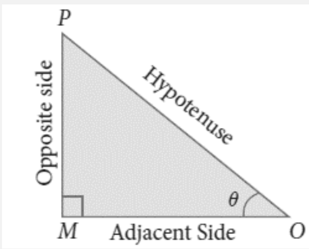
6. Trigonometry

Introduction for Exercise 6.1

Concept corner

Trigonometric Ratios:



Let $0^\circ < \theta < 90^\circ$

In right anlge triangle OMP,

$$\sin\theta = \frac{Opposite\ side}{Hypotenuse} = \frac{MP}{OP}$$

$$\cos\theta = \frac{Adjacent\ side}{Hypotenuse} = \frac{OM}{OP}$$

From the above two ratios we can obtain other four trigonometric ratios as follows.

$$tan\theta = \frac{sin\theta}{cos\theta}; \quad cot\theta = \frac{cos\theta}{sin\theta}; \quad cosec\theta = \frac{1}{sin\theta}; \quad sec\theta = \frac{1}{cos\theta}$$

Note: All right triangles with θ as one of the angle are similar. Hence the trigonometric ratios defined through such right angle triangles do not depend on the triangle chosen.

Complementary angle

$sin(90^\circ - \theta) = cos\theta$	$cos(90^\circ - \theta) = sin\theta$	$tan(90^\circ - \theta) = cot\theta$
$cosec(90^\circ - \theta) = sec\theta$	$sec(90^\circ - \theta) = cosec\theta$	$cot(90^\circ - \theta) = tan\theta$

Table of Trigonometric Ratios for $0^0, 30^0, 45^0, 60^0, 90^0$

Trigonometric Ratio \ θ	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined
$\text{cosec}\theta$	undefined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	undefined
$\cot\theta$	undefined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Note:

$(\sin\theta)^2 = \sin^2\theta$

$(\cos\theta)^2 = \cos^2\theta$

$(\tan\theta)^2 = \tan^2\theta$

$(\text{cosec}\,\theta)^2 = \text{cosec}^2\theta$

$(\sec\theta)^2 = \sec^2\theta$

$(\cot\theta)^2 = \cot^2\theta$

Trigonometric Identities:

Identity	Equal forms
$\sin^2\theta + \cos^2\theta = 1$	$\sin^2\theta = 1 - \cos^2\theta$ (or) $\cos^2\theta = 1 - \sin^2\theta$
$1 + \tan^2\theta = \sec^2\theta$	$\tan^2\theta = \sec^2\theta - 1$ (or) $\sec^2\theta - \tan^2\theta = 1$
$1 + \cot^2\theta = cosec^2\,\theta$	$\cot^2\theta = cosec^2\theta - 1$ (or) $cosec^2\theta - \cot^2\theta = 1$

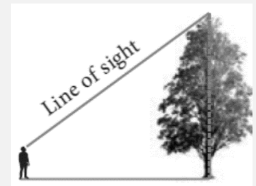
Note: Though the above identities are true for any angle θ, we will consider the six trigonometric ratios only for $0^\circ < \theta < 90^\circ$

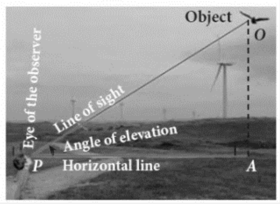
Introduction for Exercise 6.2

Concept corner

Line of sight:

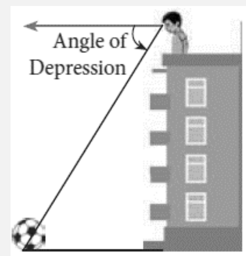
The line of sight is the line drawn from the eye of an observer to the point in the object viewed by the observer.





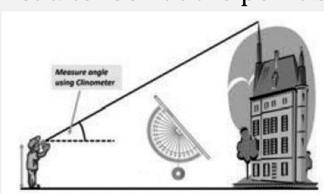
Angle of elevation:

The angle of elevation is an angle formed by the line of sight with the horizontal when the point being viewed is above the horizontal level. That is, the case when we raise our head to look at the object.



Angle of Depression:

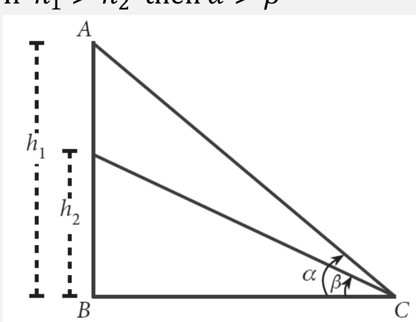
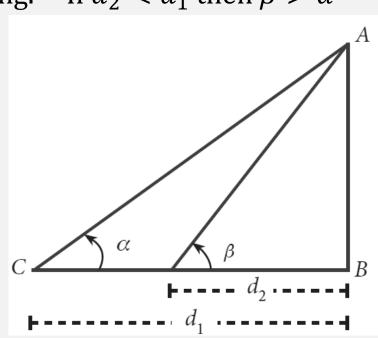
The angle of depression is an angle formed by the line of sight with the horizontal when the point is below the horizontal level. That is, the case when we lower our head to look at the point being viewed.



Clinometer:

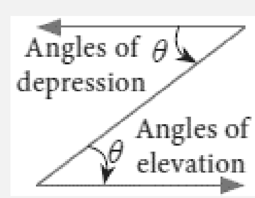
The angle of elevation and depression are usually measured by a device called clinometer.

Note:

From a given point, when height of a object increases the angle of elevation increases. If $h_1 > h_2$ then $\alpha > \beta$ 	The angle of elevation increases as we move towards the foot of the vertical object like tower or building. If $d_2 < d_1$ then $\beta > \alpha$ 

Introduction for Exercise 6.3

Concept corner



Note: Angle of Depression and angle of Elevation are equal because they are alternative angles.