Public Exam Question Paper June - 2023

Time Allowed : 3 hrs **Maximum Marks : 90**

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

PART - I

Note : (i) Answer all the 20 questions **20 x 1 = 20**

(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. If $A^T A^{-1}$ is symmetric, then A^2

(1) A^{-1} (2) $(A^T)^2$ (3) A^T (4) $(A^{-1})^2$

2. The rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 6 & 8 \\ -1 & -2 & -3 & -4 \end{bmatrix}$ is

(1) 1 (2) 2 (3) 4 (4) 3

3. If $|z-2+i| \leq 2$, then the greatest value of $|z|$ is

(1) $\sqrt{3}-2$ (2) $\sqrt{3}+2$ (3) $\sqrt{5}-2$ (4) $\sqrt{5}+2$

4. If $|z_1|=1, |z_2|=2, |z_3|=3$ and $|9z_1z_2+4z_1z_3+z_2z_3|=12$, then the value of $|z_1+z_2+z_3|$ is

(1) 1 (2) 2 (3) 3 (4) 4

5. A zero of x^3+64 is

(1) 0 (2) 4 (3) $4i$ (4) -4

6. The number of positive zeros of the polynomial $\sum_{r=0}^{n} nC_r (-1)^r x^r$ is

(1) 0 (2) n (3) $< n$ (4) r

7. If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in \mathbb{R}$, then the value of $\tan^{-1} x$ is

(1) $\frac{-\pi}{10}$ (2) $\frac{\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $\frac{-\pi}{5}$

8. The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is

(1) 1 (2) 3 (3) $\sqrt{10}$ (4) $\sqrt{11}$

9. If $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\vec{c}\cdot\vec{a}=0$, then the value of $[\vec{a},\vec{b},\vec{c}]$ is

(1) $|\vec{a}||\vec{b}||\vec{c}|$ (2) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ (3) 1 (4) -1

10. If $\sin^{-1} x + \cot^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{2}$, then the value of x is

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{5}}$ (3) $\frac{2}{\sqrt{5}}$ (4) $\frac{\sqrt{3}}{2}$

11. The number given by the mean value theorem for the function $\frac{1}{x}$, $x \in [1,9]$ is

(1) 2 (2) 2.5 (3) 3 (4) 3.5

12. The curve $y = ax^4 + bx^2$ with $a, b > 0$

(1) has no horizontal tangent (2) is concave up

(3) is concave down (4) has no points of inflection

13. If $u(x,y) = e^{x^2+y^2}$ then $\frac{\partial u}{\partial x}$ is

(1) $e^{x^2+y^2}$ (2) $2xu$ (3) x^2u (4) y^2u

14. The value of $\int_{-1}^{2}|x|\,dx$ is

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

15. Linear approximation for $g(x) = \cos x$ at $x = \frac{\pi}{2}$ is

(1) $x + \frac{\pi}{2}$ (2) $-x + \frac{\pi}{2}$ (3) $x - \frac{\pi}{2}$ (4) $-x - \frac{\pi}{2}$

16. The value of $\int_{0}^{\pi} \sin^4 x\,dx$ is

(1) $\frac{3\pi}{10}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{4}$ (4) $\frac{3\pi}{2}$

17. The solution of the differential equation $\frac{dy}{dx} = 2xy$ is

(1) $y = Ce^{x^2}$ (2) $y = 2x^2 + C$ (3) $y = Ce^{-x^2}$ (4) $y = x^2 + C$

18. The population P in any year t is such that the rate of increase in the population is proportional to the population then

(1) $P = Ce^{kt}$ (2) $P = Ce^{-kt}$ (3) $P = Ckt$ (4) $P = C$

19. A random variable X-has binomial distribution with $n = 25$ and $p = 0.8$ then standard deviation of X is

(1) 6 (2) 4 (3) 3 (4) 2

20. If a compound statement involves 3 simple statements, then the number of rows in the truth table is

(1) 9 (2) 8 (3) 6 (4) 3

PART - II

Answer any 7 questions. Question no. 30 is compulsory. **7 x 2 = 14**

21. If $adj(A) = \begin{bmatrix} 0 & -2 & 0 \\ 6 & 2 & -6 \\ -3 & 0 & 6 \end{bmatrix}$, then find A^{-1}.

22. Find the principal argument Arg z, when $z = \frac{-2}{1+i\sqrt{3}}$.

23. Find the equation of the hyperbola with vertices $(0, \pm 4)$ and foci $(0, \pm 6)$.

24. Find the distance from a point $(2, 5, -3)$ to the plane $\vec{r}.\left(6\hat{\imath} - 3\hat{\jmath} + 2\hat{k}\right) = 5$.

25. Prove that the function $f(x) = x^2 - 2x - 3$ is strictly increasing in $(2, \infty)$.

26. Show that the percentage error in the n^{th} root of a number is approximately $\frac{1}{n}$ times the percentage error in the number.

27. Evaluate: $\int_{0}^{\infty} x^5 e^{-3x}\,dx$

28. Assume that a spherical rain drop evaporates at a rate proportional to its surface area. From a differential equation involving the rate of change of the radius of the rain drop.

29. A pair of fair dice is rolled once. Find the probability mass function to get the number of four.

30. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$.

PART – III

Answer any 7 questions. Question no. 40 is compulsory. **7 x 3 = 21**

31. Solve the system of linear equations $2x + 5y = -2, x + 2y = -3$ using matrix inversion method.

32. State and prove triangle inequality.

33. Find the value of $\sin^{-1}(\sin\frac{5\pi}{9}\cos\frac{\pi}{9} + \cos\frac{5\pi}{9}\sin\frac{\pi}{9})$.

34. With usual notations in any triangle ABC, prove by vector method $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

35. Find two positive numbers whose sum is 12 and their product is maximum.

36. If $U(x,y,z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$.

37. Prove that the point of intersection of the tangents at $'t_1'$ and $'t_2'$ on the parabola $y^2 = 4ax$ is $[at_1t_2, a(t_1 + t_2)]$.

38. Find the population of a city at any time t, given that the rate of increase of population is proportional to the population at that instant and that in a period of 40 years the population increased from 3,00,000 to 4,00,000.

39. Establish the equivalence property connecting the bi-conditional with conditional $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$.

40. Show that the polynomial equation $9x^9 + 2x^5 - x^4 - 7x^2 + 2 = 0$ has at least six imaginary roots.

PART - IV

Answer all the questions. **7 x 5 = 35**

41. (a) Solve the system of linear equations by Cramer's Rule.

$x_1 - x_2 = 3,\ 2x_1 + 3x_2 + 4x_3 = 17,\ x_2 + 2x_3 = 7$ **(OR)**

(b) Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$

42. (a) If $z = x + iy$ and $arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$. **(OR)**

(b) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.

43. (a) Evaluate: $\lim_{x\to\frac{\pi}{2}} (\sin x)^{\tan x}$ **(OR)**

(b) If $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k},\quad \vec{b} = 3\hat{\imath} + 5\hat{\jmath} + 2\hat{k},\quad \vec{c} = -\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, verify that

$(\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}$

44. (a) Solve the equation $2x^3 + 11x^2 - 9x - 18 = 0$. **(OR)**

(b) Solve $(1 + x^2)\frac{dy}{dx} = 1 + y^2$

45. (a) Find the equation of the circle passing through the points $(1, 0), (-1, 0)$ and $(0, 1)$. **(OR)**

(b) The mean and variance of a binomial variate X are 2 and 1.5 respectively. Find

(i) $P(X = 0)$, (ii) $P(X = 1)$, (iii) $P(X \geq 1)$

46. (a) Find the foot of the perpendicular drawn from the point $(5,\ 4,\ 2)$ to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$. Also, find the equation of the perpendicular. **(OR)**

(b) Find the area of the region bounded by the lines $5x - 2y = 15$, $x + y + 4 = 0$ and the x-axis using integration.

47. (a) A particle moves along a line according to the law $s(t) = 2t^3 - 9t^2 + 12t - 4$, where $t \geq 0$.

(i) At what time the particle changes direction?

(ii) Find the total distance travelled by the particle in the first 4 seconds.

(iii) Find the particles' acceleration each time the velocity is zero. **(OR)**

(b) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in \mathbb{R} - \{0\}\right\}$ and $*$ be the matrix multiplication. Examine the closure, associative, existence of identity, existence of inverse for the operation $*$ on M.

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Public Exam Question Paper March - 2023

Time Allowed : 3 hrs **Maximum Marks : 90**

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

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PART - I

Note : (i) Answer all the 20 questions **20 x 1 = 20**

(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. A square matrix A of order n has inverse if and only if

(1) $\rho(A) > n$ (2) $\rho(A) = n$ (3) $\rho(A) \neq n$ (4) $\rho(A) < n$

2. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is

(1) 2 (2) 0 (3) 3 (4) 1

3. If $3\cos^{-1} x = \cos^{-1}(4x^3 - 3x)$,

(1) $x\epsilon\left(\frac{1}{2}, 1\right)$ (2) $x\epsilon\left[\frac{1}{2}, 1\right]$ (3) $x\epsilon(-\infty, 1]$ (4) $x\epsilon[\frac{1}{2}, \infty)$

4. The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ is

(1) $y = kx$ (2) $xy = k$ (3) $\log y = kx$ (4) $y = k \log x$

5. The number of normals that can be drawn from a point to the parabola $y^2 = 4ax$ is

(1) 3 (2) 2 (3) 0 (4) 1

6. If $\vec{a}$ and $\vec{b}$ are parallel vectors, then $[\vec{a}, \vec{c}, \vec{b}]$ is equal to

(1) 1 (2) 2 (3) 0 (4) -1

7. The number of real numbers in $[0,2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

(1) 1 (2) 2 (3) ∞ (4) 4

8. Suppose that X takes 0,1,2. If for some constant k, $P(X = i) = k\,P(X = i - 1)$ for $i = 1,2$ and $P(X = 0) = \frac{1}{7}$ then the value of k is

(1) 3 (2) 1 (3) 4 (4) 2

9. The maximum value of the function $x^2 e^{-2x}, x > 0$ is

(1) $\frac{1}{e^2}$ (2) $\frac{1}{e}$ (3) $\frac{4}{e^4}$ (4) $\frac{1}{2e}$

10. The operation $*$ defined by $a * b = \frac{ab}{7}$ is not a binary operation on

(1) $\mathbb{R}$ (2) $\mathbb{Q}^+$ (3) $\mathbb{C}$ (4) $\mathbb{Z}$

11. The area between $y^2 = 4x$ and its latus rectum is

(1) $\frac{8}{3}$ (2) $\frac{2}{3}$ (3) $\frac{5}{3}$ (4) $\frac{4}{3}$

12. Angle between the curves $y^2 = x$ and $x^2 = y$ at the origin is

(1) $\frac{\pi}{2}$ (2) $\tan^{-1}\left(\frac{3}{4}\right)$ (3) $\frac{\pi}{4}$ (4) $\tan^{-1}\left(\frac{4}{3}\right)$

13. $|adj(adjA)| = |A|^{16}$, then the order of the square matrix A is

(1) 2 (2) 3 (3) 5 (4) 4

14. The value of $\left(\frac{1+i}{\sqrt{2}}\right)^8 + \left(\frac{1-i}{\sqrt{2}}\right)^8$ is

(1) 8 (2) 4 (3) 2 (4) 6

15. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

(1) $\frac{1}{z}$ (2) z (3) 1 (4) $\bar{z}$

16. The abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is -0.25?

(1) -2 (2) -8 (3) 0 (4) -4

17. The value of $\int_0^{\pi/3} tan\ x\ dx$ is

(1) $-\log 2$ (2) $\log 2$ (3) $-\log 3$ (4) $\log 3$

18. The number of positive zeros of the polynomial $\sum_{r=0}^{n} nC_r (-1)^r\, x^r$ is

(1) $< n$ (2) 0 (3) r (4) n

19. The principal value of $\sin^{-1}\left(\frac{-1}{2}\right)$ is

(1) $\frac{-\pi}{6}$ (2) 0 (3) $\frac{-\pi}{2}$ (4) $\frac{\pi}{2}$

20. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(1) $\sqrt{ab}$ (2) $2ab$ (3) $\frac{a}{b}$ (4) ab

PART - II

Answer any 7 questions. Question no. 30 is compulsory. 7 x 2 = 14

21. If $|z| = 2$ show that $3 \leq |z+3+4i| \leq 7$
22. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$
23. If $y = 4x + c$ is a tangent to the circle $x^2 + y^2 = 9$, find c
24. If the radius of a sphere, with radius 10 cm, has to decrease by $0.1cm$, approximately how much will its volume decrease?
25. Evaluate: $\int_b^{\infty} \frac{1}{a^2+x^2} dx$, $a > 0$, $b \in \mathbb{R}$
26. Find the vector equation of a plane which is at a distance of 7 units from the origin having $(3, -4, 5)$ as direction ratios of a normal to it.
27. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two Boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$
28. Prove that $\begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ is orthogonal.
29. Find the equation of tangent to the curve $y = x^2 + 3x - 2$ at the point $(1, 2)$
30. Express $e^{cos\theta + i\, sin\theta}$ in $a + ib$ form.

PART – III

Answer any 7 questions. Question no. 40 is compulsory. 7 x 3 = 21

31. Find the equation of the parabola with vertex $(-1, -2)$, axis parallel to y-axis and passing through $(3, 6)$
32. The maximum and minimum distances of the Earth from the Sun respectively are $152 \times 10^6 km$ and $94.5 \times 10^6 km$. The Sun is at one focus of the elliptical orbit. Find the distance from the Sun to the other focus.
33. For what value of x, the inequality $\frac{\pi}{2} < \cos^{-1}(3x-1) < \pi$ holds?

34. Find the angle made by the straight line $\frac{x+3}{2}=\frac{y-1}{2}=-z$ with coordinate axes.

35. Use the linear approximation to find approximate value of $(123)^{\frac{2}{3}}$

36. Slove: $x\cos y\ dy = e^x(x\log x+1)\,dx$

37. If $F(\alpha)=\begin{bmatrix} cos\alpha & 0 & sin\alpha \\ 0 & 1 & 0 \\ -sin\alpha & 0 & cos\alpha \end{bmatrix}$, show that $[F(\alpha)]^{-1}=F(-\alpha)$

38. Show that $p\to q$ and $q\to p$ are not equivalent.

39. If $z=(2+3i)(1-i)$, then find z^{-1}

40. If $a+b+c=0$ and a,b,c are rational numbers then, prove that the roots of the equation $(\mathrm{b}+\mathrm{c}-\mathrm{a})x^2+(c+a-b)x+(a+b-c)=0$ are rational numbers.

PART - IV

Answer all the questions. **7 x 5 = 35**

41. (a) Solve the equation $z^3+8i=0$, where $z\in\mathbb{C}$ **(OR)**

    (b) Solve: $(1+x+xy^2)\frac{dy}{dx}+(y+y^3)=0$

42. (a) Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$ **(OR)**

    (b) Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function of random variable X is $f(x)=\begin{cases} k & 200\le x\le 600 \\ 0 & \text{otherwise}\end{cases}$ Find (i) the value of k (ii) The distribution function (iii) the probability that daily sales will fall between 300 litres and 500 litres

43. (a) Identify the type of conic and find centre, foci and vertices of $18x^2+12y^2-144x+48y+120=0$ **(OR)**

    (b) If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$ and $0<x,y,z<1$, show that $x^2+y^2+z^2+2xyz=1$

44. (a) A boy is walking along the path $y=ax^2+bx+c$ through the points $(-6,8)$, $(-2,-12)$ and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method.) **(OR)**

    (b) Prove that the ellipse $x^2+4y^2=8$ and the hyperbola $x^2-2y^2=4$ intersect orthogonally.

45. (a) Find the parametric form of vector equation and Cartesian equations of the plane containing the line $\vec{r}=(\hat{\imath}-\hat{\jmath}+3\hat{k})+t(2\hat{\imath}-\hat{\jmath}+4\hat{k})$ and perpendicular to the plane $\vec{r}\cdot(\hat{\imath}+2\hat{\jmath}+\hat{k})=8$ **(OR)**

    (b) Solve the equation $6x^4-5x^3-38x^2-5x+6=0$ if it is known that $\frac{1}{3}$ is a solution.

46. (a) Prove that $p\to(\neg q\vee r)\equiv\neg p\vee(\neg q\vee r)$ using truth table. **(OR)**

    (b) Suppose a person deposits ₹10,000 in a bank account at the rate of 5% per annum compounded continuously. How much money will be in his bank account 18 months later?

47. (a) Find the maximum value of $\frac{\log x}{x}$ **(OR)**

    (b) Find the area of the region common to the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the straight line $\frac{x}{a}+\frac{y}{b}=1$

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Public Exam Question Paper July - 2022

Time Allowed : 3 hrs **Maximum Marks : 90**

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

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PART - I

Note : (i) Answer all the 20 questions **20 x 1 = 20**

(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. If $A = \begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ x & \frac{3}{5} \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is :

(1) $-\frac{4}{5}$ (2) $\frac{-3}{5}$ (3) $\frac{3}{5}$ (4) $\frac{4}{5}$

2. If A is a non-singular matrix such that $A^{-1} = \begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$, then $(A^T)^{-1} =$

(1) $\begin{bmatrix} -5 & 3 \\ 2 & 1 \end{bmatrix}$ (2) $\begin{bmatrix} 5 & 3 \\ -2 & -1 \end{bmatrix}$ (3) $\begin{bmatrix} -1 & -3 \\ 2 & 5 \end{bmatrix}$ (4) $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$

3. If $z = x + iy$ is a complex number such that $|z+2| = |z-2|$, then the locus of z is :

(1) real axis (2) imaginary axis (3) ellipse (4) circle

4. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is :

(1) 0 (2) 1 (3) -1 (4) i

5. A zero of $x^3 + 64$ is :

(1) 0 (2) 4 (3) $4i$ (4) -4

6. The principal value of $\cos^{-1}\left(\cos\frac{\pi}{6}\right)$ is :

(1) $\frac{\pi}{6}$ (2) $\frac{5\pi}{6}$ (3) $\frac{-\pi}{6}$ (4) $\frac{\pi}{3}$

7. The equation of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ having centre at (0,3) is :

(1) $x^2 + y^2 - 6y - 7 = 0$ (2) $x^2 + y^2 - 6y + 7 = 0$

(3) $x^2 + y^2 - 6y - 5 = 0$ (4) $x^2 + y^2 - 6y + 5 = 0$

8. The eccentricity of the hyperbola $\frac{x^2}{16} - \frac{(y-3)^2}{4} = 1$ is :

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{\sqrt{5}}{2}$ (3) $\sqrt{5}$ (4) $\frac{1}{2}$

9. If a vector $\vec{\alpha}$ lies in the plane of $\vec{\beta}$ and $\vec{\gamma}$, then :

(1) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 1$ (2) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = -1$ (3) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 0$ (4) $[\vec{\alpha}, \vec{\beta}, \vec{\gamma}] = 2$

10. Distance from the origin to the plane $3x - 6y + 2z + 7 = 0$ is :

(1) 0 (2) 1 (3) 2 (4) 3

11. A stone is thrown up vertically. The height it reaches at time t seconds is given by $x = 80t - 16t^2$. The stone reaches the maximum height in time t seconds is given by

(1) 2 (2) 2.5 (3) 3 (4) 3.5

12. The angle between the parabolas $y^2 = x$ and $x^2 = y$ at the origin is :

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) 0

13. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31?

(1) $\frac{1}{31}$ (2) $\frac{1}{5}$ (3) 5 (4) 31

14. The value of $\int_{-1}^{2} |x|\, dx$ is :

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

15. The area between $y^2 = 4x$ and its latus rectum is :

(1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{8}{3}$ (4) $\frac{5}{3}$

16. The order of the differential equation of all circles with centre at (h, k) and radius 'a' is ______. (where h and k are arbitrary constants)

(1) 2 (2) 3 (3) 4 (4) 1

17. The differential equation representing the family of curves $y = A\cos(x + B)$, where A and B are parameters, is :

(1) $\frac{d^2y}{dx^2} - y = 0$ (2) $\frac{d^2y}{dx^2} + y = 0$ (3) $\frac{d^2y}{dx^2} = 0$ (4) $\frac{d^2x}{dy^2} = 0$

18. If a fair die is thrown once then the probability to get a prime number on the face is :

(1) 0 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{1}{6}$

19. A random variable X takes the probability mass function :

X	-2	3	1
$P(X = x)$	$\frac{\lambda}{6}$	$\frac{\lambda}{4}$	$\frac{\lambda}{12}$

The value of λ is

(1) 1 (2) 2 (3) 3 (4) 4

20. Which one of the following is a binary operation on $\mathbb{N}$?

(1) Subtraction (2) Multiplication (3) Division (4) All the above

PART - II

Answer any 7 questions. Question no. 30 is compulsory. **7 x 2 = 14**

21. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 3$ and $dx = 0.02$.

22. If α and β are the roots of $x^2 + 5x + 6 = 0$, then show that $\alpha^2 + \beta^2 = 13$.

23. Find the value of $\sin^{-1}(1) + \cos^{-1}(1)$.

24. Find the acute angle between the straight lines $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$.

25. Find the tangent to the curve $y = x^2 - x^4$ at $(1, 0)$.

26. If $z_1 = 3, z_2 = -7i$ and $z_3 = 5 + 4i$, show that $z_1(z_2 + z_3) = z_1 z_2 + z_1 z_3$.

27. Show that $y = ae^x + be^{-x}$ is a solution of the differential equation $y'' - y = 0$.

28. A random variable X has the following probability mass function.

x	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Show that the value of k is $\frac{1}{6}$.

29. Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function X is :

$$f(x) = \begin{cases} k & 200 \le x \le 600 \\ 0 & \text{otherwise} \end{cases}$$ Find the value of k.

30. Form the differential equation of the curve $y = ax^2 + bx + c$ where a, b and c are arbitrary constants.

PART – III

Answer any 7 questions. Question no. 40 is compulsory. 7 x 3 = 21

31. Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.

32. Find the rank of the matrix $\begin{bmatrix} 1 & -2 & 3 \\ 2 & 4 & -6 \\ 5 & 1 & -1 \end{bmatrix}$.

33. Show that the square roots of $6 - 8i$ are $\pm(2\sqrt{2} - i\sqrt{2})$.

34. Prove that the roots of the equation $x^4 - 3x^2 - 4 = 0$ are $\pm 2, \pm i$.

35. Find centre and radius of the circle $x^2 + y^2 + 6x - 4y + 4 = 0$.

36. A particle acted on by constant forces $8\hat{i} + 2\hat{j} - 6\hat{k}$ and $6\hat{i} + 2\hat{j} - 2\hat{k}$ is displaced from the point $(1, 2, 3)$ to the point $(5, 4, 1)$. Find the total work done by the forces.

37. Show that $\lim_{x \to 0^+} x \log x$ is 0.

38. A circular plate expands uniformly under the influence of heat. If its radius increases from 10.5 cm to 10.75 cm, then find an approximate change in the area.

39. Verify (i) closure property (ii) commutative property of the following operation on the given set $(a * b) = a^b; \forall a, b \in \mathbb{N}$ (exponentiation property)

40. Prove that $\int_0^1 xe^x \, dx = 1$.

PART - IV

Answer all the questions. 7 x 5 = 35

41. (a) Solve the system of linear equations by Cramer's Rule

$3x + 3y - z = 11, 2x - y + 2z = 9, 4x + 3y + 2z = 25$.

(OR)

(b) A particle is fired straight up from ground to reach a height of s feet in t seconds, where $s(t) = 128t - 16t^2$.

(i) Compute the maximum height of the particle reached.

(ii) What is the velocity when the particle hits the ground?

42. (a) Show that $\left(2+i\sqrt{3}\right)^{10}-\left(2-i\sqrt{3}\right)^{10}$ is purely imaginary.

(OR)

(b) Find the area of the region bounded by the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ using integration.

43. (a) Show that the value of $\sin^{-1}\left(\sin\frac{5\pi}{9}\cos\frac{\pi}{9}+\cos\frac{5\pi}{9}\sin\frac{\pi}{9}\right)$ is $\frac{\pi}{3}$.

(OR)

(b) The parabolic communication antenna has a focus at 2 mts. distance from the vertex of the antenna. Show that the width of the antenna 3 mts. from the vertex is $4\sqrt{6}\,mts$.

44. (a) Solve the differential equation $\frac{dy}{dx}+\frac{y}{x}=\sin x$.

(OR)

(b) Verify whether the following compound proposition is tautology or contradiction or contingency. $(p\rightarrow q)\leftrightarrow(\neg p\rightarrow q)$

45. (a) Prove by using vector method that $\cos(A-B)=\cos A\cos B+\sin A\sin B$.

(OR)

(b) Prove that among all the rectangles of the given perimeter, the square has the maximum area.

46. (a) Find the eccentricity, foci, vertices and centre for the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ and draw the rough diagram.

(OR)

(b) The cumulative distribution function of a discrete random variable is given by :

$$F(x)=\begin{cases}0 & \text{for } -\infty<x<0\\ \frac{1}{2} & \text{for } 0\le x<1\\ \frac{3}{5} & \text{for } 1\le x<2\\ \frac{4}{5} & \text{for } 2\le x<3\\ \frac{9}{10} & \text{for } 3\le x<4\\ 1 & \text{for } 4\le x<\infty\end{cases}$$

Find : (i) The probability mass function (ii) $P(X<3)$ and (iii) $P(X\ge 2)$

47.(a) Show that the area between the parabola $y^2=16x$ and its latus rectum (using integration) is $\frac{128}{3}$.

(OR)

(b) Show that the Cartesian equation of the plane passing through the points $(a,0,0),(0,b,0),(0,0,C)$ is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

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Public Exam Question Paper May - 2022

Time Allowed : 3 hrs **Maximum Marks : 90**

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

PART - I

Note : (i) Answer all the 20 questions **20 x 1 = 20**

(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. If $f(x) = \begin{cases} 2x & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases}$ is a probability density function of a random variable, then the value of a is :
(1) 3 (2) 1 (3) 4 (4) 2

2. Which one of the following is not true in the case of discrete random variable X?
(1) $\lim_{x\to\infty} F(x) = F(\infty) = 1$ (2) $0 \le F(x) \le 1$ $for\ all\ x \epsilon R$
(3) $F(x)$ is real valued decreasing function. (4) $\lim_{x\to-\infty} F(x) = F(-\infty) = 0$

3. If $f(x) = \frac{x}{x+1}$ then its differentials is :
(1) $\frac{1}{x+1}dx$ (2) $\frac{-1}{(x+1)^2}dx$ (3) $\frac{-1}{x+1}dx$ (4) $\frac{1}{(x+1)^2}dx$

4. The value of $\int_0^1 x(1-x)^{99}dx$ is :
(1) $\frac{1}{10010}$ (2) $\frac{1}{11000}$ (3) $\frac{1}{10001}$ (4) $\frac{1}{10100}$

5. The principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is :
(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{5\pi}{6}$ (4) $\frac{\pi}{6}$

6. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is :
(1) 19 (2) 17 (3) 21 (4) 14

7. If $\alpha, \beta,$and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is :
(1) $\frac{q}{r}$ (2) $-\frac{q}{r}$ (3) $-\frac{q}{p}$ (4) $-\frac{p}{r}$

8. If $(1+i)(1+2i)(1+3i) \ldots\ldots (1+ni) = x + iy$ then the value 2.5.10 … … … . $(1+n^2)$ is :
(1) $x^2 + y^2$ (2) 1 (3) $1 + n^2$ (4) i

9. The minimum value of the function $|3 - x| + 9$ is :
(1) 6 (2) 0 (3) 9 (4) 3

10. The value of $\sum_{n=1}^{12} i^n$ is : (1) 0 (2) 1 (3) -1 (4) i

11. If the vectors $2\hat{\imath} - \hat{\jmath} + 3\hat{k}, 3\hat{\imath} + 2\hat{\jmath} + \hat{k}, \hat{\imath} + m\hat{\jmath} + 4\hat{k}$ are coplanar, then the value of m is :
(1) 2 (2) 3 (3) -2 (4) -3

12. The general equation of a circle with centre $(-3,-4)$ and radius 3 units is :
(1) $x^2 + y^2 - 6x + 8y - 16 = 0$ (2) $x^2 + y^2 - 6x - 8y + 16 = 0$
(3) $x^2 + y^2 + 6x - 8y + 16 = 0$ (4) $x^2 + y^2 + 6x + 8y + 16 = 0$

13. The solution of $\frac{dy}{dx} + p(x)y = 0$ is :
(1) $x = ce^{-\int pdy}$ (2) $y = ce^{\int pdx}$ (3) $x = ce^{\int pdy}$ (4) $y = ce^{-\int pdx}$

14. The value of $\int_0^\infty e^{-3x}x^2\,dx$ is :

(1) $\frac{4}{27}$ (2) $\frac{7}{27}$ (3) $\frac{2}{27}$ (4) $\frac{5}{27}$

15. The point of inflection of the curve $y=(x-1)^3$ is :

(1) (1,0) (2) (0,0) (3) (1,1) (4) (0,1)

16. The angle between the lines $\frac{x-4}{2}=\frac{y}{1}=\frac{z+1}{-2}$ and $\frac{x-1}{4}=\frac{y+1}{-4}=\frac{z-2}{2}$ is :

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{4}$ (3) $\frac{2\pi}{3}$ (4) $\frac{\pi}{3}$

17. Which one of the following is a binary operation on $\mathbb{N}$?

(1) Multiplication (2) Division (3) Subtraction (4) All the above

18. Which one of the following is incorrect?

(1) If A is a square matrix of order n and λ is a scalar, then $Adj(\lambda A)=\lambda^n(Adj\ A)$.

(2) Adjoint of a symmetric matrix is also a symmetric matrix.

(3) $A(Adj\ A)=(Adj\ A)A=|A|I$.

(4) Adjoint of a diagonal matrix is also a diagonal matrix.

19. If $\sin x$ is the integrating factor of the linear differential equation $\frac{dy}{dx}+Py=Q$, then P is :

(1) $\tan x$ (2) $\log\sin x$ (3) $\cot x$ (4) $\cos x$

20. The length of the latus rectum of the parabola $x^2=24y$ is :

(1) 8 (2) 24 (3) 6 (4) 12

PART - II

Answer any 7 questions. Question no. 30 is compulsory. 7 x 2 = 14

21. Prove the following properties : $\text{Re}(z)=\frac{z+\bar{z}}{2}$ and $\text{Im}(z)=\frac{z-\bar{z}}{2i}$

22. Find a polynomial equation of minimum degree with rational coefficients, having $2-\sqrt{3}$ as a root.

23. Find the principal value of $\tan^{-1}(\sqrt{3})$.

24. Find the points on the curve $y=x^3-3x^2+x-2$ at which the tangent is parallel to the line $y=x$.

25. Find df for $f(x)=x^2+3x$ and evaluate it for $x=2$ and $dx=0.1$.

26. Show that the differential equation of the family curves $=Ae^x+Be^{-x}$, where A and B are arbitrary constants, is $\frac{d^2y}{dx^2}-y=0$.

27. Solve: $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

28. A random variable X has the following probability mass function.

x	1	2	3	4	5	6
$f(x)$	k	$2k$	$6k$	$5k$	$6k$	$10k$

Find k

29. X is the number of tails occurred when three fair coins are tossed simultaneously. Find the values of the random variable X and number of points in its reverse images.

30. Show that the distance from the origin to the plane $3x+6y+2z+7=0$ is 1.

PART – III

Answer any 7 questions. Question no. 40 is compulsory. 7 x 3 = 21

31. Show that the rank of the matrix $\begin{bmatrix}1 & 2 & -1\\3 & -1 & 2\\1 & -2 & 3\\1 & -1 & 1\end{bmatrix}$ is 3.

32. Solve the following system of linear equations, using matrix inversion method :
$5x+2y=3, 3x+2y=5$.
33. Which one of the points $10-8i, 11+6i$ is closest to $1+i$
34. Solve the equation $2x^3-9x^2+10x=3$, if 1 is a root, find the other roots.
35. Find the magnitude and the direction cosines of the torque about the point $(2,0,-1)$ of a force $2\hat{\imath}+\hat{\jmath}-\hat{k}$, whose line of action passes through the origin.
36. Evaluate: $\lim\limits_{x\to\infty}\frac{2x^2-3}{x^2-5x+3}$
37. Assume that the cross section of the artery of human is circular. A drug is given to a patient to dilate his arteries. If the radius of an artery is increased from 2 mm to 2.1 mm, how much is cross-sectional area increased approximately?
38. Show that $\int_0^{\frac{\pi}{3}}\frac{\sec x\tan x}{1+\sec^2 x}\,dx=\tan^{-1}(2)-\frac{\pi}{4}$.
39. Let $*$ be defined on $\mathbb{R}$ by $(a*b)=a+b+ab-7$. Is $*$ binary on $\mathbb{R}$? If so, find $3*\left(-\frac{7}{15}\right)$
40. Prove that the general equation of the circle whose diameter is the line segment joining the points $(-4,-2)$ and $(-1,-1)$, is $x^2+y^2+5x+3y+6=0$.

PART - IV

Answer all the questions. **7 x 5 = 35**

41. (a) Cramer's rule is not applicable to solve the system $3x+y+z=2, x-3y+2z=1$, $7x-y+4z=5$. Why? **(OR)**
(b) Prove that the local minimum values for the function $f(x)=4x^6-6x^4$ attain at -1 and 1
42. (a) Show that the locus of $z=x+iy$ if $|z+i|=|z-1|$, is $x+y=0$. **(OR)**
(b) Show that $\int_0^a\frac{f(x)}{f(x)+f(a-x)}dx=\frac{a}{2}$.
43. (a) Show that the equation of the parabola with focus $(-\sqrt{2},0)$ and directrix $x=\sqrt{2}$ is $y^2=-4\sqrt{2}x$. **(OR)**
(b) Find the value of $\cot^{-1}(1)+\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)-\sec^{-1}(-\sqrt{2})$.
44. (a) The maximum and minimum distances of the Earth from the Sun respectively are $152\times10^6 km$ and $94.5\times10^6 km$. The Sun is at one focus of the elliptical orbit. Show that the distance from the Sun to the other focus is $575\times10^5\ km$. **(OR)**
(b) Prove by vector method $\sin(A+B)=\sin A.\cos B+\cos A.\sin B$
45. (a) Find the vector equation (any form) or Cartesian equation of a plane passing through the points $(2,2,1)$, $(9,3,6)$ and perpendicular to the plane $2x+6y+6z=9$. **(OR)**
(b) Show that the angle between the curves $y=x^2$ and $x=y^2$ at $(1,1)$ is $\tan^{-1}\left(\frac{3}{4}\right)$.
46. (a) The distribution function of a continuous random variable X is:
$$F(x)=\begin{cases}0\ , & x<1\\ \frac{x-1}{4}\ , & 1\le x\le 5\\ 1\ , & x>5\end{cases}$$
Find (i) $P(X<3)$ (ii) $P(2<X<4)$ (iii) $P(3\le X)$ **(OR)**
(b) Show that the area of the region bounded by $3x-2y+6=0; x=-3,\ x=1$ and x-axis, is $\frac{15}{2}$.
47. (a) Show that the solution of the differential equation $(1+x^2)\frac{dy}{dx}=1+y^2$ is $\tan^{-1}y=\tan^{-1}x+C$ (or) $\tan^{-1}x=\tan^{-1}y+C$. **(OR)**
(b) Prove $p\to(q\to r)\equiv(p\wedge q)\to r$ using truth table.

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Public Exam Question Paper September - 2021

Time Allowed : 3 hrs **Maximum Marks : 90**

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

PART - I

Note : (i) Answer all the 20 questions **20 x 1 = 20**

(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. The inverse of $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$ is

(1) $\begin{bmatrix} 3 & -1 \\ -5 & -3 \end{bmatrix}$ (2) $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$ (3) $\begin{bmatrix} -3 & 5 \\ 1 & -2 \end{bmatrix}$ (4) $\begin{bmatrix} -2 & 5 \\ 1 & -3 \end{bmatrix}$

2. The centre of the hyperbola $\frac{(x-1)^2}{16} - \frac{(y+1)^2}{25} = 1$ is

(1) $\left(\frac{1}{2}, -\frac{1}{2}\right)$ (2) $(-1,1)$ (3) $(1,-1)$ (4) $(0,0)$

3. The order and degree of the differential equation $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^{\frac{1}{3}} + x^{\frac{1}{4}} = 0$ are

(1) 2,6 (2) 2,3 (3) 2,4 (4) 3,3

4. A pair of dice numbered 1,2,3,4,5,6 of a six sided die and 1,2,3,4 of a four sided die is rolled and the sum is determined. If the random variable X denote the sum, then the number of elements in the inverse image of 7 is

(1) 3 (2) 1 (3) 4 (4) 2

5. If $|z| = 1$, then value of $\frac{1+z}{1+\bar{z}}$ is

(1) $\frac{1}{z}$ (2) z (3) 1 (4) $\bar{z}$

6. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} sin^2x\ cosx\ dx$ is

(1) 0 (2) $\frac{3}{2}$ (3) $\frac{2}{3}$ (4) $\frac{1}{2}$

7. The function $f(x) = x^2$, in the interval $[0, \infty]$ is

(1) cannot be determined (2) increasing function
(3) increasing and decreasing function (4) decreasing function

8. The volume of the parallelepiped with its edges represented by the vectors $\hat{\imath} + \hat{\jmath},\ \hat{\imath} + 2\hat{\jmath}, \hat{\imath} + \hat{\jmath} + \pi\hat{k}$ is

(1) π (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{3}$

9. In the set R of real numbers '*' is defined as follows. Which one of the following is not a binary operation on R?

(1) $a * b = a$ (2) $a * b = \min(a, b)$
(3) $a * b = a^b$ (4) $a * b = \max(a, b)$

10. The position of a particle 's' moving at any time t is given by $s(t) = 5t^2 - 2t - 8$. The time at which the particle is at rest, is

(1) 1 (2) 0 (3) 3 (4) $\frac{1}{3}$

11. If the function $f(x) = \frac{1}{12}$ for $a < x < b$, represents a probability density function of a continuous random variable X, then which of the following cannot be the values of a and b?

(1) 7 and 19 (2) 0 and 12 (3) 16 and 24 (4) 5 and 17

12. If $P(x, y)$ be any point on $16x^2 + 25y^2 = 400$ with foci $F_1(3,0)$ and $F_2(-3,0)$, then $PF_1 + PF_2$ is

(1) 10 (2) 8 (3) 12 (4) 6

13. If the planes $\vec{r} \cdot (2\hat{\imath} - \lambda\hat{\jmath} + \hat{k}) = 3$ and $\vec{r} \cdot (4\hat{\imath} + \hat{\jmath} - \mu\hat{k}) = 5$ are parallel, then the values of λ and μ are respectively

(1) $-\frac{1}{2}, -2$ (2) $\frac{1}{2}, -2$ (3) $\frac{1}{2}, 2$ (4) $-\frac{1}{2}, 2$

14. A zero of $x^3 + 64$ is

(1) $4i$ (2) 0 (3) -4 (4) 4

15. The solution of $\frac{dy}{dx} + P(x)y = 0$ is

(1) $x = ce^{-\int Pdy}$ (2) $y = ce^{\int Pdx}$ (3) $x = ce^{\int Pdy}$ (4) $y = ce^{-\int Pdx}$

16. $\int_0^{\frac{\pi}{2}} sin^7 x \ dx =$

(1) $\frac{\pi}{2}$ (2) $\int_0^{\frac{\pi}{2}} \cos^7 x \ dx =$ (3) 0 (4) 1

17. The value of $\sin^{-1}\left(\frac{1}{2}\right) + \cos^{-1}\left(\frac{1}{2}\right)$ is

(1) 0 (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{3}$ (4) π

18. If A, B and C are invertible matrices of some order, then which one of the following is not true?

(1) $detA^{-1} = (detA)^{-1}$ (2) $adjA = |A|A^{-1}$

(3) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$ (4) $adj(AB) = (adjA)(adjB)$

19. The value of the complex number $(i^{25})^3$ is equal to

(1) 1 (2) i (3) $-i$ (4) -1

20. If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in calculation of the volume is (in cubic cm)

(1) 2 (2) 0.4 (3) 4.8 (4) 0.45

PART - II

Answer any 7 questions. Question no. 30 is compulsory. **7 x 2 = 14**

21. If $z = (2 + 3i)(1 - i)$ then prove that $z^{-1} = \frac{5}{26} - i\frac{1}{26}$.

22. If α and β are the roots of $x^2 - 5x + 6 = 0$ then prove that $\alpha^2 - \beta^2 = \pm 5$.

23. For what value of x does $\sin x = \sin^{-1} x$?

24. Show that the three vectors $2\hat{\imath} + 3\hat{\jmath} + \hat{k}, \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $3\hat{\imath} + \hat{\jmath} + 3\hat{k}$ are coplanar.

25. Prove that $\lim_{x\to\infty}\left(\frac{e^x}{x^m}\right)$, where m is a positive integer, is ∞.

26. If $g(x) = x^2 + \sin x$, then find dg

27. Show that the solution of $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$ is $\sin^{-1} y = \sin^{-1} x + C$ (or) $\sin^{-1} x = \sin^{-1} y + C$

28. If X is the random variable with distribution function $F(x)$, given by

$$F(x) = \begin{cases} 0 & ; -\infty < x < 0 \\ \frac{1}{2}(x^2 + x); & 0 \le x < 1 \\ 1 & ; 1 \le x < \infty \end{cases}$$ then prove that the p.d.f is $$f(x) = \begin{cases} \frac{1}{2}(2x+1); & 0 \le x < 1 \\ 0 & ; \text{ otherwise} \end{cases}$$

29. The probability density function of X is given by $f(x) = \begin{cases} k\,x\,e^{-2x} & for\ x > 0 \\ 0 & for\ x \le 0 \end{cases}$

    Prove that the value of k is 4.

30. Show that the differential equation corresponding to $y = \mathrm{A}\sin x$, where A is an arbitrary constant, is $y = y' \tan x$.

PART – III

Answer any 7 questions. Question no. 40 is compulsory. 7 x 3 = 21

31. Show that the rank of the matrix $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 0 & 2 & 4 & 3 \\ 8 & 1 & 0 & 2 \end{bmatrix}$ is 3

32. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(adjA) = (adjA)A = |A|I$

33. Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$, and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle of side length $\sqrt{3}$

34. If the sides of a cubic box are increased by 1,2,3 units respectively to form a cuboid, then the volume is increased by 52 cubic units. Show that the volume of the cuboid is 60 cubic units.

35. Prove that the equation of the parabola with focus (4,0) and directix $x = -4$ is $y^2 = 16x$

36. A force $13\hat{\imath} + 10\hat{\jmath} - 3\hat{k}$ acts on a particle which is displaced from the point with position vector $4\hat{\imath} - 3\hat{\jmath} - 2\hat{k}$ to the point with position vector $6\hat{\imath} + \hat{\jmath} - 3\hat{k}$ Show that the work done by the force is 69 units.

37. Show that the point on the curve $y = x^2 - 5x + 4$ at which the tangent is parallel to the line $3x + y = 7$, is (1,0)

38. An egg of a particular bird is spherical in shape. If radius to the inside of the shell is 4 mm and radius to the outside of the shell is 4.2 mm , prove that the approximate volume of the shell is $12.8\pi\ mm^3$

39. Define an operation $*$ on $\mathbb{Q}$, the set of all rational numbers, as follows: $a * b = \left(\frac{a+b}{2}\right)$; $a, b \in \mathbb{Q}$

    Examine the closure and commutative properties satisfied by $*$ on $\mathbb{Q}$

40. Show that $\int_0^1 \frac{\sqrt{x}}{\sqrt{1-x}+\sqrt{x}} dx = \frac{1}{2}$

PART - IV

Answer all the questions. **7 x 5 = 35**

41. (a) Solve the system of equations $x-y+2z=2,\ 2x+y+4z=7,\ 4x-y+z=4$ by Cramer's rule.

(OR)

(b) A camera is accidentaly knocked off an edge of a cliff 400 ft. high. The camera falls a distance of $s=16t^2$ in t seconds. Show that the camera hits the ground when t = 5 seconds and also prove that the velocity when it hits the ground is -160 ft./sec.

42. (a) If $z=x+iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right|=1$, show that the locus of z is real axis or $y=0$

(OR)

(b) Show that $\int_0^1 \sqrt{\frac{1-x}{1+x}}\,dx=\frac{\pi}{2}-1$

43. (a) Prove that $\cos^{-1}\left(\cos\left(\frac{4\pi}{3}\right)\right)+\cos^{-1}\left(\cos\left(\frac{5\pi}{4}\right)\right)=\frac{17\pi}{12}$

(OR)

(b) Find the eccentricity, centre, vertices and foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and also draw the rough diagram.

44. (a) Solve $(e^y+1)\cos x\ dx+e^y\sin x\ dy=0$

(OR)

(b) Show that $\neg(p\rightarrow q)\equiv p\wedge\neg q$

45. (a) Using vector method, prove that $\cos(A-B)=\cos A\cos B+\sin A\sin B$

(OR)

(b) Find two positive numbers whose product is 20 and their sum is minimum.

46. (a) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4 m when it is 6 m away from the point of projection. Finally it reaches the ground 12 m away from the starting point. Show that the angle of projection is $\tan^{-1}\left(\frac{4}{3}\right)$

(OR)

(b) A random variable X has the following probability mass function:

X	1	2	3	4	5
$f(x)$	k^2	$2k^2$	$3k^2$	$2k$	$3k$

Find (i) the value of k (ii) $P(2\le X<5)$ (iii) $P(3<X)$

47.(a) Show that the area of the region bounded by $3x-2y=0, x=-3$ and $x=1$ is $\frac{15}{2}$

(OR)

(b) Show that the Cartesian equation of the plane passing through the points (1,2,3) and (2,3,1) and also perpendicular to the plane $3x-2y+4z+5=0$ is $2y+z-7=0$

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Public Exam Question Paper September - 2020

Time Allowed : 3 hrs **Maximum Marks : 90**

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

PART - I

Note : (i) Answer all the 20 questions **20 x 1 = 20**

(ii) Choose the most appropriate answer from the given four alternatives and write the option code and the corresponding answer.

1. If $A^T A^{-1}$ is symmetric, then $A^2 =$
   (1) A^{-1} (2) $(A^T)^2$ (3) A^T (4) $(A^{-1})^2$
2. If $x^a y^b = e^m, x^c y^d = e^n, \Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}, \Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}, \Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, then the values of x and y are respectively.
   (1) $e^{\left(\frac{\Delta_2}{\Delta_1}\right)}, e^{\left(\frac{\Delta_3}{\Delta_1}\right)}$ (2) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_3)$
   (3) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ (4) $e^{\left(\frac{\Delta_1}{\Delta_3}\right)}, e^{\left(\frac{\Delta_2}{\Delta_3}\right)}$
3. If z is a non zero complex number, such that $2iz^2 = \bar{z}$ then $|z|$ is
   (1) $\frac{1}{2}$ (2) 1 (3) 2 (4) 3
4. If $a = 3 + i$ and $z = 2 - 3i$, then the points on the Argand diagram representing $az, 3az$ and $-az$ are
   (1) Vertices of a right angled triangle (2) Vertices of an equilateral triangle
   (3) Vertices of an isosceles triangle (4) Collinear
5. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies
   (1) $|k| \le 6$ (2) $k = 0$ (3) $|k| > 6$ (4) $|k| \ge 6$
6. If $\cot^{-1} x = \frac{2\pi}{5}$ for some $x \in \mathbb{R}$, the value of $\tan^{-1} x$ is
   (1) $-\frac{\pi}{10}$ (2) $\frac{\pi}{5}$ (3) $\frac{\pi}{10}$ (4) $-\frac{\pi}{5}$
7. $\sin^{-1}(\cos x) = \frac{\pi}{2} - x$ is valid for
   (1) $-\pi \le x \le 0$ (2) $0 \le x \le \pi$ (3) $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ (4) $-\frac{\pi}{4} \le x \le \frac{3\pi}{4}$
8. The length of the latus rectum of the parabola $y^2 - 4x + 4y + 8 = 0$ is
   (1) 8 (2) 6 (3) 4 (4) 2
9. If the co-ordinates at one end of a diameter of the circle $x^2 + y^2 - 8x - 4y + c = 0$ are $(11, 2)$, the co-ordinates of the other end is
   (1) $(-5, 2)$ (2) $(2, -5)$ (3) $(5, -2)$ (4) $(-2, 5)$
10. If $\vec{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{b} = \hat{\imath} + \hat{\jmath}$, $\vec{c} = \hat{\imath}$ and $(\vec{a} \times \vec{b}) \times \vec{c} = \lambda\vec{a} + \mu\vec{b}$, then the value of $\lambda + \mu$ is
    (1) 0 (2) 1 (3) 6 (4) 3
11. The co-ordinates of the point where the line $\vec{r} = (6\hat{\imath} - \hat{\jmath} - 3\hat{k}) + t(-\hat{\imath} + 4\hat{k})$ meets the plane $\vec{r} \cdot (\hat{\imath} + \hat{\jmath} - \hat{k}) = 3$ are
    (1) $(2, 1, 0)$ (2) $(7, -1, -7)$ (3) $(1, 2, -6)$ (4) $(5, -1, 1)$
12. Angle between $y^2 = x$ and $x^2 = y$ at the origin is
    (1) $\tan^{-1}\frac{3}{4}$ (2) $\tan^{-1}\left(\frac{4}{3}\right)$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

13. The number given by the Rolle's theorem for the function $x^3 - 3x^2,\ x \in [0,3]$ is

(1) 1 (2) $\sqrt{2}$ (3) $\frac{3}{2}$ (4) 2

14. If $W(x,y) = x^y, x > 0$ then $\frac{\partial W}{\partial x}$ is equal to

(1) $x^y \log x$ (2) $y \log x$ (3) yx^{y-1} (4) $x \log y$

15. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{\sin x}{2+\cos x}\right) dx$ is

(1) 0 (2) 2 (3) $\log 2$ (4) $\log 4$

16. If $f(x) = \int_0^x t \cos t\, dt$, then $\frac{df}{dx} =$

(1) $\cos x - x \sin x$ (2) $\sin x + x \cos x$ (3) $x \cos x$ (4) $x \sin x$

17. If $\cos x$ is an integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$, then $P =$

(1) $-\cot x$ (2) $\cot x$ (3) $\tan x$ (4) $-\tan x$

18. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\varphi\left(\frac{y}{x}\right)}{\varphi'\left(\frac{y}{x}\right)}$ is

(1) $x.\varphi\left(\frac{y}{x}\right) = k$ (2) $\varphi\left(\frac{y}{x}\right) = kx$ (3) $y\varphi\left(\frac{y}{x}\right) = k$ (4) $\varphi\left(\frac{y}{x}\right) = ky$

19. In 16 throws of a die getting an even number is considered a success, then the variance of the successes is

(1) 4 (2) 6 (3) 2 (4) 256

20. The operation $*$ defined by $a * b = \frac{ab}{7}$ is not a binary operation on

(1) $\mathbb{Q}^+$ (2) $\mathbb{Z}$ (3) $\mathbb{R}$ (4) $\mathbb{C}$

PART - II

Answer any 7 questions. Question no. 30 is compulsory. 7 x 2 = 14

21. Find the least positive integer n such that $\left(\frac{1+i}{1-i}\right)^n = 1$.
22. Obtain the Cartesian form of the locus of $z = x + iy$ in $|z + i| = |z - 1|$.
23. If α, β, γ and δ are the roots of the polynomial equation $2x^4 + 5x^3 - 7x^2 + 8 = 0$, find a quadratic equation with integer coefficients whose roots are $\alpha + \beta + \gamma + \delta$ and $\alpha\beta\gamma\delta$.
24. Find the principal value of $\tan^{-1}(\sqrt{3})$.
25. If $\hat{a},\ \hat{b},\ \hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a} \times (\hat{b} \times \hat{c}) = \frac{1}{2}\hat{b}$, find the angle between $\hat{a}$ and $\hat{c}$
26. Evaluate the limit: $\lim_{x \to 0} \left(\frac{\sin mx}{x}\right)$
27. Evaluate $\int_3^4 \frac{dx}{x^2-4}$
28. Find the differential equation of the family of $y = ax^2 + bx + c$ where a, b are parameters and c is a constant.
29. Examine the binary operation of the operation $a * b = \left(\frac{a-1}{b-1}\right), \forall\, a, b \in Q$.
30. Show that, if $x = r \cos\theta, y = r \sin\theta$, then $\frac{\partial r}{\partial x}$ is equal to $\cos\theta$.

PART – III

Answer any 7 questions. Question no. 40 is compulsory. 7 x 3 = 21

31. Verify $(AB)^{-1} = B^{-1}A^{-1}$ with $A = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} -2 & -3 \\ 0 & -1 \end{bmatrix}$.
32. If $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(adjA) = (adjA)A = |A|I$
33. Obtain the condition that the roots of $x^3 + px^2 + qx + r = 0$ are in A.P.
34. A circle of area 9π square units has two of its diameters along the lines $x + y = 5$ and $x - y = 1$ Find the equation of the circle.
35. Prove that with usual notations $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ by using area of the triangle property.
36. Find the absolute extrema of the function $f(x) = x^2 - 12x + 10$ on $[1,2]$
37. Suppose that $z = ye^{x^2}$ where $x = 2t$ and $y = 1 - t$ then find $\frac{dz}{dt}$
38. Two fair coins are tossed simultaneously. Find the probability mass function for number of heads occured.
39. The mean and variance of a binomial variate X are respectively 2 and 1.5 Find $P(X = 0)$
40. Show that $((\neg q) \wedge p) \wedge q$ is a contradiction.

PART - IV

Answer all the questions. 7 x 5 = 35

41. (a) Test for consistency and if possible, solve the following system of equations by rank method. $2x + 2y + z = 5,\ x - y + z = 1,\ 3x + y + 2z = 4$ **(OR)**
    (b) Prove that $arg(z_1z_2) = arg(z_1) + arg(z_2)$
42. (a) Draw the graph of $\tan x$ in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ and $\tan^{-1} x$ in $(-\infty, \infty)$ **(OR)**
    (b) Find the centre, foci, and eccentricity of the hyperbola $11x^2 - 25y^2 - 44x + 50y - 256 = 0$
43. (a) A rod of length $1.2m$ moves with its ends always touching the co-ordinate axes. The locus of a point P on the rod, which is $0.3m$ from the end in contact with x -axis is an ellipse. Find the eccentricity. **(OR)**
    (b) Using vector method, prove that $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$
44. (a) Find the Vector and Cartesian equations of the plane passing through the point $(1, -2, 4)$ and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$ **(OR)**
    (b) Prove that among all the rectangles of the given perimeter, the square has the maximum area.
45. (a) Show that $\int_0^1 (\tan^{-1} x + \tan^{-1}(1 - x))dx = \frac{\pi}{2} - \log 2$ **(OR)**
    (b) A pot of boiling water at $100°C$ is removed from the stove at time $t = 0$ and left to cool in the kitchen. After 5 minutes, the water temperature has decreased to $80°C$, and another 5 minutes later it has dropped to $65°C$. Determine the temperature of the kitchen.
46. (a) Solve $\frac{dy}{dx} = e^{x+y} + x^3 e^y$ **(OR)**
    (b) If $X \sim B(n, p)$ such that $4P(X = 4) = P(X = 2)$ and $n = 6$, find the distribution, mean and standard deviation of X.
47. (a) A Car A is travelling from west at 50 km/hr and Car B is travelling towards north at 60 km/hr. Both are headed for the intersection of the two roads. At what rate are the Cars approaching each other when Car A is 0.3 kilometers and Car B is 0.4 kilometers from the intersection? **(OR)**
    (b) Find the area of the region bounded by the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its latus rectums.

Public Exam Question Paper March - 2020

Time Allowed : 3 hrs. **Maximum Marks : 90**

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.

(2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

PART - I

Note : (i) Answer all the 20 questions **20 x 1 = 20**

(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. If $u(x, y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

(1) y^2u (2) $e^{x^2+y^2}$ (3) $2xu$ (4) x^2u

2. Subtraction is not a binary operation in

(1) $\mathbb{Q}$ (2) $\mathbb{R}$ (3) $\mathbb{Z}$ (4) $\mathbb{N}$

3. The value of $\int_0^{\pi} \sin^4 x\, dx$ is

(1) $\frac{3\pi}{2}$ (2) $\frac{3\pi}{10}$ (3) $\frac{3\pi}{8}$ (4) $\frac{3\pi}{4}$

4. A polynomial equation of degree n always has

(1) exactly n roots (2) n distinct roots

(3) n real roots (4) n imaginary roots

5. If $\rho(A) = \rho([A|B])$, then the system $AX = B$ of linear equations is

(1) inconsistent (2) consistent and has a unique solution

(3) consistent (4) consistent and has infinitely many solutions

6. The vertex of the parabola $x^2 = 8y - 1$ is :

(1) $\left(0, -\frac{1}{8}\right)$ (2) $\left(-\frac{1}{8}, 0\right)$ (3) $\left(\frac{1}{8}, 0\right)$ (4) $\left(0, \frac{1}{8}\right)$

7. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$; then $\cos^{-1} x + \cos^{-1} y$ is equal to

(1) π (2) $\frac{2\pi}{3}$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

8. The value of $\Sigma_{i=1}^{13}(i^n + i^{n-1})$ is :

(1) 0 (2) $1 + i$ (3) i (4) 1

9. $\vec{r} = s\hat{\imath} + t\hat{\jmath}$ is the equation of (s, t are parameters)

(1) zox plane (2) a straight line joining the points $\hat{\imath}$ and $\hat{\jmath}$

(3) xoy plane (4) yoz plane

10. The order of the differential equation of all circles with centre at (h, k) and radius 'a', where h, k and a are arbitrary constants, is

(1) 1 (2) 2 (3) 3 (4) 4

11. arg (0) is :
(1) ∞ (2) 0 (3) π (4) undefined

12. $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)$ is :
(1) $\tan^{-1}\left(\frac{1}{2}\right)$ (2) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ (3) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ (4) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$

13. The position of a particle moving along a horizontal line of any time t is given by $s(t)=3t^2-2t-8$. The time at which the particle is at rest is :
(1) $t=3$ (2) $t=0$ (3) $t=\frac{1}{3}$ (4) $t=1$

14. The least possible perimeter (in meter) of a rectangle of area $100m^2$ is :
(1) 50 (2) 10 (3) 20 (4) 40

15. A random variable X has binomial distribution with $n=25$ and $p=0.8$, then the standard deviation of X is :
(1) 2 (2) 6 (3) 4 (4) 3

16. The radius of the circle $3x^2+by^2+4bx-6by+b^2=0$ is :
(1) $\sqrt{11}$ (2) 1 (3) 3 (4) $\sqrt{10}$

17. The distance between the planes $x+2y+3z+7=0$ and $2x+4y+6z+7=0$ is :
(1) $\frac{7}{2\sqrt{2}}$ (2) $\frac{\sqrt{7}}{2\sqrt{2}}$ (3) $\frac{7}{2}$ (4) $\frac{\sqrt{7}}{2}$

18. If $(AB)^{-1}=\begin{bmatrix}12 & -17\\ -19 & 27\end{bmatrix}$ and $A^{-1}=\begin{bmatrix}1 & -1\\ -2 & 3\end{bmatrix}$, then $B^{-1}=$
(1) $\begin{bmatrix}8 & -5\\ -3 & 2\end{bmatrix}$ (2) $\begin{bmatrix}2 & -5\\ -3 & 8\end{bmatrix}$ (3) $\begin{bmatrix}8 & 5\\ 3 & 2\end{bmatrix}$ (4) $\begin{bmatrix}3 & 1\\ 2 & 1\end{bmatrix}$

19. The value of $\int_0^{\frac{2}{3}}\frac{dx}{\sqrt{4-9x^2}}$ is :
(1) π (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

20. The order and degree of the differential equation $\frac{dx}{dy}+\frac{dy}{dx}=0$ are
(1) 2, degree not defined (2) 1, 2 (3) 2, 1 (4) 2, 2

PART - II

Answer any 7 questions. Question no. 30 is compulsory. **7 x 2 = 14**

21. Prove that $\left(\frac{1+i}{1-i}\right)^3-\left(\frac{1-i}{1+i}\right)^3=-2i$.

22. If $(1+i)(1+2i)\ldots\ldots(1+ni)=x+iy$, then prove that $2.5.10\ldots\ldots\ldots.(1+n^2)=x^2+y^2$.

23. Find the values of $\sin^{-1}\left[\sin\left(\frac{5\pi}{4}\right)\right]$.

24. Find the magnitude and the direction cosines of the torque about the point $(2,0,-1)$ of a force $2\hat{\imath}+\hat{\jmath}-\hat{k}$, whose line of action passes through the origin.

25. Find the value in the interval $\left(\frac{1}{2},2\right)$ satisfied by the Rolle's theorem for the function $f(x)=x+\frac{1}{x}, x\in\left[\frac{1}{2},2\right]$.

26. For the function $f(x) = x^2 + 3x$, calculate the differential df when $x = 2$ and $dx = 0.1$.
27. Prove that $\int_0^{\frac{\pi}{2}} \frac{f(\sin x)}{f(\sin x)+f(\cos x)} dx = \frac{\pi}{4}$.
28. Find the differential equation of the family of parabolas $y^2 = 4ax$, where 'a' is an arbitrary constant.
29. Prove that the identity element is unique if it exists.
30. Find the equation of the parabola if the curve is open leftward, vertex is $(2, 1)$ and passing through the point $(1, 3)$.

PART – III

Answer any 7 questions. Question no. 40 is compulsory. 7 x 3 = 21

31. If $A = \begin{bmatrix} 2 & 9 \\ 1 & 7 \end{bmatrix}$ then prove that $(A^T)^{-1} = (A^{-1})^T$.
32. If p is real, discuss the nature of the roots of the equation $4x^2 + 4px + p + 2 = 0$, in terms of p.
33. A concrete bridge is designed as a parabolic arch. The road over bridge is 40 m long and the maximum height of the arch is 15 m. Write the equation of the parabolic arch. Take $(0, 0)$ as the vertex.
34. Find the Vector and Cartesian equations of a straight line passing through the points $(-5, 7, -4)$ and $(13, -5, 2)$. Find the point where the straight line crosses the xy-plane.
35. Find the critical numbers (only x values) of the function $f(x) = x^{\frac{4}{5}}(x-4)^2$.
36. If $U = \log(x^3 + y^3 + z^3)$ then find $\frac{\partial U}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$.
37. A random variable X has the following probability mass function

X	1	2	3	4	5	6
P(X=x)	k	$2k$	$6k$	$5k$	$6k$	$10k$

Then find $p(2 < X < 6)$

38. Let X be a continuous random variable and $f(x)$ is defined as: $f(x) = \begin{cases} kx(1-x)^{10}, & 0 < x < 1 \\ 0 \quad , & \text{Otherwise} \end{cases}$

Find the value of k.

39. Prove that $p \to q \equiv \neg p \vee q$
40. If the lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ lie on the same plane, then write the number of ways to find the Cartesian equation of the above plane and explain in detail.

PART - IV

Answer all the questions. 7 x 5 = 35

41. (a) Test the consistency of the following system of linear equations by rank method.

$x - y + z = -9,\ 2x - y + z = 4,\ 3x - y + z = 6,\ 4x - y + 2z = 7$

(OR)

(b) If $2\,cos\alpha = x + \frac{1}{x}$ and $2cos\beta = y + \frac{1}{y}$, show that

(i) $\frac{x^m}{y^n} - \frac{y^n}{x^m} = 2i\, sin(m\alpha - n\beta)$

(ii) $x^m y^n + \frac{1}{x^m y^n} = 2\, cos(m\alpha + n\beta)$

42. (a) Draw the graph of $\cos x$ in$[0, \pi]$and $\cos^{-1} x$ in $[-1, 1]$.

(OR)

(b) Find the equation of the circle passing through the points $(1, 1), (2, -1)$ and $(3, 2)$

43. (a) Assume that water issuing from the end of a horizontal pipe, $7.5m$ above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position $2.5m$ below the line of the pipe, the flow of water has curved outward $3m$ beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

(OR)

(b) By vector method, prove that, $\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$.

44. (a) Find the vector and Cartesian equation of the plane passing through the point $(0, 1, -5)$ and parallel to the straight lines

$\vec{r} = (\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + s(2\hat{\imath} + 3\hat{\jmath} + 6\hat{k})$ and $\vec{r} = (\hat{\imath} - 3\hat{\jmath} + 5\hat{k}) + t(\hat{\imath} + \hat{\jmath} - \hat{k})$

(OR)

(b) Evaluate : $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$

45. (a) A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6km north of the intersection and the car is 0.8km to the east, the police determine with a radar that the distance between the jeep and the car is increasing at 20km/hr. If the jeep is moving at 60km/hr at the instant of measurement, what is the speed of the car?

(OR)

(b) Find the area of the region bounded by x-axis, the curve $y = |\cos x|$, the lines $x = 0$ and $x = \pi$.

46. (a) A square shaped thin material with area196 sq. units to make into an open box by cutting small equal squares from the four corners and folding the sides upward. Prove that the length of the side of a removed square is $\frac{7}{3}$when the volume of the box is maximum.

(OR)

(b) If F is the constant force generated by the motor of an automobile of mass M, its velocity V is given by $M\frac{dV}{dt} = F - kV$, where k is a constant. Prove that $V = \frac{F}{k}\left(1 - e^{\frac{-kt}{M}}\right)$ when $t = 0$ and $V = 0$.

47.(a) In an investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F. If the room temperature is 50°F, and assuming that the body temperature of the person before death was 98.6°F , prove that the time of death is 5.26 p.m. (5 hrs 26 minutes) (app.). $\left[\frac{\log(2.43)}{\log(2)} \simeq 1.28\right]$

(OR)

(b) Three fair coins are tossed once. Find the probability mass function, mean and variance for number of heads occurred. Verify the results by binomial distribution.

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Government Model Question Paper

Time Allowed : 3 hrs **Maximum Marks : 90**

Instructions : (1) Check the question paper for fairness of printing. If there is any lack of fairness, inform the Hall Supervisor immediately.
(2) Use Blue or Black ink to write and underline and pencil to draw diagrams.

PART - I

Note : (i) Answer all the 20 questions **20 x 1 = 20**
(ii) Choose the most suitable answer from the given four alternatives and write the option code and the corresponding answer.

1. If $A = \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}, B = adj\ A$ and $C = 3A$, then $\frac{|adj\ B|}{|C|} =$
(1) $\frac{1}{3}$ (2) $\frac{1}{9}$ (3) $\frac{1}{4}$ (4) 1

2. If the inverse of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & -5 \end{bmatrix}$ is $\frac{1}{11}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then ascending order of a, b, c, d is
(1) a, b, c, d (2) d, b, c, a (3) c, a, b, d (4) b, a, c, d

3. The least value of n satisfying $\left[\frac{\sqrt{3}}{2} + \frac{i}{2}\right]^n = 1$ is
(1) 30 (2) 24 (3) 12 (4) 18

4. The principal argument of $\frac{3}{-1+i}$ is
(1) $\frac{-5\pi}{6}$ (2) $\frac{-2\pi}{3}$ (3) $\frac{-3\pi}{4}$ (4) $\frac{-\pi}{2}$

5. The polynomial equation $x^3 + 2x + 3 = 0$ has
(1) One negative and two real roots (2) One positive and two imaginary roots
(3) three real roots (4) no solution

6. The domain of the function defined by $f(x) = \sin^{-1}(\sqrt{x-1})$ is
(1) $[1, 2]$ (2) $[-1, 1]$ (3) $[0, 1]$ (4) $[-1, 0]$

7. If $x + y = k$ is a normal to the parabola $y^2 = 12x$, then value of k is
(1) 3 (2) -1 (3) 1 (4) 9

8. The circle passing through $(1, -2)$ and touching the x-axis of x at $(3, 0)$, again passing through the point is
(1) $(-5, 2)$ (2) $(2, -5)$ (3) $(5, -2)$ (4) $(-2, 5)$

9. The volume of the parallelepiped with its edges represented by the vectors $\hat{\imath} + \hat{\jmath}, \hat{\imath} + 2\hat{\jmath}$, $\hat{\imath} + \hat{\jmath} + \pi\hat{k}$ is
(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) π (4) $\frac{\pi}{4}$

10. If the line $\frac{x-2}{3} = \frac{y-1}{-5} = \frac{z+2}{2}$ lies in the plane $x + 3y - \alpha z + \beta = 0$ then (α, β) is
(1) $(-5, 5)$ (2) $(-6, 7)$ (3) $(5, -5)$ (4) $(6, -7)$

11. The function $\sin^4 x + \cos^4 x$ is increasing in the interval
(1) $\left[\frac{5\pi}{8}, \frac{3\pi}{4}\right]$ (2) $\left[\frac{\pi}{2}, \frac{5\pi}{8}\right]$ (3) $\left[\frac{\pi}{4}, \frac{\pi}{2}\right]$ (4) $\left[0, \frac{\pi}{4}\right]$

12. The curve $y = ax^4 + bx^2$ with $ab > 0$
(1) has no horizontal tangent (2) is concave up
(3) is concave down (4) has no points of inflection

13. If $u = (x - y)^2$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$ is
(1) 1 (2) -1 (3) 0 (4) 2

14. The value of $\int_0^{\pi} \frac{dx}{1+5^{cosx}}$ is

(1) $\frac{\pi}{2}$ (2) π (3) $\frac{3\pi}{2}$ (4) 2π

15. The volume of solid of revolution of the region bounded by $y^2 = x(a-x)$ about x-axis is

(1) πa^3 (2) $\frac{\pi a^3}{4}$ (3) $\frac{\pi a^2}{5}$ (4) $\frac{\pi a^3}{6}$

16. If m, n are the order and degree of the differential equation $\left[\frac{d^4y}{dx^4} + \frac{d^2y}{dx^2}\right]^{\frac{1}{2}} = a\frac{d^2y}{dx^2}$ respectively, then the value of $4m - n$ is

(1) 15 (2) 12 (3) 14 (4) 13

17. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \frac{\emptyset(\frac{y}{x})}{\emptyset'(\frac{y}{x})}$ is

(1) $x\emptyset\left(\frac{y}{x}\right) = k$ (2) $\emptyset\left(\frac{y}{x}\right) = kx$ (3) $y\emptyset\left(\frac{y}{x}\right) = k$ (4) $\emptyset\left(\frac{y}{x}\right) = ky$

18. A random variable X has the following distribution.

x	1	2	3	4
$P(X = x)$	c	2c	3c	4c

Then the value of c is

(1) 0.1 (2) 0.2 (3) 0.3 (4) 0.4

19. If $P\{X = 0\} = 1 - P\{X = 1\}$ and $E[X] = 3Var(X)$, then $P\{X = 0\}$ is

(1) $\frac{2}{3}$ (2) $\frac{2}{5}$ (3) $\frac{1}{3}$ (4) $\frac{1}{5}$

20. Which one is the contrapositive of the statement $(p \vee q) \to r$?

(1) $\neg r \to (\neg p \wedge \neg q)$ (2) $\neg r \to (P \vee q)$ (3) $r \to (p \wedge q)$ (4) $p \to (q \vee r)$

PART - II

Answer any 7 questions. Question no. 30 is compulsory. **7 x 2 = 14**

21. Solve the following system of linear equations by Cramer's rule $2x - y = 3, x + 2y = -1$
22. If z_1, z_2 and z_3 are the complex numbers such that $|z_1| = |z_2| = |z_3| = |z_1 + z_2 + z_3| = 1$, find the value of $\left|\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}\right|$
23. Find the value of $\sin\left(\frac{\pi}{3} + \cos^{-1}\left(-\frac{1}{2}\right)\right)$
24. Find the equation of the parabola with vertex $(-1, -2)$, axis parallel to y-axis and passing through $(3,6)$
25. If $\hat{a}, \hat{b}, \hat{c}$ are three unit vectors such that $\hat{b}$ and $\hat{c}$ are non-parallel and $\hat{a} \times (\hat{b} \times \hat{c}) = \frac{1}{2}\hat{b}$, find the angle between $\hat{a}$ and $\hat{c}$
26. If the mass $m(x)$ (in kilograms) of a thin rod of length x (in metres) is given by, $m(x) = \sqrt{3x}$ then what is the rate of change of mass with respect to the length when it is $x = 27$ metres.
27. Evaluate: $\int_0^{\infty} e^{-ax} x^n dx$, where $a > 0$
28. Show that $y = ax + \frac{b}{x}, x \neq 0$ is a solution of the differential equation $x^2y'' + xy' - y = 0$
29. Find the mean of a random variable X, whose probability density function is

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

30. Let $*$ be a binary operation on set Q of rational numbers defined as $a * b = \frac{ab}{8}$. Write the identity for $*$, if any

PART - III

Answer any 7 questions. Question no. 40 is compulsory. **7 x 3 = 21**

31. Find the inverse of $\begin{bmatrix} 2 & -1 \\ 5 & -2 \end{bmatrix}$ by Gauss Jordan method.

32. If $w \neq 1$ is a cube root of unity, show that the roots of the equation $(z-1)^3 + 8 = 0$ are -1, $1-2w, 1-2w^2$
33. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$.
34. Find the centre, foci and eccentricity of the hyperbola $12x^2 - 4y^2 - 24x + 32y - 127 = 0$
35. Find the image of the point whose position vector is $\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ in the plane $\vec{r}.(\hat{\imath} + 2\hat{\jmath} + 4\hat{k}) = 38$
36. Evaluate : $\lim_{x \to 0^+} x \log x$.
37. Find a linear approximation for the following functions at the indicated points. $f(x) = x^3 - 5x + 12, x_0 = 2$.
38. By using the properties of definite integrals, evaluate $\int_0^3 |x-1| dx$.
39. Solve : $\frac{dy}{dx} + 2y \cot x = 3x^2 \operatorname{cosec}^2 x$.
40. A fair coin is tossed a fixed number of times. If the probability of getting seven heads is equal to that of getting nine heads, find the probability of getting exactly two heads.

PART - IV

Answer all the questions. **7 x 5 = 35**

41. (a) By using Gaussian elimination method, balance the chemical reaction equation: $C_2H_6 + O_2 \rightarrow H_2O + CO_2$ **(OR)**
 (b) If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^3 + y^2 + 3x - 3y + 2 = 0$
42. (a) Solve the equation: $3x^4 - 16x^3 + 26x^2 - 16x + 3 = 0$ **(OR)**
 (b) Solve : $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$
43. (a) A rod of length 1.2 m moves with its ends always touching the coordinate axes. The locus of a point P on the rod, which is 0.3m from the end in contact with x-axis is an ellipse. Find the eccentricity. **(OR)**
 (b) Find the non-parametric and Cartesian equations of the plane passing through the points (4,2,4) and is perpendicular to the planes $2x + 5y + 4z + 1 = 0$ and $4x + 7y + 6z + 2 = 0$
44. (a) A steel plant is capable of producing x tonnes per day of a low – grade steel and y tonnes per day of a high –grade steel, where $y = \frac{40-5x}{10-x}$. If the fixed market price of low-grade steel is half that of high-grade steel, then what should be optimal productions in low-grade steel and high-grade steel in order to have maximum receipts. **(OR)**
 (b) Let $z(x,y) = xe^y + ye^{-x}, x = e^{-t}, y = st^2, s, t \in \mathbb{R}$. Find $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$
45. (a) Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$ **(OR)**
 (b) Water temperature $100°C$ cools in 10 minutes to $80°C$ in a room temperature of $25°C$
 Find (i) The temperature of water after 20 minutes
 (ii) The time when ten temperature is $40°C$ $\left[\log_e \frac{11}{15} = -0.3101; \log_e 5 = 1.6094\right]$
46. (a) Suppose a discrete random variable can take only the values 0, 1 and 2. The probability mass function is defined by
 $$f(x) = \begin{cases} \frac{x^2+1}{k}, & \text{for } x = 0,1,2 \\ 0 & \text{otherwise} \end{cases}$$
 Find (i) the value of k (ii) cumulative distribution function (iii) $P(X \geq 1)$ **(OR)**
 (b) Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent.
47. (a) Prove by vector method that $\sin(\alpha + \beta) = sin\alpha\ cos\beta + cos\alpha\ sin\beta$ **(OR)**
 (b) Find the equations of tangent and normal to the curve $y^2 - 4x - 2y + 5 = 0$ at the point where it cuts the x–axis.