of juice that can be transferred from the bowl into the cylindrical vessel.** 5M

Given

Radius of hemisphere $= r$

Radius of cylinder $= r$

Also, Radius $(r) = 50\%$ more than height (h)



$$r = h + \frac{50}{100}h \Rightarrow r = h + \frac{1}{2}h \Rightarrow r = \frac{3h}{2}$$

$$\therefore \; h = \frac{2r}{3}$$

Volume of hemispherical bowl $= \frac{2}{3}\pi r^3$ ………….. (1)

Volume of cylindrical vessel $= \pi r^2 h$

$$= \pi \times r^2 \times \frac{2r}{3} = \frac{2}{3}\pi r^3 \text{ ……….. (2)}$$

$(1) = (2) \Rightarrow$ **100%** Juice that can be transferred from bowl into the cylinderical vessel.

Creative Questions

1. **Find the number of spherical lead shots, each of diameter $6cm$ that can be made from a solid cuboids of lead having dimensions $24cm \times 22cm \times 12cm$** PTA-2 2M

$$\frac{\text{Volume of cuboids}}{\text{Volume of sphere}} = \frac{l \times b \times h}{\frac{4}{3}\pi r^3}$$

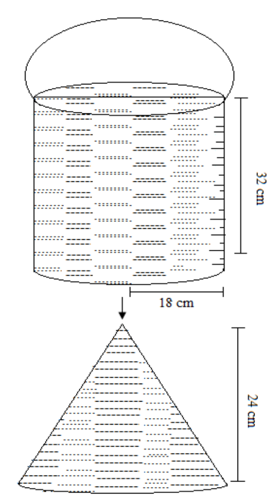
$$= \frac{24 \times 22 \times 12}{\frac{4}{3} \times \frac{22}{7} \times 3 \times 3 \times 3}$$

$$= 8 \times 7$$

$$= \mathbf{56}$$

2. **A cylindrical bucket, $32cm$ high and with radius of base $18cm$, is filled with sand completely. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is $24cm$, find the radius and slant height of the heap.** MDL, PTA-4 5M

Cylindrical bucket :	Conical heap :
Radius $r = 18$ cm	Height $h_1 = 24$cm
Height $h = 32$cm	



Volume of the conical heap

= Volume of the sand in the cylindrical bucket.

$$\frac{1}{3}\pi {r_1}^2 h_1 = \pi r^2 h$$

$$\frac{1}{3} \times \cancel{\pi} \times {r_1}^2 \times 24 = \cancel{\pi} \times 18 \times 18 \times 32$$

$${r_1}^2 = \frac{18 \times 18 \times 32 \times 3}{24}$$

$$r_1 = \sqrt{18 \times 18 \times 4} = 18 \times 2 = 36$$

Radius $r_1 = 36$cm

Slant height $l_1 = \sqrt{{r_1}^2 + {h_1}^2} = \sqrt{36^2 + 24^2} = 12\sqrt{3^2 + 2^2}$

$$l_1 = 12\sqrt{9 + 4} = \mathbf{12\sqrt{13}\ cm}$$

Unit Exercise - 7

Note for Unit Exercise – 7
Q.No: 2 – Similar to Exercise 7.4 -2nd Question
Q.No: 7 – Similar to Exercise 7.1 – 10th Question
Q.No: 9 – Similar to Exercise 7.2 – 3rd Question

1. The barrel of a fountain-pen cylindrical in shape, is $7\ cm$ long and $5\ mm$ in diameter. A full barrel of ink in the pen will be used for writing 330 words on an average. How many words can be written using a bottle of ink containing one fifth of a litre? (5M)

Volume of cylindrical pen $= \pi r^2 h$

$= \frac{22}{7} \times \frac{5}{2\times10} \times \frac{5}{2\times10} \times 7$

$= \frac{11}{8}\ cm^3$ [used for 330 words]

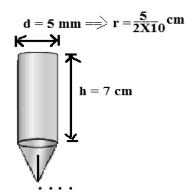
Number of words by $\frac{1}{5^{th}}$ litre

(ie., $200\ ml = 200\ cm^3$)

$= \frac{330}{\frac{11}{8}} \times 200$

$= \frac{330\times200\times8}{11}$

$= 48000$



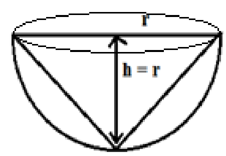
Number of words can be written using one fifth of a litre Ink = **48000 words**

3. Find the maximum volume of a cone that can be carved out of a solid hemisphere of radius r units. (2M)

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times r^2 \times r$

Maximum volume of cone $\boldsymbol{= \frac{1}{3}\pi r^3}$ **cu.units**



4. An oil funnel of tin sheet consists of a cylindrical portion $10\ cm$ long attached to a frustum of a cone. If the total height is $22\ cm$, the diameter of the cylindrical portion be $8\ cm$ and the diameter of the top of the funnel be $18\ cm$, then find the area of the tin sheet required to make the funnel. (5M)

$l = \sqrt{12^2 + (9-4)^2}$ $\left[\because\ l = \sqrt{h^2 + (R-r)^2}\right]$

$= \sqrt{144 + 25} = \sqrt{169}$

$l = 13\ cm$

Area of tin sheet = CSA of frustum + CSA of cylinder

$= \pi\,(R + r)l + 2\pi rh$

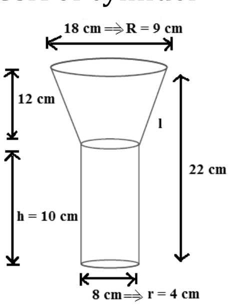
$= \pi\,[(R + r)l + 2rh]$

$= \frac{22}{7}[(9 + 4)13 + 2(4)(10)]$

$= \frac{22}{7}[169 + 80]$

$= \frac{22}{7} \times 249 = \frac{5478}{7}$

$\cong \mathbf{782.57\ \ cm^2}$



Area of tin sheet to make the funnel $\simeq 782.57\ \ cm^2$

Similar Problems (Solve Your Self)
Eg. 7.27: A funnel consists of a frustum of a cone attached to a cylindrical portion 12 cm long attached at the bottom. If the total height be 20 cm, diameter of the cylindrical portion be 12 cm and the diameter of the top of the funnel be 24 cm. Find the outer surface area of the funnel. **(PTA-1)** (5M)

5. **Find the number of coins, $1.5\ cm$ in diameter and $2\ mm$ thick, to be melted to form a right circular cylinder of height $10\ cm$ and diameter $4.5\ cm$.** 5M PTA-1

$$\text{Number of coins} = \frac{\text{volume of cylinder } (\pi r^2 h)}{\text{volume of a coin } (\pi r^2 h)}$$

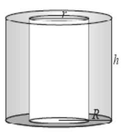
$$= \frac{\pi\times45\times45\times10\times10\times2\times10\times2\times10}{2\times10\times2\times10\times\pi\times15\times15\times2}$$

Number of coins to be melted $= \mathbf{450}$ **coins**

Similar Problems (Solve Your Self)
Eg. 7.29: A metallic sphere of radius 16 cm is melted and recast into small spheres each of radius 2 cm. How many small spheres can be obtained? **(JUN-23)** 5M

6. **A hollow metallic cylinder whose external radius is $4.3\ cm$ and internal radius is $1.1\ cm$ and whole length is $4\ cm$ is melted and recast into a solid cylinder of $12\ cm$ long. Find the diameter of solid cylinder.** 2M

Volume of solid cylinder



= Volume of hollow cylinder

$$\pi r^2 h = \pi(R^2 - r^2)h$$

$$\pi \times r^2 \times 12 = \pi[(4.3)^2 - (1.1)^2] \times 4$$

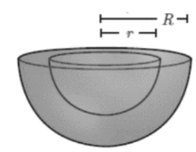
$$r^2 = \frac{\cancel{\pi}\times17.28\times4}{\cancel{\pi}\times12} = 5.76$$

Radius of the cylinder $r = 2.4\ cm$

Diameter of the solid cylinder $= \mathbf{4.8\ cm}$

8. **A hemi-spherical hollow bowl has material of volume $\frac{436\pi}{3}$ cubic cm. Its external diameter is $14\ cm$. Find its thickness.** 2M

Volume of hollow hemisphere



$$\frac{2}{3}\pi(R^3 - r^3) = \frac{436\pi}{3}$$

$$\frac{2}{3} \times \pi \times (7^3 - r^3) = \frac{436\pi}{3}$$

$$343 - r^3 = \frac{436\pi\times3}{3\times2\times\pi}$$

$$-r^3 = 218 - 343$$

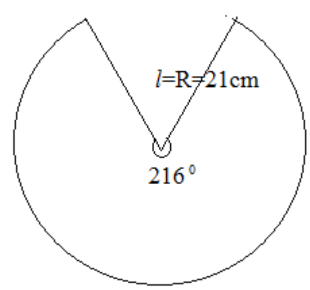
$$-r^3 = -125 = 5^3$$

$$r = 5\ cm$$

$\Rightarrow$ Thickness $= R - r = 7 - 5 = 2\ cm$

Thickness of the bowl $= \mathbf{2\ cm}$

10. **A metallic sheet in the form of a sector of a circle of radius $21\ cm$ has central angle of 216°. The sector is made into a cone by bringing the bounding radii together. Find the volume of the cone formed.** PTA-2 5M



Arc length $L = \frac{2\pi R}{360} \times 216$

$$L = \frac{2\pi\times21\times3}{5}$$

Circum of base of the cone = Arc length

i.e, $2\pi r = \frac{2\pi\times21\times3}{5} = \frac{63}{5}$

$$r = 12.6\ cm$$

$$h = \sqrt{l^2 - r^2}$$

$$= \sqrt{21^2 - 12.6^2} = \sqrt{441 - 158.76} = \sqrt{282.24}$$

$$h = 16.8\ cm$$

Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 12.6 \times 12.6 \times 16.8$$

$$= 2794.176\ cm^3$$

Volume of the cone formed $= \mathbf{2794.176\ cm^3}$

8. Statistics and Probability

Exercise 8.1

1. Find the range and coefficient of range of the following data. 2M

(i) $63, 89, 98, 125, 79, 108, 117, 68$

Arrange in Ascending order: APR-23, SEP-20

63,68,79,89,98,108,117,125

$$Range = L - S = 125 - 63 = 62$$

$$\text{Coefficient of Range} = \frac{L-S}{L+S} = \frac{125-63}{125+63} = \frac{62}{188}$$

$$= 0.3297 = \mathbf{0.33}$$

Similar Problems (Solve Your Self) 2M

1. Find the range and coefficient of range of the following data (ii) 43.5, 13.6, 18.9, 38.4, 61.4, 29.8

Eg. 8.1: Find the range and coefficient of range of the following data: 25, 67, 48, 53, 18, 39, 44.

UE-7: If the range and coefficient of range of the data are 20 & 0.2 respectively, then find the largest and smallest values of the data.

2. If the range and the smallest value of a set of data are 36.8 and 13.4 respectively, then find the largest value. 2M

$R = 36.8,$

$S = 13.4, L = ?$

$R = L - S$

$R + S = L$

$L = 36.8 + 13.4$

$\boldsymbol{L = 50.2}$

Similar Problems (Solve Your Self)

Eg. 8.3: The range of a set of data is 13.67 and the largest value is 70.08. Find the smallest value. **(PTA-4)** 2M

3. Calculate the range of the following data.

Income	400-450	450-500	500-550	550-600	600-650
Number of workers	8	12	30	21	6

Given: 2M

Income	No.of workers
400-450	8
450-500	12
500-550	30
550-600	21
600-650	6

$L = 650, S = 400, R = ?$

$R = L - S$

$= 650 - 400 = \mathbf{250}$

5. Find the variance and standard deviation of the wages of 9 workers given below:

₹ 310, ₹ 290, ₹ 320, ₹ 280, ₹ 300, ₹ 290, ₹ 320, ₹ 310, ₹ 280. 5M

Given: ₹ 310, ₹ 290, ₹ 320, ₹ 280, ₹ 300, ₹ 290, ₹ 320, ₹ 310, ₹ 280.

$$\bar{X} = \frac{310+290+320+280+300+290+320+310+280}{9}$$

$$\bar{X} = \frac{2700}{9} = 300$$

x_i	$d = (x_i - \bar{X})$ $(X_i - 300)$	d^2
280	-20	400
280	-20	400
290	-10	100
290	-10	100
300	0	0
310	10	100
310	10	100
320	20	400
320	20	400
	$\Sigma d = 0$	$\Sigma d^2 = 2000$

$$\sigma = \sqrt{\frac{2000}{9}}$$

$= \sqrt{222.2}$

$= 14.91$

$\sigma^2 = 222.22$

$\therefore$ Variance $= 222.22$

Standard deviation $= \sqrt{222.2} = \mathbf{14.91}$

Similar Problems (Solve Your Self) 2M

Eg. 8.2: Find the range of the following distribution. **(PTA-5)**

Age (in years)	16 - 18	18 - 20	20 - 22	22 - 24	24 - 26	26 – 28
Number of students	0	4	6	8	2	2

Similar Problems (Solve Your Self) 5M

4. A teacher asked the students to complete 60 pages of a record note book. Eight students have completed only 32,35,37,30,33,36, 35 and 37 pages. Find the standard deviation of the pages completed by them.

Eg.8.4: The number of televisions sold in each day of a week are 13, 8, 4, 9, 7, 12, 10. Find its standard deviation.

Eg. 8.5: The amount of rainfall in a particular season for 6 days are given as 17.8 cm, 19.2 cm, 16.3 cm, 12.5 cm, 12.8 cm and 11.4 cm. Find its standard deviation.

Eg. 8.6: The marks scored by 10 students in a class test are 25, 29, 30, 33, 35, 37, 38, 40, 44, 48. Find the standard deviation.

Eg. 8.7: The amount that the children have spent for purchasing some eatables in one day trip of a school are 5, 10, 15, 20, 25, 30, 35, 40. Using step deviation method, find the standard deviation of the amount they have spent.

6. A wall clock strikes the bell once at 1 o' clock, 2 times at 2 o' clock, 3 times at 3 o' clock and so on. How many times will it strike in a particular day. Find the standard deviation of the number of strikes the bell make a day. 5M

A wall clock strikes. The bell,

Once at $1'o$ clock,

2 times at $2'o$ clock,

3 times at $3'o$ clock and so on.

$\therefore$ The series

$(1 + 2 + 3 + 4 \ldots\ldots + 12)(1 + 2 + \cdots + 12)$

$= \frac{12\times13}{2} = 78$

$\Rightarrow 78 \times 2 = 156$

The clock **156** times strike in a day.

The S.D of first $'n'$ natural numbers is

$\sigma = \sqrt{\frac{n^2-1}{12}}, n = 12$

$= \sqrt{\frac{12^2-1}{12}}$

$= \sqrt{\frac{144-1}{12}}$

$= \sqrt{\frac{143}{12}}$

$= \sqrt{11.9}$

$= 3.44$

From 1 to 12'o clock $\sigma = 3.44$

Again 1 to 12'o clock $\sigma = 3.44$

$\sigma = 3.44 \times 2 = 6.88$

$\therefore$ The standard deviation of the number of strikes the bell make a day.

$\boldsymbol{\sigma = 6.9}$

7. Find the standard deviation of first 21 natural numbers. JUN-23, PTA-6 2M

Standard deviation of first 21 natural numbers.

$\sigma = \sqrt{\frac{n^2-1}{12}}; \ n = 21$

$= \sqrt{\frac{(21)^2-1}{12}}$

$= \sqrt{\frac{441-1}{12}}$

$= \sqrt{\frac{440}{12}}$

$= \sqrt{36.67}$

$\sigma = 6.049$

$\boldsymbol{\sigma = 6.05}$

Similar Problems (Solve Your Self) 5M

Eg. 8.10: Find the mean and variance of the first n natural numbers.

8. If the standard deviation of a data is 4.5 and if each value of the data is decreased by 5, then find the new standard deviation.

$\sigma = 4.5$, decreased by 5 for each value.

New $\sigma =$? 2M

The standard deviation will not change when we subtract some fixed constant to all the values.

New $\boldsymbol{\sigma = 4.5}$

Similar Problems (Solve Your Self)

Eg. 8.8: Find the standard deviation of the following data 7, 4, 8, 10, 11. Add 3 to all the values then find the standard deviation for the new values. 2M

9. If the standard deviation of a data is 3.6 and each value of the data is divided by 3, then find the new variance and new standard deviation. 2M

$\sigma = 3.6, \quad \div\, by\ 3$

New variance & new $\sigma = ?$

New $\sigma = \frac{3.6}{3} = 1.2,$

variance $\sigma^2 = (1.2)^2$

$= 1.44$

$\therefore$ new variance $= 1.44$

New $\boldsymbol{\sigma = 1.2}$

Similar Problems (Solve Your Self) 2M

Eg. 8.9: Find the standard deviation of the data 2, 3, 5, 7, 8. Multiply each data by 4. Find the standard deviation of the new values.

10. The rainfall recorded in various places of five districts in a week are given below. Find its standard deviation. 5M

Rainfall (in mm)	45	50	55	60	65	70
Number of places	5	13	4	9	5	4

x	f_i	$d = x_i - A = x_i - 60$	d_i^2	$f_i d_i$	$f_i d_i^2$
45	5	-15	225	-75	1125
50	13	-10	100	-130	1300
55	4	-5	25	-20	100
60	9	0	0	0	0
65	5	5	25	25	125
70	4	10	100	40	400
	$N = 40$			$\Sigma f_i d_i = -160$	$\Sigma f_i d_i^2 = 3050$

$$\sigma = \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

$$= \sqrt{\frac{3050}{40} - \left(\frac{-160}{40}\right)^2}$$

$$= \sqrt{76.25 - (-4)^2}$$

$$= \sqrt{76.25 - 16}$$

$$= \sqrt{60.25}$$

$$= 7.76$$

$\boldsymbol{\sigma \cong 7.76}$

Similar Problems (Solve Your Self) 5M

Eg. 8.11: 48 students were asked to write the total number of hours per week they spent on watching television. With this information find the standard deviation of hours spent for watching television.

x	6	7	8	9	10	11	12
f	3	6	9	13	8	5	4

Eg. 8.12: The marks scored by the students in a slip test are given below. Find the standard deviation of their marks. **(PTA-1)**

x	4	6	8	10	12
f	7	3	5	9	5

13. The time taken by 50 students to complete a 100 meter race are given below. Find its standard deviation.

Time taken (sec)	8.5 -9.5	9.5 -10.5	10.5 -11.5	11.5 -12.5	12.5 -13.5
Number of students	6	8	17	10	9

PTA-5

5M

$A = 11, c = 1$

Time taken (sec)	Mid value x_i	No.of students f_i	$d = X_i - A$	d^2	$f_i d_i$	$f_i d_i^2$
8.5 - 9.5	9	6	-2	4	-12	24
9.5 - 10.5	10	8	-1	1	-8	8
10.5 - 11.5	11	17	0	0	0	0
11.5 - 12.5	12	10	1	1	10	10
12.5 - 13.5	13	9	2	4	18	36
		$N = 50$	$\Sigma d_i = 0$		$\Sigma f_i d_i = -8$	$\Sigma f_i d_i^2 = 78$

$$\sigma = c \times \sqrt{\frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2}$$

$$= 1 \times \sqrt{\frac{78}{50} - \left(\frac{-8}{50}\right)^2}$$

$$= \sqrt{1.56 - (-0.16)^2}$$

$$= \sqrt{1.56 - 0.0256}$$

$$= \sqrt{1.5344}$$

$$= 1.238$$

$$\boldsymbol{\sigma \cong 1.24}$$

Similar Problems (Solve Your Self)

5M

11. In a study about viral fever, the number of people affected in a town were noted as

Age in years	0 - 10	10 - 20	20 -30	30 - 40	40 - 50	50 - 60	60 - 70
Number of people affected	3	5	16	18	12	7	4

Find its standard deviation.

12. The measurements of the diameters (in cms) of the plates prepared in a factory are given below. Find its standard deviation.

Diameter (cm)	21 - 24	25 - 28	29 - 32	33 - 36	37 - 40	41- 44
Number of plates	15	18	20	16	8	7

Eg. 8.13: Marks of the students in a particular subject of a class are given below. Find its standard deviation.

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Number of students	8	12	17	14	9	7	4

UE-2: The diameter of circles (in mm) drawn in a design are given below.

Diameters	33-36	37-40	41-44	45-48	49-52
Number of circles	15	17	21	22	25

Calculate the standard deviation.

14. For a group of 100 candidates the mean and standard deviation of their marks were found to be 60 and 15 respectively. Later on it was found that the scores 45 and 72 were wrongly entered as 40 and 27. Find the correct mean and standard deviation.

5M

$n = 100,\ \bar{X} = 60,\ \sigma = 15$

$\bar{X} = \frac{\Sigma x}{n} \Rightarrow 60 = \frac{\Sigma x}{100}$

$\Sigma x = 60 \times 100 \quad = 6000$

Wrong observation value $= 40 + 27 = 67$

Correct observation value $= 45 + 72 = 117$

Correct total $= 6000 - 67 + 117 = 6050$

Correct mean $\bar{X} = \frac{6050}{100} = 60.5$

$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$

Incorrect value $\sigma = 15 = \sqrt{\frac{\Sigma x^2}{100} - (60)^2}$

$15^2 = \frac{\Sigma x^2}{100} - 3600$

$225 + 3600 = \frac{\Sigma x^2}{100} \quad \Rightarrow \quad 3825 = \frac{\Sigma x^2}{100}$

$\Sigma x^2 = 382500$

Incorrect $\Sigma x^2 = 382500$

Correct value of Σx^2

$= 382500 - 40^2 - 27^2 + 45^2 + 72^2$

$= 382500 - 1600 - 729 + 2025 + 5184$

Correct $\Sigma x^2 = 387380$

Correct $\sigma = \sqrt{\frac{387380}{100} - (60.5)^2}$

$= \sqrt{3873.8 - 3660.25}$

$= \sqrt{213.55}$

$\boldsymbol{\sigma \cong 14.61}$

Similar Problems (Solve Your Self)

Eg. 8.14: The mean and standard deviation of 15 observations are found to be 10 and 5 respectively. On rechecking it was found that one of the observation with value 8 was incorrect. Calculate the correct mean and standard deviation if the correct observation value was 23? (5M)

15. The mean and variance of seven observations are 8 and 16 respectively. If five of these are $2, 4, 10, 12$ *and* 14, then find the remaining two observations. (5M)

The mean and variance of 7 observations $\overline{X} = 8$, $\sigma^2 = 16$

Five of these are 2,4,10,12, &14

Let X and Y be the remaining two observations, then mean = 8

$\frac{2+4+10+12+14+X+Y}{7} = 8$

$42 + x + y = 56$

$x + y = 56 - 42$

$x + y = 14$ ....(1)

Variance = 16 ,

$\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2 = \frac{\Sigma x^2}{n} - (\bar{X})^2 = 16$

$\frac{1}{7}(2^2 + 4^2 + 10^2 + 12^2 + 14^2 + x^2 + y^2) - (\text{Mean})^2 = 16$

$\frac{4+16+100+144+196+x^2+y^2}{7} - (8)^2 = 16$

$\frac{460+x^2+y^2}{7} - 64 = 16$

$\frac{460+x^2+y^2}{7} = 16 + 64$

$\frac{460+x^2+y^2}{7} = 80$

$460 + x^2 + y^2 = 80 \times 7$

$x^2 + y^2 = 560 - 460$

$x^2 + y^2 = 100$ ....(2)

$(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$

Using (1) & (2)

$196 + (x - y)^2 = 2(100)$

$(x - y)^2 = 200 - 196$

$(x - y)^2 = 4$

$(x - y) = \pm 2$

If $x - y = 2$, then $x + y = 14$ &

$x - y = 2 \Rightarrow x = 2 + y$

$2 + y + y = 14$

$2 + 2y = 14$

$2y = 14 - 2$

$y = \frac{12}{2} = 6$

Sub $y = 6$ in $x - y = 2$

$x = 2 + 6$

$x = 8$

$\therefore x = 8, y = 6$

If $x - y = -2$, then $x + y = 14$

$x = -2 + y \Rightarrow -2 + y + y = 14$

$-2 + 2y = 14$

$2y = 14 + 2$

$y = \frac{16}{2} = 8$

Sub $y = 8$ in $x - y = -2$

$x = -2 + 8$

$x = 6$

$\therefore$ **The remaining 2 observations are 6 & 8**

Creative Questions

1. The scores of a cricketer in 7 matches are 70, 80, 60, 50, 40, 90, 95. Find the Standard Deviation. SEP-20 (5M)

x	d	d^2
40	-30	900
50	-20	400
60	-10	100
70	0	0
80	10	100
90	20	400
95	25	625
	$\sum d = -1$	$\sum d^2 = 2525$

$\sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2} = \sqrt{\frac{2525}{7} - \left(\frac{-1}{7}\right)^2}$

$= \sqrt{(360.71) - 0.0204}$

$= 18.99 \cong 19$

$\therefore \sigma \cong 19$

2. The standard deviation of 20 observations is $\sqrt{6}$. Is each observation is multiplied by 3, find the standard deviation and variance of the resulting observations PTA-1 (2M)

Standard deviation= $\sqrt{6}$

SD of new data= $3\times \sqrt{6}= 3\sqrt{6}$

Variance of new data = $(3\sqrt{6})^2$

$= 9 \times 6 = 54$

Exercise 8.2

1. The standard deviation and mean of a data are 6.5 and 12.5 respectively. Find the coefficient of variation. (2M)

$\sigma = 6.5,\ \bar{X} = 12.5,\ C.V = ?$

$$\text{Coefficient of variation} = \frac{\sigma}{\bar{X}} \times 100\%$$
$$= \frac{6.5}{12.5} \times 100\%$$
$$= 0.52 \times 100\%$$
$$\boldsymbol{C.V = 52\%}$$

Similar Problems (Solve Your Self) (2M)

Eg. 8.15: The mean of a data is 25.6 and its coefficient of variation is 18.75. Find the standard deviation.(PTA-3)

2. The standard deviation and coefficient of variation of a data are 1.2 & 25.6 respectively. Find the value of mean.
3. If the mean and coefficient of variation of a data are 15 & 48 respectively, then find the value of standard deviation.

5. Find the coefficient of variation of $24, 26, 33, 37, 29, 31$. JUN-23 (5M)

24,26,33,37,29,31, C.V= ?

$$\bar{X} = \frac{24+26+33+37+29+31}{6} = \frac{180}{6} = 30$$

x	$d = x - \bar{x}$ $d = x - 30$	d^2
24	-6	36
26	-4	16
33	3	9
37	7	49
29	-1	1
31	-1	1
		$\Sigma d^2 = 112$

$$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{112}{6}} = \sqrt{18.66} = 4.3197$$
$$C.V = \frac{\sigma}{\bar{X}} \times 100\% = \frac{4.321}{30} \times 100\%$$
$$= \frac{432.1}{30} = 14.399$$
$$\boldsymbol{C.V = 14.4\%}$$

Similar Problems (Solve Your Self) (2M)

4. If $n = 5, \bar{x} = 6, \Sigma x^2 = 765,$ then calculate the coefficient of variation.
6. The time taken (in minutes) to complete a homework by 8 students in a day are given by 38,40,47,44,46,43,49,53. Find the coefficient of variation. (5M)

8. The mean and standard deviation of marks obtained by 40 students of a class in three subjects Mathematics, Science and Social Science are given below.

Subject	Mean	SD
Mathematics	**56**	**12**
Science	**65**	**14**
Social science	**60**	**10**

Which of the three subjects shows more consistent and which shows less consistent in marks? (5M)

<u>**Mathematics**</u>

$n = 40,\ \bar{X} = 56,\ \sigma = 12$

$$C.V = \frac{\sigma}{\bar{X}} \times 100\%$$
$$= \frac{12}{56} \times 100\% = \frac{1200}{56} = \mathbf{21.42}\ \%$$

<u>**Science**</u>

$\bar{X} = 65, \sigma = 14$

$$C.V = \frac{14}{65} \times 100\% = \frac{1400}{65} = \mathbf{21.53}\ \%$$

<u>**Social**</u>

$\bar{X} = 60, \sigma = 10$

$$C.V = \frac{10}{60} \times 100 = \frac{1000}{60} = \mathbf{16.66}\ \%$$

The subject <u>**Social**</u> shows highest variation in marks. The subject <u>**Science**</u> shows lowest variation in marks.

Similar Problems (Solve Your Self) (5M)

7. The total marks scored by two students Sathya and Vidhya in 5 subjects are 460 and 480 with standard deviation 4.6 and 2.4 respectively. Who is more consistent in performance?

Eg. 8.16: The following table gives the values of mean and variance of heights and weights of the 10th standard students of a school.

	Height	Weight
Mean	155 cm	46.50 kg
Variance	72.25 cm^2	28.09 kg

Which is more varying than the other? **(PTA-5)**

Creative Questions

1. The temperature of two cities A and B in a winter season are given below. 5M PTA-2

Temperature of city A (in degree Celsius)	18	20	22	24	26
Temperature of city B (in degree Celsius)	11	14	15	17	18

Find which city is more consistent in temperature changes?

City A

$\bar{X} = \frac{18+20+22+24+26}{5} = \frac{110}{5} = 22$

x	$d = x - \bar{x}$	d^2
18	-4	16
20	-2	4
22	0	0
24	2	4
26	4	16
		$\Sigma d^2 = 40$

$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{40}{5}} = \sqrt{8}$

$\simeq 2.8$

$C.V = \frac{\sigma}{\bar{x}} \times 100\%$

$= \frac{2.8}{22} \times 100\%$

$= \frac{280}{22} = 12.7$

$C.V = 12.7\%$

City B

$\bar{X} = \frac{11+14+15+17+18}{5} = 15$

x	$d = x - \bar{x}$	d^2
11	-4	16
14	-1	1
15	0	0
17	2	4
18	3	9
		$\Sigma d^2 = 30$

$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{30}{5}} = \sqrt{6}$

$\simeq 2.4$

$= \frac{2.4}{15} \times 100\%$

$= \frac{240.0}{15}$

$= 16$

$C.V = 16\%$

City A is <u>**more consistent**</u> in temperature changes.

2. Find the co-efficient of variation of the data $18, 20, 15, 12, 25$. PTA-3 5M

$\bar{x} = \frac{\Sigma x}{n}$

$\bar{x} = \frac{18+20+15+12+25}{5} = \frac{90}{5} = 18$

x	$d = x - \bar{x}$	d^2
18	0	0
20	2	4
15	-3	9
12	-6	36
25	7	49
		$\sum d^2 = 98$

$\sigma = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{98}{5}} = \sqrt{19.6}$

$= 4.427$

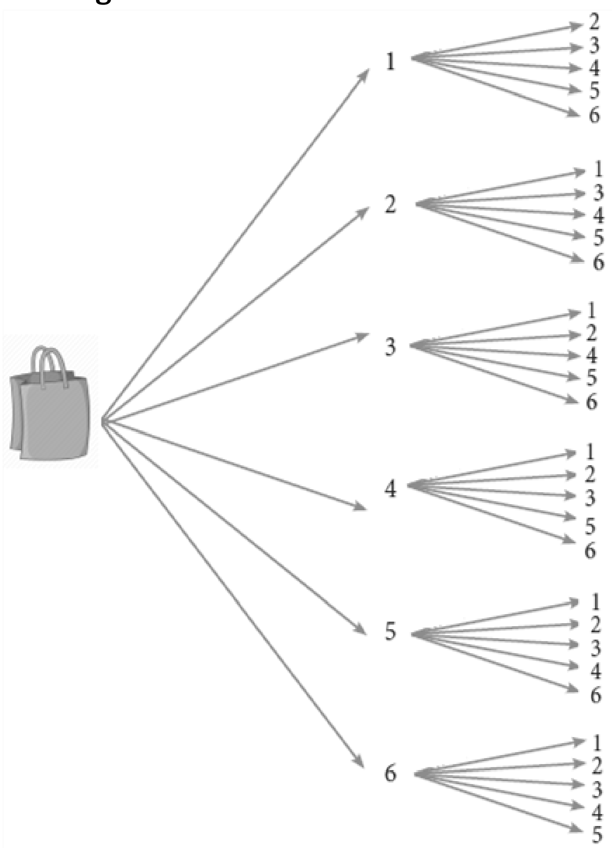
C.V $= \frac{\sigma}{\bar{x}} \times 100\% = \frac{4.427}{18} \times 100 = \frac{442.7}{18} = 24.59$

Exercise 8.3

2. Write the sample space for selecting two balls at a time from a bag containing 6 balls numbered 1 to 6 (using tree diagram). PTA-4

When we select two balls from a bag containing 6 balls numbered 1,2,3,4,5,6. 2M

Tree diagram:



Hence the sample space can be written as,

$$S = \{(1,2), (1,3), (1,4), (1,5), (1,6)$$
$$(2,1), (2,3), (2,4), (2,5), (2,6)$$
$$(3,1), (3,2), (3,4), (3,5), (3,6)$$
$$(4,1), (4,2), (4,3), (4,5), (4,6)$$
$$(5,1), (5,2), (5,3), (5,4), (5,6)$$
$$(6,1), (6,2), (6,3), (6,4), (6,5)\}$$

Similar Problems (Solve Your Self) 2M

1. Write the sample space for tossing three coins using tree diagram.

Eg. 8.17: Express the sample space for rolling two dice using tree diagram.

3. If A is an event of a random experiment such that $P(A) : P(\bar{A}) = 17 : 15$ and $n(S) = 640$ then find PTA-3

(i) $P(\bar{A})$ (ii) $n(A)$ 5M

$P(A) : P(\bar{A}) = 17 : 15,$

$n(S) = 640$

(i) $P(\bar{A}) = ?$

$$\frac{P(A)}{P(\bar{A})} = \frac{17}{15}$$

$$\frac{P(A)}{1-P(A)} = \frac{17}{15}$$

$$15P(A) = 17(1 - P(A))$$

$$15P(A) = 17 - 17P(A)$$

$$15P(A) + 17P(A) = 17$$

$$32P(A) = 17$$

$$P(A) = \frac{17}{32};$$

$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - \frac{17}{32}$$

$$P(\bar{A}) = \frac{32-17}{32}$$

$$P(\bar{A}) = \mathbf{\frac{15}{32}}$$

(ii) $n(A)$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A) \times n(S) = n(A)$$

$$n(A) = \frac{17}{32} \times 640$$

$$n(A) = 340$$

5. At a fete, cards bearing numbers 1 to 1000, one number on one card are put in a box. Each player selects one card at random and that card is not replaced. If the selected card has a perfect square number greater than 500, the player wins a prize. What is the probability that (i) the first player wins a prize (ii) the second player wins a prize, if the first has won? 5M

At a fete, cards bearing numbers from 1 *to* 1000

(i.e) $n(S) = 1000$

(i) Let A be the selected card has a perfect square number greater than 500.

$A = \{529, 576, 625, 676, 729, 784, 841, 900, 961\}$

$n(A) = 9$

$\therefore$ The probability that the first player wins a prize

$P(A) = \frac{n(A)}{n(S)} = \mathbf{\frac{9}{1000}}$

(ii) Let B be the second player wins a prize,

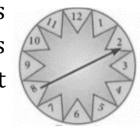
$n(B) = 8, \; n(S) = 999$

$\therefore$ The probability of the second player wins a prize

$P(B) = \frac{n(B)}{n(S)} = \mathbf{\frac{8}{999}}$

Similar Problems (Solve Your Self)

Eg. 8.24: A game of chance consists of spinning an arrow which is equally likely to come to rest pointing to one of the numbers 1, 2, 3, …. 12. What is the probability that it will point to (i) 7 (ii) a prime number (iii) a composite number? 5M



7. Two unbiased dice are rolled once. Find the probability of getting APR-23, SEP-20, JUL-22

(i) a doublet (equal numbers on both dice) (ii) the product as a prime number

(iii) the sum as a prime number (iv) the sum as 1 5M

Two unbiased dice are rolled once.

$$S = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6)\\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6)\\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6)\\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6)\\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$$

$n(S) = 36$

(i) Let the A be event of getting a doublet.

$A = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$

$n(A) = 6$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \mathbf{\frac{1}{6}}$

(ii) Let B be the event of getting the product as a prime number.

$B = \{(1,2), (1,3), (1,5), (2,1), (3,1), (5,1)\}$

$n(B) = 6$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{6}{36} = \mathbf{\frac{1}{6}}$

(iii) Let C be the event of getting the sum as a prime number.

$$C = \{(1,1), (1,2), (1,4), (1,6), (2,1),\\ (2,3), (2,5), (3,2), (3,4), (4,1),\\ (4,3), (5,2), (5,6), (6,1), (6,5)\}$$

$n(C) = 15$

$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{15}{36} = \mathbf{\frac{5}{12}}$

(iv) Let D be the event of getting the sum as 1.

$D = \{ \; \}$

$n(D) = 0$

$\therefore P(D) = \frac{n(D)}{n(S)} = \frac{0}{36} = 0$

$\therefore P(D) = \mathbf{0}$

Similar Problems (Solve Your Self)

Eg. 8.19: Two dice are rolled. Find the probability that the sum of outcomes is (i) equal to 4 (ii) greater than 10 (iii) less than 13 **(SEP-21)** 5M

Eg. 8.22: A die is rolled and a coin is tossed simultaneously. Find the probability that the die shows an odd number and the coin shows a head. **(JUN-23, SEP-21)** 2M

8. Three fair coins are tossed together. Find the probability of getting 5M
(i) all heads (ii) atleast one tail PTA-5 **(iii) atmost one head** PTA-5 **(iv) atmost two tails**

Three fair coins are tossed together.

$S = \{HHH, HHT, HTH, HTT, TTT, TTH, THT, THH\}$

$n(S) = 8.$

i) Let A be the event of getting all heads.

$A = \{HHH\}; n(A) = 1$

$P(A) = \frac{n(A)}{n(S)} = \mathbf{\frac{1}{8}}$

ii) Let B be the event of getting atleast one tail.

$B = \{HHT, HTH, HTT, TTT, TTH, THT, THH\}$

$n(B) = 7$

$\therefore P(B) = \frac{n(B)}{n(S)} = \mathbf{\frac{7}{8}}$

iii) Let C be the event of getting atmost one head.

$C = \{HTT, TTH, THT, TTT\}$

$n(C) = 4$

$P(C) = \frac{n(C)}{n(S)} = \mathbf{\frac{4}{8}}$

$\therefore P(C) = \frac{1}{2}$

iv) Let D be the event of getting atmost 2 tails.

$D = \{HHH, HHT, HTH, HTT, TTH, THT, THH\}$

n (D)=7

$\therefore P(D) = \frac{n(D)}{n(S)} = \mathbf{\frac{7}{8}}$

Similar Problems (Solve Your Self)

4. A coin is tossed thrice. What is the probability of getting two consecutive tails? 2M

13. In a game, the entry fee is ₹150. The game consists of tossing a coin 3 times. Dhana bought a ticket for entry. If one or two heads show, she gets her entry fee back. If she throws 3 heads, she receives double the entry fees. Otherwise she will lose. Find the probability that she (i) gets double entry fee (ii) just gets her entry fee (iii) loses the entry fee. 5M

Eg. 8.20: Two coins are tossed together. What is the probability of getting different faces on the coins? **(MAY-22)** 2M

9. A bag contains 5 red balls, 6 white balls, 7 green balls, 8 black balls. One ball is drawn at random from the bag. Find the probability that the ball drawn is. JUL-22 5M
(i) white (ii) black or red (iii) not white (iv) neither white nor black

$n(R) = 5, n(W) = 6, n(G) = 7, n(B) = 8 \quad n(S) = 5 + 6 + 7 + 8 \quad = 26$

i) Let A be the event of drawn white ball $n(A) = 6$

$P(A) = \frac{n(A)}{n(S)}$

$P(A) = \frac{6}{26} = \mathbf{\frac{3}{13}}$

ii) Let B & R be the event of drawn black or red ball.

$P(B) = \frac{8}{26}$

$P(R) = \frac{5}{26}$

$P(B \cup R) = P(B) + P(R) \; = \frac{8}{26} + \frac{5}{26}$

$\therefore \; P(B \cup R) = \frac{13}{26} = \mathbf{\frac{1}{2}}$

iii) Let $\bar{A}$ be the event of getting not white ball.

$P(A) = \frac{3}{13}$

$P(A) + P(\bar{A}) = 1$

$P(\bar{A}) = 1 - P(A)$

$= 1 - \frac{3}{13} = \frac{13-3}{13}$

$\therefore P(\bar{A}) = \frac{10}{13}$

iv) Let C be the event of neither white nor black.

$n(C) = 26 - (6 + 8) \quad = 26 - 14 \quad = 12$

$P(\text{neither white not black}) = \mathrm{P}(C) = \frac{n(C)}{n(S)} = \frac{12}{26} = \mathbf{\frac{6}{13}}$

Similar Problems (Solve Your Self)

6. A bag contains 12 blue balls and x red balls. If one ball is drawn at random (i) what is the probability that it will be a red ball? (ii) If 8 more red balls are put in the bag, and if the probability of drawing a red ball will be twice that of the probability in (i), then find x. 5M

Eg. 8.18: A bag contains 5 blue balls and 4 green balls. A ball is drawn at random from the bag. Find the probability that the ball drawn is (i) blue (ii) not blue. 5M

Eg. 8.23: A bag contain 6 green balls, some black and red balls. Number of black balls is as twice as the number of red balls. Probability of getting a green ball is thrice the probability of getting a red ball. Find (i) number of black balls (ii) total number of balls. 5M

UE-10. A bag contains 5 white and some black balls. If the probability of drawing a black ball from the bag is twice the probability of drawing a white ball then find the number of black balls. 2M

10. In a box there are 20 non-defective and some defective bulbs. If the probability that a bulb selected at random from the box found to be defective is $\frac{3}{8}$ then, find the number of defective bulbs. (2M)

In a box, $n(\text{non defective bulbs}) = 20$

$n(\text{defective}) = x$

$P(D) = \frac{3}{8}; n(D) = ?$

$P(\bar{D}) = 1 - \frac{3}{8} \Rightarrow \frac{8-3}{8} = \frac{5}{8}$

$P(\text{non defective bulbs}) = \frac{20}{x+20} = \frac{20}{x+20} = \frac{5}{8}$

$5x + 100 = 160$

$5x = 160 - 100$

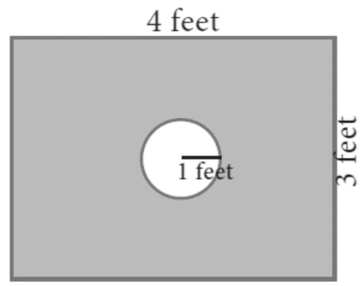
$5x = 60$

$x = \frac{60}{5}$

$x = \mathbf{12}$

$\therefore$ The number of defective bulbs $= 12$

11. Some boys are playing a game, in which the stone thrown by them landing in a circular region (given in the figure) is considered as win and landing other than the circular region is considered as loss. What is the probability to win the game? (5M)



Area of the rectangle $(l \times b) = 4 \times 3 = 12\ feet$

$n(S) = 12\ feet$

Let A be the event of success in the game.

$n(A) = \text{circular region}$

$ie\ n(A) = \pi r^2 \Rightarrow r = 1$ feet

$\Rightarrow \pi \times 1^2 = \pi$

$\therefore$ The probability of success in the game

$P(A) = \frac{n(A)}{n(S)} = \frac{\pi}{12} = \frac{3.14}{12} = \frac{3.14 \times 100}{12 \times 100} = \frac{314}{1200} = \mathbf{\frac{157}{600}}$

12. Two customers Priya and Amuthan are visiting a particular shop in the same week (Monday to Saturday). Each is equally likely to visit the shop on any one day as on another day. What is the probability that both will visit the shop on. (5M)

(i) the same day (ii) different days (iii) consecutive days?

$S = \{$(mon, mon), (mon, tues), (mon, wed), (mon, thurs), (mon, fri), (mon, sat)
(tues, tues), (tues, mon), (tues, wed), (tues, thurs), (tues, frid), (tues, sat)
(wed, wed), (wed, mon), (wed, tues), (wed, thurs), (wed, fri), (wed, sat)
(thurs, thurs), (thurs, mon), (thurs, tues), (thurs, wed), (thurs, fri), (thurs, sat)
(fri, fri), (fri, mon), (fri, tues), (fri, wed), (fri, thurs), (fri, sat)
(sat, sat), (sat, mon), (sat, tues), (sat, wed), (sat, thurs), (sat, fri)$\}$

$n(S) = 36$

i) Let A be the event of both will visit the shop on the same day.

$A = \{$(mon, mon), (tues, tues), (wed, wed), (thurs, thurs), (frid, fri), (sat, sat)$\}$

$n(A) = 6$

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \mathbf{\frac{1}{6}}$

ii) Let B be the event of both will visit the shop on different days.

$B = \{$different days except same days ie (mon, tues), (mon, wed) …… (sat, fri)$\}$

$n(B) = 30$

$P(B) = \frac{30}{36} = \mathbf{\frac{5}{6}}$

$\bar{A}$ means considered as different days.

> **Similar Problems** (Solve Your Self) (2M)
> **Eg. 8.21:** What is the probability that a leap year selected at random will contain 53 Saturdays.

iii) Let C be the event of both will visit the shop on consecutive days.

$C = \{$(Mon, Tue), (Tue, wed), (wed, thurs), (thurs, fri), (fri, sat)$\}$

$n(C) = 5$

$\therefore P(C) = \frac{n(C)}{n(S)} = \mathbf{\frac{5}{36}}$

Creative Questions

1. **Two dice, one blue and one grey, are thrown at the same time. Write down all the possible outcomes. What is the probability that the sum of the two numbers appearing on the top of the dice is (i) 8 (ii) 13 (iii) less than or equal to 12** MDL 5M

Total out comes $= 6 \times 6 = 36$

$n(S) = 36$

(i) sum of two dice 8

Let A be the event getting sum 8 of two dice

$A = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

$n(A) = 5$

$P(A) = \frac{n(A)}{n(S)} = \frac{5}{36}$

(ii) sum of two dice 13

Let B be the event getting sum 13 of two dice

$B = \{\ \}$

$n(B) = 0$

$P(B) = \frac{n(B)}{n(S)} = \frac{0}{36} = 0$

(iii) less than or equal to 12

Let C be the event getting sum of two dice is less than or equal to 12

$C = \{(1,1) \ldots (6,6)\}$

$n(C) = 36$

$P(C) = \frac{n(C)}{n(S)} = \frac{36}{36} = 1$

Exercise 8.4

1. **If $P(A) = \frac{2}{3}, P(B) = \frac{2}{5}, P(A \cup B) = \frac{1}{3}$ then find $P(A \cap B)$.** 5M PTA-1

$P(A) = \frac{2}{3}, P(B) = \frac{2}{5}, P(A \cup B) = \frac{1}{3}$

$P(A \cap B) = ?$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{1}{3} = \frac{2}{3} + \frac{2}{5} - P(A \cap B)$

$P(A \cap B) = \frac{2}{3} + \frac{2}{5} - \frac{1}{3} = \frac{1}{3} + \frac{2}{5} = \frac{5+6}{15} = \mathbf{\frac{11}{15}}$

> **Similar Problems** (Solve Your Self)
> **Eg. 8.25:** If $P(A) = 0.37$, $P(B) = 0.42$, $P(A \cap B) = 0.09$ then find $P(A \cup B)$. 2M

3. **If A and B are two mutually exclusive events of a random experiment and $P(not A) = 0.45$, $P(A \cup B) = 0.65$, then find $P(B)$.** 5M PTA-2

$P(not\ A) = 0.45 = P(\bar{A}), P(A \cup B) = 0.65$

$P(B) = ?$

$P(A) = 1 - P(\bar{A})$

$= 1 - 0.45$

$= 0.55$

$P(A \cup B) = P(A) + P(B)$

$0.65 = 0.55 + P(B)$

$0.65 - 0.55 = P(B)$

$0.10 = P(B)$

$\therefore P(B) = \mathbf{0.1}$

> **Similar Problems** (Solve Your Self)
> 2. A and B are two events such that, $P(A) = 0.42, P(B) = 0.48$, and $P(A \cap B) = 0.16$. find (i) $P(not\ A)$ (ii) $P(not\ B)$ (iii) $P(A\ or\ B)$ 5M
> 4. The probability that at least one of A and B occur is 0.6. If A and B occur simultaneously with probability 0.2, then find $P(\bar{A}) + P(\bar{B})$. 5M
> 5. The probability of happening of an event A is 0.5 and that of B is 0.3. If A and B are mutually exclusive events, then find the probability that neither A nor B happen. 5M
> **Eg. 8.28:** If A and B are two events such that $P(A) = \frac{1}{4}$, $P(B) = \frac{1}{2}$ and $P(A \text{ and } B) = \frac{1}{8}$, find (i) $P(A\ or\ B)$ (ii) $P(\text{not } A \text{ and not } B)$. 5M

6. **Two dice are rolled once. Find the probability of getting an even number on the first die or a total of face sum 8.** 5M JUN-23

Two dice are rolled,

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6)$$
$$(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$$
$$(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)$$
$$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$$
$$(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)$$
$$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$n(S) = 36$

(i) Let A be the even number on the first die,

$$A = \{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6)$$
$$(4,1),(4,2),(4,3),(4,4),(4,5),(4,6)$$
$$(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$n(A) = 18$

$\therefore P(A) = \frac{n(A)}{n(S)} = \mathbf{\frac{18}{36}}$

(ii) Let B be the event of total face sum 8.

$B = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}$

$n(B) = 5$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{5}{36}$

$A \cap B = \{(2,6),(4,4),(6,2)\}, n(A \cap B) = 3$

$\therefore P(A \cap B) = \frac{n(A\cap B)}{n(S)} = \frac{3}{36}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{23-3}{36} = \frac{20}{36} = \mathbf{\frac{5}{9}}$

Similar Problems (Solve Your Self) 5M

Eg. 8.27: Two dice are rolled together. Find the probability of getting a doublet or sum of faces as 4.

UE-8: If two dice are rolled, then find the probability of getting the product of face value 6 or the difference of face values 5.

7. **A box contains cards numbered $3, 5, 7, 9, \ldots .35, 37$. A card is drawn at random from the box. Find the probability that the drawn card have either multiples of 7 or a prime number.** 5M

A box contains card numbered

$\{3,5,7,9,11,13,15,17,19,21,23,25,27,29,31,33,35,37\}$

$\therefore n(S) = 18$

i) Let A be the event of multiples of 7

$A = \{7,21,35\}$

$n(A) = 3$

$\therefore P(A) = \mathbf{\frac{3}{18}}$

ii) Let B be the event of prime number.

$B = \{3,5,7,11,13,17,19,23,29,31,37\}$

$n(B) = 11$

$P(B) = \frac{n(B)}{n(S)}$

$= \frac{11}{18}$

$A \cap B = \{7\}, \; n(A \cap B) = 1,$

$P(A \cap B) = \frac{n(A\cap B)}{n(S)} = \frac{1}{18}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{3}{18} + \frac{11}{18} - \frac{1}{18}$

$= \frac{13}{18}$

Similar Problems
(Solve Your Self) 5M

Eg. 8.26: A flower is selected at random from a basket containing 80 yellow, 70 red and 50 white flowers. Find the probability of selecting a yellow or red flower?

Eg. 8.29: In an apartment, in selecting a house from door numbers 1 to 100 randomly, find the probability of getting the door number of the house to be an even number or a perfect square number or a perfect cube number.

9. The probability that a person will get an electrification contract is $\frac{3}{5}$ and the probability that he will not get plumbing contract is $\frac{5}{8}$. The probability of getting atleast one contract is $\frac{5}{7}$. What is the probability that he will get both? (5M)

The probability of get an electrification contract is $\frac{3}{5}$, $P(A) = \frac{3}{5}$

The probability that he will not get plumbing contract is $\frac{5}{8}$, $P(\bar{B}) = \frac{5}{8}$

The probability of getting atleast one contract is $\frac{5}{7}$.

$P(A \cup B) = \frac{5}{7}; P(A \cap B) = ?$

$P(\bar{B}) = 1 - P(B)$

$P(B) = 1 - P(\bar{B})$

$= 1 - \frac{5}{8} = \frac{8-5}{8} = \frac{3}{8}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{5}{7} = \frac{3}{5} + \frac{3}{8} - P(A \cap B)$

$P(A \cap B) = \frac{3}{5} + \frac{3}{8} - \frac{5}{7}$

$= \frac{(56\times3)+(35\times3)-(40\times5)}{280}$

$= \frac{168+105-200}{280}$

$P(A \cap B) = \frac{273-200}{280}$

$P(A \cap B) = \mathbf{\frac{73}{280}}$

Similar Problems (Solve Your Self) (5M)

Eg. 8.30: In a class of 50 students, 28 opted for NCC, 30 opted for NSS and 18 opted both NCC and NSS. One of the students is selected at random. Find the probability that

(i) The student opted for NCC but not NSS.

(ii) The sutdent opted for NSS but not NCC.

(iii) The student opted for exactly one of them.

(PTA-1,4, MAY-22)

Eg. 8.31: A and B are two candidates seeking admission to IIT. The probability that A getting selected is 0.5 and the probability that both A and B getting selected is 0.3. Prove that the probability of B being selected is atmost 0.8. **(APR-23, PTA-6)**

UE-11: The probability that a student will pass the final examination in both English and Tamil is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Tamil examination?

10. In a town of 8000 people, 1300 are over 50 years and 3000 are females. It is known that 30% of the females are over 50 years. What is the probability that a chosen individual from the town is either a female or over 50 years? (5M)

$n(S) = 8000$

Let A be the event of choosing female

$\therefore P(A) = \frac{n(A)}{n(S)}$

$= \frac{3000}{8000}$

$= \frac{30}{80}$

Let B be the event of randomly chosen individual over 50 years old.

$n(B) = 1300$

$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{1300}{8000} = \frac{13}{80}$

$P(A \cap B) = P($An individual is female over 50 years old)

$= \frac{30\% of\ 3000}{8000} = \frac{\frac{30}{100}\times3000}{8000} = \frac{900}{8000} = \mathbf{\frac{9}{80}}$

$\therefore$ The required probability is

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{30}{80} + \frac{13}{80} - \frac{9}{80}$

$= \frac{30+13-9}{80}$

$= \frac{43-9}{80}$

$= \frac{34}{80}$

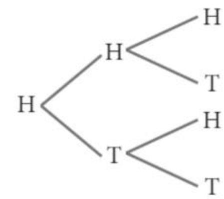
$P(A \cup B) = \mathbf{\frac{17}{40}}$

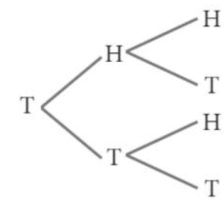
11. A coin is tossed thrice. Find the probability of getting exactly two heads or atleast one tail or consecutive two heads. 5M PTA-2, 6

A coin is tossed thrice,

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(S) = 8$





i) Let A be the event of getting exactly two heads.

$A = \{HHT, HTH, THH\}$

$n(A) = 3$

$\therefore P(A) = \frac{n(A)}{n(S)} = \mathbf{\frac{3}{8}}$

ii) Let B be the event of the getting atleast one tail.

$B = \{HHT, HTH, HTT, THH, THT, TTH, TTT\}$

$n(B) = 7$

$\therefore P(B) = \frac{n(B)}{n(S)} = \mathbf{\frac{7}{8}}$

iii) Let C be the event of getting consecutive two heads.

$C = \{HHH, HHT, THH\}$

$n(C) = 3$

$\therefore P(C) = \frac{n(C)}{n(S)} = \mathbf{\frac{3}{8}}$

$A \cap B = \{HHT, HTH, THH\}$

$n(A \cap B) = 3, \;\; P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{3}{8}$

$B \cap C = \{HHT, THH\}, \;\; n(B \cap C) = 2$

$\therefore P(B \cap C) = \frac{n(B \cap C)}{n(S)} = \frac{2}{8}$

$A \cap C = \{HHT, THH\}, \;\; n(A \cap C) = 2$

$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{2}{8}$

$A \cap B \cap C = \{HHT, THH\}$

$n(A \cap B \cap C) = 2,$

$P(A \cap B \cap C) = \frac{n(A \cap B \cap C)}{n(S)} = \frac{2}{8}$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + n(A \cap B \cap C)$

$P(A \cup B \cup C) = \cancel{\frac{3}{8}} + \frac{7}{8} + \frac{3}{8} - \cancel{\frac{3}{8}} - \frac{2}{8} - \cancel{\frac{2}{8}} + \cancel{\frac{2}{8}}$

$= \frac{3}{8} + \frac{7}{8} - \frac{2}{8}$

$= \frac{3}{8} + \frac{7}{8} - \frac{2}{8}$

$= \frac{10-2}{8}$

$= \frac{8}{8} = \mathbf{1}$

Similar Problems (Solve Your Self)

8. Three unbiased coins are tossed once. Find the probability of getting atmost 2 tails or atleast 2 heads. 5M

13. In a class of 35, students are numbered from 1 to 35. The ratio of boys to girls is 4 : 3. The roll numbers of students begin with boys and end with girls. Find the probability that a student selected is either a boy with prime roll number or a girl with composite roll number or an even number.

5M

Given: In a class 35 students

$n(S) = 35$

The ratio of boys to girls 4: 3

$4x + 3x = 35$

$7x = 35$

$x = \frac{35}{7} = 5$

$\therefore$ Boys $4x \Rightarrow 4(5) = 20$;

Girls $3x = 3(5) = 15$

Number of Boys $= 20$, Number of Girls $= 15$

i) Let A be the event of a student selected is either a boy with prime roll number;

$A = \{2,3,5,7,11,13,17,19\}, \quad n(A) = 8$

$\therefore P(A) = \frac{n(A)}{n(S)} = \mathbf{\frac{8}{35}}$

ii) Let B be the event of a girl with composite roll number,

$B = \{21,22,24,25,26,27,28,30,32,33,34,35\}, n(B) = 12$

$\therefore P(B) = \frac{n(B)}{n(S)} = \mathbf{\frac{12}{35}}$

iii) Let C be the event of an even roll number,

$C = \{2,4,6,8,10,12,14,16,18,20, 22,24,26,28,30,32,34\}, n(C) = 17$

$P(C) = \frac{n(C)}{n(S)} = \frac{17}{35}$

$A \cap B = \{\quad\}, \ n(A \cap B) = 0$

$P(A \cap B) = 0$

$B \cap C = \{22,24,26,28,30,32,34\}$

$n(B \cap C) = 7$

$\therefore P(B \cap C) = \frac{n(B \cap C)}{n(S)} = \frac{7}{35}$

$A \cap C = \{2\}, \ n(A \cap C) = 1$

$P(A \cap C) = \frac{n(A \cap C)}{n(S)} = \frac{1}{35}$

$A \cap B \cap C = \{\quad\}, n(A \cap B \cap C) = 0, \ P(A \cap B \cap C) = \frac{n(A \cap B \cap C)}{n(S)} = \frac{0}{35} = 0$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

$= \frac{8}{35} + \frac{12}{35} + \frac{17}{35} - 0 - \frac{7}{35} - \frac{1}{35} + 0$

$= \frac{8+12+17-7-1}{35}$

$= \frac{8-8+29}{35}$

$P(A \cup B \cup C) = \mathbf{\frac{29}{35}}$

Similar Problems (Solve Your Self)

5M

12. If A, B, C are any three events such that probability of B is twice as that of probability of A and probability of C is thrice as that of probability of A and if $P(A \cap B) = \frac{1}{6}$, $P(B \cap C) = \frac{1}{4}$, $P(A \cap C) = \frac{1}{8}$, $P(A \cup B \cup C) = \frac{9}{10}$, $P(A \cap B \cap C) = \frac{1}{15}$, then find $P(A), P(B)$ and $P(C)$?

Unit Exercise - 8

1. **The mean of the following frequency distribution is 62.8 and the sum of all frequencies is 50. Compute the missing frequencies f_1 and f_2.** 5M

Class interval	$0-20$	$20-40$	$40-60$	$60-80$	$80-100$	$100-120$
Frequency	5	f_1	10	f_2	7	8

$\bar{X} = 62.8,\ \Sigma f = 50, A = 50, h = 20$

$C.I$	Frequency f_i	Midvalue (x_i)	$u_i = \frac{x-A}{h}$	$f_i u_i$
$0-20$	5	10	-2	-10
$20-40$	f_1	30	-1	$-f_1$
$40-60$	10	50	0	0
$60-80$	f_2	70	1	f_2
$80-100$	7	90	2	14
$100-120$	8	110	3	24
	$N = \Sigma f_i = 30 + f_1 + f_2$			$\Sigma f_i u_i = 28 - f_i + f_2$

$N = \Sigma f_i = 50$

$30 + f_1 + f_2 = 50$

$f_1 + f_2 = 50 - 30$

$f_1 + f_2 = 20$ ------(1)

Mean $= 62.8$

$A + c\left\{\frac{1}{N}\Sigma f_i u_i\right\} = 62.8$

$50 + 20\left\{\frac{28 - f_1 + f_2}{50}\right\} = 62.8$ (here $c = 20$)

$50 + \frac{2}{5}[28 - f_1 + f_2] = 62.8 - 50$

$\frac{2}{5}[28 - f_1 + f_2] = 62.8 - 50$

$28 - f_1 + f_2 = 32.0$

$-f_1 + f_2 = 32 - 28$

$f_2 - f_1 = 4$ ------ (2)

Solving (1) & (2)

$$\begin{aligned} f_2 + \cancel{f_1} &= 20 \\ f_2 - \cancel{f_1} &= 4 \\ \hline 2f_2 &= 24 \end{aligned} \qquad \Rightarrow f_2 = \frac{24}{2} = 12$$

$\boldsymbol{f_2 = 12}$

Sub: $f_2 = 12$ in (1)

$f_1 + f_2 = 20$

$f_1 + 12 = 20$

$f_1 = 20 - 12;$

$\boldsymbol{f_1 = 8}$

3. **The frequency distribution is given below.**

x	k	$2k$	$3k$	$4k$	$5k$	$6k$
f	2	1	1	1	1	1

In the table, k is a positive integer, has a variance of 160. Determine the value of k. 5M

The frequency distribution:

x_i	f_i	$f_i x_i$	$f_i x_i^2$
k	2	$2k$	$2k^2$
$2k$	1	2k	$4k^2$
$3k$	1	$3k$	$9k^2$
$4k$	1	$4k$	$16k^2$
$5k$	1	$5k$	$25k^2$
$6k$	1	$6k$	$36k^2$
	$\Sigma f_i = 7$	$\Sigma f_i x_i = 22k$	$\Sigma f_i x_i^2 = 92k^2$

$N = 7, \quad \Sigma f_i x_i = 22k,\ \Sigma f_i x_i^2 = 92k^2$

Variance $= 160$, $\sigma^2 = 160$

$\sigma^2 = \frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2$

$160 = \frac{92k^2}{7} - \left(\frac{22k}{7}\right)^2$

$160 = \frac{92k^2}{7} - \frac{484k^2}{49}$

$160 = \frac{644k^2 - 484k^2}{49}$

$160 = \frac{160k^2}{49}$

$k^2 = 49 \Rightarrow \quad k = \sqrt{49} = \mathbf{7}$

4. **The standard deviation of some temperature data in degree Celsius (°C) is 5. If the data were converted into degree Farenheit (°F) then what is the variance?** 2M

$\sigma = 5°c$, variance $= 25$, $F = \frac{9}{5}c + 32$

Variance of $(aX + b) = a^2$ variance of X

$$\text{Variance of } F = \frac{9}{5}c + 32$$
$$= \left(\frac{9}{5}\right)^2 \times 25 = \frac{81}{25} \times 25 \Rightarrow \text{Variance of } F = \mathbf{81}$$

5. **If for a distribution, $\Sigma(x-5) = 3$, $\Sigma(x-5)^2 = 43$, and total number of observations is 18, find the mean and standard deviation.** 5M

$$\Sigma(x-5) = 3, \quad \Sigma(x-5)^2 = 43$$
$$N = 18$$
$$\Sigma(x-5) = 3$$
$$\Sigma x - \Sigma 5 = 3$$
$$\Sigma x - (18 \times 5) = 3$$
$$\Sigma x - 90 = 3$$
$$\Sigma x = 3 + 90$$
$$\Sigma x = 93$$
$$\bar{X} = \frac{\Sigma x}{n} = \frac{93}{18} = 5.166666 = 5.17$$
$$\Sigma(x-5)^2 = 43$$
$$\Sigma(x^2 - 10x + 25) = 43$$
$$\Sigma x^2 - \Sigma 10x + \Sigma 25 = 43$$
$$\Sigma x^2 - (10 \times 93) + (25 \times 18) = 43$$
$$\Sigma x^2 - 930 + 450 = 43$$
$$\Sigma x^2 - 480 = 43$$
$$\Sigma x^2 = 43 + 480$$
$$\Sigma x^2 = 523$$
$$\sigma = \sqrt{\frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2}$$
$$= \sqrt{\frac{523}{18} - \left(\frac{93}{18}\right)^2} = \sqrt{\frac{523}{18} - \frac{93 \times 93}{18 \times 18}}$$
$$= \sqrt{\frac{523 \times 18 - (93 \times 93)}{18 \times 18}} = \frac{1}{18}\sqrt{9414 - 8649}$$
$$= \frac{1}{18}\sqrt{765} = \frac{27.66}{18}$$
$$\sigma \simeq 1.53$$

6. **Prices of peanut packets in various places of two cities are given below. In which city, prices were more stable?** 5M

Prices in city A	**20**	**22**	**19**	**23**	**16**
Prices in city B	**10**	**20**	**18**	**12**	**15**

Prices in city A

$\bar{X} = \frac{20+22+19+23+16}{5} = \frac{100}{5} = 20$

$\bar{X} = 20$

x	$d = x - \bar{x}$ $x - 20$	d^2
20	0	0
22	2	4
19	-1	1
23	3	9
16	-4	16
		$\Sigma d^2 = 30$

$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{30}{5}} = \sqrt{6} = 2.44$

$\sigma \simeq 2.44$

Prices in city B

$\bar{X} = \frac{10+20+18+12+15}{5} = \frac{75}{5} = 15$

$\bar{X} = 15$

x	$d = x - \bar{x}$ $x - 15$	d^2
10	-5	25
20	5	25
18	3	9
12	-3	9
15	0	0
		$\Sigma d^2 = 68$

$\sigma = \sqrt{\frac{\Sigma d^2}{n}} = \sqrt{\frac{68}{5}} = \sqrt{13.6} = 3.68$

$\sigma \simeq 3.68$

City A $\Rightarrow \sigma \simeq 2.44$, City B $\Rightarrow \sigma \simeq 3.68$

∴ City A is more stable.

9. **In a two children family, find the probability that there is at least one girl in a family.** 2M

$S = \{BB, BG, GG, GB\}$

B →boys, G →girls

$n(S) = 4$

Let A be the event of at least one girl

$A = \{BG, GG, GB\}$

$n(A) = 3$

$P(A) = \frac{n(A)}{n(S)} = \frac{3}{4}$

$\therefore P(A) = \frac{3}{4}$

Note for Unit Exercise – 8

Q.No: 2 – Similar to Exercise 8.1- 13th Question

Q.No: 7 - Similar to Exercise 8.1 – 1st Question

Q.No: 8 - Similar to Exercise 8.4 – 6th Question

Q.No: 10 - Similar to Exercise 8.3 – 9th Question

Q.No: 11 - Similar to Exercise 8.4 – 10th Question

State and prove Addition Theorem of Probability 5M

Statement:

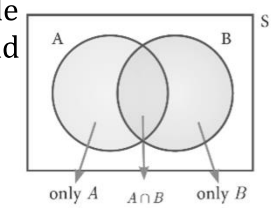
(i) If A and B are any two events then, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(ii) If A, B and C are any three events then,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$$

Proof:

(i) Let A and B be any two events of a random experiment with sample space S. From the Venn diagram, we have the events only A, $A \cap B$ and only B are mutually exclusive and their union is $A \cup B$



Therefore, $P(A \cup B) = P[(\text{only } A) \cup (A \cap B) \cup P(\text{only } B)]$

$= P(\text{only } A) + P(A \cap B) + P(\text{only } B)$

$= [P(A) - P(A \cap B)] + P(A \cap B) + [P(B) - P(A \cap B)]$

$= P(A) + P(B) - P(A \cap B)$

(ii) Let A, B, C are any three events of a random experiment with sample space S.

Let $D = B \cup C$

$P(A \cup B \cup C) = P(A \cup D)$

$= P(A) + P(D) - P(A \cap D)$

$= P(A) + P(B \cup C) - P[A \cap (B \cup C)]$

$= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)]$

$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P[(A \cap B) \cap (A \cap C)]$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P[A \cap B \cap C]$