WAY TO SUCCESS

Leads to Success

12^{th} Standard

Maths

FIRST REVISION 2023

Various District
Question Paper Collection

TNJ

FIRST REVISION TEST - 2023

12 - Std **MATHEMATICS**

Time : 3.00 hrs. Marks : 90

I All questions are compulsory. 20 X 1 = 20

1. If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$, then adj (adj A) is

a) $\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{pmatrix}$ c) $\begin{pmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{pmatrix}$ d) $\begin{pmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{pmatrix}$

2. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is a) 1 b) -1 c) i d) 0

3. A zero of $x^3 + 64$ is a) 0 b) 4 c) 4i d) - 4

4. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

5. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

a) [-1, 0] b) [0,1] c) [-1, 1] d) [1,2]

6. If x + y = k is a normal to the parabola y^2 - 12x, then the value of k is

a) 1 b) 9 c) 3 d) - 1

7. If the coordinates at one end of a diameter of the circle $x^2+y^2-8x-4y + c = 0$ are (11,2) the cordinates of the other end are

a) (-5, 2) b) (-3, 2) c) (5,-2) d) (-2,5)

8. Distance from the origin to the plane 3x - 6y +2z + 7 = 0 is

a) 0 b) 1 c) 2 d) 3

9. The value of $(1+i)^4 + (1-i)^4$ is

a) 8 b) 4 c) -8 d) -4

10. If A is a square matrix of order n, then |adj A| =

a) $|A|^{n-1}$ b) $|A|^{n-2}$ c) $|A|^n$ d) None

11. The point of inflection of the curve $y = (x -1)^3$ is

a) (0,0) b) (0,1) c) (1,0) d) (1,1)

12. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

a) $e^{x^2+y^2}$ b) 2xu c) x^2u d) y^2u

13. The value of $\int_0^{\pi} \sin^4 x\,dx$ is a) $\frac{3\pi}{10}$ b) $\frac{3\pi}{8}$ c) $\frac{3\pi}{4}$ d) $\frac{3\pi}{2}$

14. The value of $\int_0^1 x(1-x)^{99}\,dx$ is

a) $\frac{1}{11000}$ b) $\frac{1}{10100}$ c) $\frac{1}{10010}$ d) $\frac{1}{10001}$

15. The solution of the differential equation ${dy}/{dx} = 2xy$ is

a) $y = Ce^{x^2}$ b) $y = 2^{x^2} + C$ c) $y = Ce^{-x^2} + C$ d) $y = x^2 + C$

16. The integrating factor of the differential equation ${dy}/{dx} + p(x)y = Q(x)$ is x, then p(x)

a) x b) ${x^2}/{2}$ c) ${1}/{x}$ d) ${1}/{x^2}$

17. A random variable X has binomial distribution with n= 25 and p = 0.8 then standard deviation X is

a) 6 b) 4 c) 3 d) 2

18. If p(x=0) = 1 - p(x = 1), if E(x) = 3 var (x) then p(x = 0) is

a) ${2}/{3}$ b) ${2}/{5}$ c) ${1}/{5}$ d) ${1}/{3}$

19. Subtraction is not a binary operation in

a) R b) Z c) N d) Q

20. If x + y = 8, then the maximum value of xy is ....

a) 8 b) 16 c) 20 d) 24

II Answer any seven question. Q.No. 30 Compulsory. 7 X 2 = 14

21. Prove that $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.

22. Prove that $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = 2i$.

23. Form a polynomial equation with integer coefficient with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

24. Find the value of $\sin^{-1}\left(\sin\left({5\pi}/{4}\right)\right)$.

25. Find the acute angle between the straight line $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$.

26. Prove that : $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+\cos(x)}\, dx = \frac{\pi}{4}$.

27. Solve : $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

28. Find the points on the curve $y = x^3 - 3x^2 + x - 2$ at which the tangent is parallel to the line $y = x$.

29. Prove that the identity is unique if it exists.

30. Find the equation of tangent to the curve $y = x^2 - x^4$ at (1,0).

III Answer any seven question. Q.No. 40 is compulsory. 7 X 3 = 21

31. If $A = \begin{pmatrix} 0 & -3 \\ 1 & 4 \end{pmatrix}$ $B = \begin{pmatrix} -2 & -3 \\ 0 & -1 \end{pmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$.

32. Find the square root of 6 - 8i.

33. Solve the equation $x^4 - 9x^2 + 20 = 0$.

34. With usual notation, in any triangle ABC prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

35. Evaluate : $\lim_{x \to \infty} \frac{2x^2 - 3}{x^2 - 5x + 3}$.

36. If $u(x,y) = \frac{x^2 + y^2}{\sqrt{x+y}}$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = \frac{3}{2}u$.

37. Let * be defined on R by (a*b) = a + b + ab - 7. If * binary an R? If so find $3 * \left(\frac{-7}{15}\right)$.

38. Show that $\int_0^{\pi/3} \frac{\sec x \tan x}{1+\sec^2 x}\, dx = \tan^{-1}(2)\frac{-\pi}{4}$.

39. The mean and variance of a binaryial variate x are respectively 2 and 1.5. Find p(x=0).

40. Obtain the equation of circle for which (3,4) and (2,-7) are the end of a diameter.

IV Answer all the questions. $7 \times 5 = 35$

41. a) Solve $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0$, $\frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0$, $\frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$ by Cramer's rule. ***(OR)*** b) If 2 cos α = x + $\frac{1}{x}$ and 2 cos β = y + $\frac{1}{y}$ show that

i) $\frac{x^m}{y^n}-\frac{y^n}{x^m}$ = 2i sin (mα - nβ) ii) $x^m y^n + \frac{1}{x^m y^n}$ = 2cos (mα + nβ)

42. a) Find the equation of the circle passing through the points (1,1), (2,-1) and (3,2). ***(OR)***

b) By vector method, prove that cos(α - β) = cosα cosβ + sinα sin β.

43. a) Solve : $6x^4 - 35x^3 + 6x^2 - 35x + 6 = 0$. ***(OR)***

b) Evaluate : $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$

44. a) Find vector and Cartesion equation of the plane passing through the point (0,1,5) and parallel to the straight line $\bar{r}$ = (i + 2j - 4k) + s (2i + 3j + 6k) and $\bar{r}$ = (i - 3j + 5k) + t(i + j - k) ***(OR)***

b) If u = $\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}$ tan u.

45. a) Find the value of $\cot^{-1}(1)+\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)-\sec^{-1}(-2)$ ***(OR)***

b) Show that the equation of the parabola with focus (-$\sqrt{2}$,0) and directrix and x = $\sqrt{2}$ is $y^2 = -4\sqrt{2}x$.

46. a) Show that the area of the region bounded by 3x -2y + 6 = 0, x =-3 x = 1 and x axis, is $\frac{15}{2}$. ***(OR)***

b) Show that the solution of the differential equation $(1+x^2)\frac{dy}{dx}=1+y^2$ is $\tan^{-1}y = \tan^{-1}x + c$ or $\tan^{-1}x = \tan^{-1}y + c$.

47. a) Prove $p \to (7qvr) = 7pr(7qvr)$ using truth table. ***(OR)***

b) The distribution function of a continuous random variable is

$$p(x)=\begin{cases} 0 & x<1 \\ \frac{x-1}{4} & 1\le x\le 5 \\ 1 & x>0 \end{cases}$$

find i) p(x < 3) ii) p(2< x < 4) ii) p (3 ≤ x) .

TNJ 12 - கணிதம் (EM) பக்கம் - 4

Tsi12M

Tenkasi District Common Examinations
First Revision Examination - January 2023

06-01-2023

Standard 12
MATHEMATICS

Time: 3.00 hrs | Marks: 90

Part - I

Note: i) *All questions are compulsory.*
ii) *Choose the correct or most suitable answer from the given four alternatives. Write the options code and the corresponding answer.* **20×1=20**

1) If A, B and C are invertible matrices of some order, then which one of the following is not true?
   a) $\text{adj } A = |A|A^{-1}$ b) $\text{adj}(AB) = (\text{adj } A)(\text{adj } B)$
   c) $\det A^{-1} = (\det A)^{-1}$ d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$
2) If $z = x+iy$ is a complex number such that $|z+2| = |z-2|$, then the locus of z is
   a) real axis b) imaginary axis c) ellipse d) circle
3) A zero of x^3+64 is
   a) 0 b) 4 c) 4i d) −4
4) The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is
   a) −2 b) −1 c) 1 d) 2
5) If $\sin^{-1}x = 2\sin^{-1}\alpha$ has a solution, then
   a) $|\alpha| \le \frac{1}{\sqrt{2}}$ b) $|\alpha| \ge \frac{1}{\sqrt{2}}$ c) $|\alpha| < \frac{1}{\sqrt{2}}$ d) $|\alpha| > \frac{1}{\sqrt{2}}$
6) Which one is the inverse of the statement $(p \vee q) \rightarrow (p \wedge q)$?
   a) $(p \wedge q) \rightarrow (p \vee q)$ b) $\neg(p \vee q) \rightarrow (p \wedge q)$
   c) $(\neg p \vee \neg q) \rightarrow (\neg p \wedge \neg q)$ d) $(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$
7) If $x+y=k$ is a normal to the parabola $y^2 = 12x$, then the value of k is
   a) 3 b) −1 c) 1 d) 9
8) Distance from the origin to the lane $3x-6y+2z+7 = 0$ is
   a) 0 b) 1 c) 2 d) 3
9) Evaluate: $\int_0^1 [2x]\,dx$ where [.] is the greatest integer function.
   a) 0 b) 2 c) $\frac{1}{2}$ d) 1
10) The minimum value of the function $|3-x|+9$ is
   a) 0 b) 3 c) 6 d) 9
11) If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to
   a) xye^{xy} b) $(1+xy)e^{xy}$ c) $(1+y)e^{xy}$ d) $(1+x)e^{xy}$
12) The differential equation representing the family of curves $y = A\cos(x+B)$, where A and B are parameters is
   a) $\frac{d^2y}{dx^2} - y = 0$ b) $\frac{d^2y}{dx^2} + y = 0$ c) $\frac{d^2y}{dx^2} = 0$ d) $\frac{d^2x}{dy^2} = 0$
13) A baloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. The rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 meters above the ground.
   a) $\frac{3}{25}$ radians/sec b) $\frac{4}{25}$ radians/sec c) $\frac{1}{5}$ radians/sec d) $\frac{1}{3}$ radians/sec

14) The area between $y^2 = 4x$ and its latus rectum is

a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{8}{3}$ d) $\frac{5}{3}$

15) If $2\bar{i} - \bar{j} + 3\bar{k}$, $3\bar{i} + 2\bar{j} + \bar{k}$, $\bar{i} + m\bar{j} + 4\bar{k}$ are coplanar, find the vlue of m.

a) 3 b) $-\frac{1}{3}$ c) -3 d) $\frac{1}{3}$

16) The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is

a) $y+\sin^{-1}x = c$ b) $x+\sin^{-1}y = 0$ c) $y^2+2\sin^{-1}x = c$ d) $x^2+2\sin^{-1}y=0$

17) If $y = 4x+c$ is a tangent to the circle $x^2+y^2 = 9$ find c

a) $\pm 3\sqrt{17}$ b) $\pm 17\sqrt{4}$ c) $\pm 17\sqrt{3}$ d) $\pm 4\sqrt{17}$

18) If A^TA^{-1} is symmetric, then $A^2 =$

a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

19) On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is

a) $\frac{11}{243}$ b) $\frac{3}{8}$ c) $\frac{1}{243}$ d) $\frac{5}{243}$

20) Subtraction is not a binary operation in

a) $\mathbb{R}$ b) $\mathbb{Z}$ c) $\mathbb{N}$ d) $\mathbb{Q}$

Part - II

Note: Answer any seven questions. Question No. 30 is compulsory **7x2=14**

21) Prove that $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ is orthogonal.

22) Find the square root of $-6+8i$

23) Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

24) Find the angle between the planes $\bar{r}.(\bar{i} + \bar{j} - 2\bar{k}) = 3$ and $2x-2y+z = 2$.

25) If $f(x, y) = x^3-3x^2+y^2+5x+6$ then find f_x at $(1, -2)$

26) Evaluate: $\int_0^{\pi/2} (\sin^2 x + \cos^4 x)dx$

27) Determine the order and degree (if exists) of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right)$$

28) Find the mean of the distribution $f(x) = \begin{cases} 3e^{-3x}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$

29) Find the equation of the parabola whose vertex is $(-2, 5)$ and focus $(-2, 2)$

30) On $\mathbb{Z}$, define $\otimes$ by $(m \otimes n) = m^n+n^m : \forall m,n \in \mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$?

Ts/12M

3

Part - III

$7 \times 3 = 21$

Note: Answer any seven questions. Question No. 40 is compulsory.

31) Solve by matrix inversion method: $5x+2y = 4$, $7x+3y = 5$.

32) Solve the equation $2x^3+11x^2-9x-18 = 0$

33) State and prove that triangle inequality.

34) Evaluate: $\int_0^{\frac{\pi}{3}} \frac{\sec x \tan x}{1+\sec^2 x} dx$

35) Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $\frac{2b^2}{a}$

36) Solve the differential equation $\frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$

37) If $u(x, y) = \frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{3}{2}u$

38) Verify whether the compound proposition $(p \to q) \leftrightarrow (\neg p \to q)$ is a tautology or contradiction or contingency.

39) The mean and variance of a binomial variate X are respectively 2 and 1.5. Find $P(X = 0)$

40) If $\vec{a}, \vec{b}, \vec{c}$ are three vectors prove that $\left[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}\right] = 2\left[\vec{a}, \vec{b}, \vec{c}\right]$

Part - IV

$7 \times 5 = 35$

Note. Answer all the questions:

41) a] Investigate for what values of λ and μ the system of linear equations $x+2y+z = 7$, $x+y+\lambda z = \mu$, $x+3y-5z = 5$ has (i) no solution (ii) a unique solution.

(OR)

b] If $z = x+iy$ and $\arg\left(\frac{z-1}{z+2}\right) = \frac{\pi}{4}$, then show that $x^2+y^2+3x-3y+2 = 0$

42) a] If the curves $ax^2+by^2 = 1$ and $cx^2+dy^2 = 1$ intersect each other orthogonally then, $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$

(OR)

b] At a water fountain, water attains a maximum height of 4 m at horizontal distance of 0.5 m from its origin. The flow is from the origin and the path of water is a parabola open upwards, find the height of water at a horizontal distance of 0.75 m from the point of origin.

2

43) a] If $\vec{a} = \hat{i} - \hat{j}$, $\vec{b} = \hat{i} - \hat{j} - 4\hat{k}$, $\vec{c} = 3\hat{j} - \hat{k}$ and $\vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$ verify that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$

(OR)

b] The equation of electromotive force for an electric circuit containing resistance an self-inductance is $E = Ri + L\frac{di}{dt}$, where E is the electromotive force given to the circuit, the resistance and L, the coefficient of induction. Find the current i at time t when $E = 0$.

44) a] Find intervals of concavity and points of inflexion for the function $f(x) = \frac{1}{2}(e^x - e^{-x})$

(OR)

b] Verify (i) closure property (ii) commutative property (iii) associative property, (iv) existence of identity and (v) existence of inverse for the operation $\times_{11}$ on a subset A = {1, 3, 4, 5, 9} of the set of remainder {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

45) a] Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$

(OR)

b] Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r} = (6\hat{i} - \hat{j} - \hat{k}) + s(-\hat{i} + 2\hat{j} + \hat{k}) + t(-5\hat{i} - 4\hat{j} - 5\hat{k})$

46) a] For the ellipse $4x^2+y^2+24x-2y+21 = 0$, find the centre, vertices, foci and the length of latus rectum.

(OR)

b] If $U = \sec^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2 \cot u$

47) a] The cumulative distribution function of a discrete random variable is given by $f(x) = \begin{cases} 0 & -\infty < x < -1 \\ 0.15 & -1 \le x < 0 \\ 0.35 & 0 \le x < 1 \\ 0.60 & 1 \le x < 2 \\ 0.85 & 2 \le x < 3 \\ 1 & 3 \le x < \infty \end{cases}$ Find (i) the probability mass function (ii) $P(X < 1)$ and (iii) $P(X \ge 2)$

(OR)

b] If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z = \pi$ and $0 < x, y, z < 1$ then show that $x^2+y^2+z^2+2xyz = 1$.

SIVAKUMAR. M, Sri Ram Matric HSS, Vallam - 627809, Tenkasi District.

CHENNAI

Register Number

Class : 12

FIRST REVISION EXAMINATION - 2022 - 23

MATHEMATICS

Time Allowed : 3.00 Hours] [Max. Marks : 90

PART - I

20 x 1 = 20

1. Answer all the questions by choosing the correct answer from the given 4 alternatives
2. Write question number, correct option and corresponding answer
3. Each question carries 1 mark

1. If $A\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}=\begin{bmatrix}6 & 0\\0 & 6\end{bmatrix}$, then $A =$

(1) $\begin{bmatrix}1 & -2\\1 & 4\end{bmatrix}$ (2) $\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}$ (3) $\begin{bmatrix}4 & 2\\-1 & 1\end{bmatrix}$ (4) $\begin{bmatrix}4 & -1\\2 & 1\end{bmatrix}$

2. If $\omega \neq 1$ is a cubic root of unity and $(1+\omega)^7 = A + B\omega$, then (A, B) equals

(1) $(1, 0)$ (2) $(-1, 1)$ (3) $(0, 1)$ (4) $(1, 1)$

3. The number of real numbers in $[0, 2\pi]$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

(1) 2 (2) 4 (3) 1 (4) ∞

4. If α, β, and γ are the zeros of $x^3 + px^2 + qx + r$, then $\sum\frac{1}{\alpha}$ is

(1) $-\frac{q}{r}$ (2) $-\frac{p}{r}$ (3) $\frac{q}{r}$ (4) $-\frac{q}{p}$

5. If $\sin^{-1} x + \sin^{-1} y = \frac{2\pi}{3}$; then $\cos^{-1} x + \cos^{-1} y$ is equal to

(1) $\frac{2\pi}{3}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{6}$ (4) π

6. An ellipse has *OB* as *semi minor axes*, *F* and *F'* its foci and the angle *FBF'* is a right angle. Then the eccentricity of the ellipse is

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{1}{\sqrt{3}}$

7. If $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{c} = \vec{c}\cdot\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$ is

(1) $|\vec{a}||\vec{b}||\vec{c}|$ (2) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ (3) 1 (4) -1

8. The abscissa of the point on the curve $f(x) = \sqrt{8-2x}$ at which the slope of the tangent is -0.25?

(1) -8 (2) -4 (3) -2 (4) 0

9. If $f(x) = \frac{x}{x+1}$, then its differential is given by

(1) $\frac{-1}{(x+1)^2}dx$ (2) $\frac{1}{(x+1)^2}dx$ (3) $\frac{1}{x+1}dx$ (4) $\frac{-1}{x+1}dx$

10. The percentage error of fifth root of 31 is approximately how many times the percentage error in 31 ?

(1) $\frac{1}{31}$ (2) $\frac{1}{5}$ (3) 5 (4) 31

11. If $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ then n is

(1) 10 (2) 5 (3) 8 (4) 9

12. The degree of the differential equation $y(x) = 1 + \frac{dy}{dx} + \frac{1}{1\cdot 2}\left(\frac{dy}{dx}\right)^2 + \frac{1}{1\cdot 2\cdot 3}\left(\frac{dy}{dx}\right)^3 + \ldots$ is

(1) 2 (2) 3 (3) 1 (4) 4

13. If in 6 trials, X is a binomial variable which follows the relation $9P(X=4) = P(X=2)$, then the probability ...

(1) 0.125 (2) 0.25 (3) 0.375 (4) 0.75

14. Which one of the following statements has the truth value T ?

(1) $\sin x$ is an even function.

(2) Every square matrix is non-singular

(3) The product of complex number and its conjugate is purely imaginary

(4) $\sqrt{5}$ is an irrational number

15. A is a non-singular matrix of order n then adj(adj A) =

(1) A (2) $|A|^{n-1} A$ (3) $|A|^{n-2} A$ (4) $|A|^{n} A$

16. $\tan^{-1}(3) + \tan^{-1}(x) = \tan^{-1}(8)$ then x=

(1) 5 (2) $\frac{1}{5}$ (3) $\frac{5}{14}$ (4) $\frac{14}{5}$

17. Focus of $\frac{x^2}{7} - \frac{y^2}{9} = 1$ is

(1) $(\pm\sqrt{2}, 0)$ (2) $(0, \pm\sqrt{2})$ (3) $(\pm 4, 0)$ (4) $(0, \pm 4)$

18. The function $f(x) = \frac{x}{\log x}$ increases in the interval

(1) $(1, \infty)$ (2) $(-1, \infty)$ (3) $(0, \infty)$ (4) none of the above

19. Area of ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is

(1) 12π (2) 16π (3) 9π (4) 144π

20. If $a * b = a + b - 1$ then identity element is

(1) 0 (2) -1 (3) 1 (4) -2

PART – II

1. **Answer any 7 questions** **7 x 2 = 14**
2. **Each question carries 2 marks**
3. **Question number 30 is compulsory**

21. Find the differential equation of the family of all the parabolas with latus rectum 4a and whose axes are parallel to the x-axis.

22. If $\frac{z+3}{z-5i} = \frac{1+4i}{2}$, find the complex number z in the rectangular form

23. Discuss the maximum possible number of positive and negative roots of the polynomial equation $9x^9 - 4x^8 + 4x^7 - 3x^6 + 2x^5 + x^3 + 7x^2 + 7x + 2 = 0$.

24. If $\cot^{-1}\left(\frac{1}{7}\right) = \theta$, find the value of $\cos\theta$.

25. If $A^3 = A$ then find A^{-1}

26. Find the angles between the straight line $\frac{x+3}{2} = \frac{y-1}{2} = -z$ with coordinate axes.

27. Find maclaurin's series for $\frac{1}{1-x}$

28. Evaluate $\lim_{(x,y)\to(1,2)} g(x,y)$, if the limit exists, where $g(x,y) = \frac{3x^2-xy}{x^2+y^2+3}$.

29. Evaluate the following integrals using properties of integration : $\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \sin^2 x \, dx$

30. Write the Properties of cumulative distribution function

PART – III

7 x 3 = 21

1. **Answer any 7 questions**
2. **Each question carries 3 marks**
3. **Question number 40 is compulsory**

31. Prove that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

32. Prove the following properties: $\text{Re}(z)=\frac{z+\bar{z}}{2}$ and $\text{Im}(z)=\frac{z-\bar{z}}{2i}$

33. If α and β are the roots of the equation $3x^2 - 4x + 2 = 0$, then find $\frac{1}{\alpha}+\frac{1}{\beta}$ and $\alpha^3+\beta^3$

34. Find the equation of the ellipse in each of the cases given below: length of latus rectum 8, eccentricity $= \frac{3}{5}$ and major axis on x -axis.

35. With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

36. Find the absolute extrema of the following functions on the given closed interval $f(x) = 6x^{\frac{4}{3}} - 3x^{\frac{1}{3}}$; $[-1,1]$

37. Evaluate: $\int_1^2 \frac{x}{(x+1)(x+2)} dx$.

38. Solve the following differential equations: $x \cos y \, dy = e^x(x \log x + 1)dx$

39. Find the probability mass function $f(x)$ of the discrete random variable X whose cumulative distribution function $F(x)$ is given by $F(x) = \begin{cases} 0 & -\infty < x < -2 \\ 0.25 & -2 \le x < -1 \\ 0.60 & -1 \le x < 0 \\ 0.90 & 0 \le x < 1 \\ 1 & 1 \le x < \infty \end{cases}$ Also find (i) $P(X < 0)$ and (ii) $P(X \ge 1)$.

40. $G = \{1, -1, i, -i\}$ Verify (i) ClosureProperty (ii) Identity property (iii) Inverse property with respect to complex number Multiplication on G

PART – IV

7 x 5 = 35

1. **Answer all the questions**
2. **Each question carries 5 marks**

41. (a) Solve the following systems of linear equations by Cramer's rule:

$\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0$, $\frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0$, $\frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$

(OR)

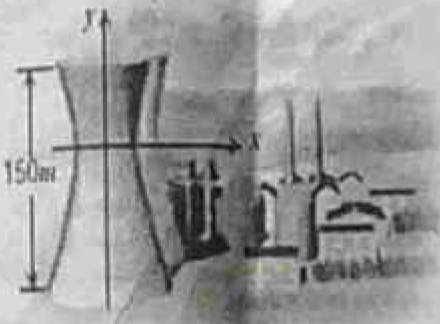
(b) Show that $\left(\frac{19+9i}{5-3i}\right)^{15} - \left(\frac{8+i}{1+2i}\right)^{15}$ is purely imaginary.

42. (a) Solve : $(2x-1)(x+3)(x-2)(2x+3)+20=0$

(OR)

(b) Prove that $\tan(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}, -1 < x < 1$

43. (a) Cross section of a Nuclear cooling tower is in the shape of a hyperbola with equation $\frac{x^2}{30^2} - \frac{y^2}{44^2} = 1$. The tower is 150m tall and the distance from the top of the tower to the centre of the hyperbola is half the distance from the base of the tower to the centre of the hyperbola. Find the diameter of the top and base of the tower.



(OR)

(b) If $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{i} - \hat{j} - 4\hat{k}, \vec{c} = 3\hat{j} - \hat{k}$ and $\vec{d} = 2\hat{i} + 5\hat{j} + \hat{k}$, verify that $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = [\vec{a}, \vec{b}, \vec{d}]\vec{c} - [\vec{a}, \vec{b}, \vec{c}]\vec{d}$

44. (a) Verify the following points (1,3,1), (1,1,-1), (-1,1,1), (2,2,-1) are coplanar

(OR)

(b) If the curves $ax^2 + by^2 = 1$ and $cx^2 + dy^2 = 1$ intersect each other orthogonally then, show that $\frac{1}{a} - \frac{1}{b} = \frac{1}{c} - \frac{1}{d}$.

45. (a)A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine (Brine is a high-concentration solution of salt (usually sodium chloride) in water) runs in a rate of 10 litres per minute, and each litre contains 5 grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time t.

(OR)

(b) Evaluate : $\lim_{x\to 0^+} x^{\sin x}$

46. (a) Father of a family wishes to divide his square field bounded by $x = 0, x = 4, y = 4$ and $y = 0$ along the curve $y^2 = 4x$ and $x^2 = 4y$ into three equal parts for his wife, daughter and son. Is it possible to divide? If so, find the area to be divided among them.

(OR)

(b) Solve $y^2 + x^2\frac{dy}{dx} = xy\frac{dy}{dx}$.

47. (a) If X is the random variable with probability density function f (x) given by, $f(x) = \begin{cases} x+1, & -1 \le x < 0 \\ -x+1, & 0 \le x < 1 \\ 0, & \text{otherwise} \end{cases}$

then find (i) the distribution function F(x) (ii) $P(-0.5 \le X \le 0.5)$

(OR)

(b) (i) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let $*$ be the matrix multiplication. Determine Whether M is closed under $*$. If so, examine the commutative and associative properties satisfied by $*$ on M.

(ii) Also examine the existence of identity, existence of inverse properties for the operation $*$ on M.

TNJ

FIRST REVISION TEST - 2023

12 - Std **MATHEMATICS**

Time : 3.00 hrs. Marks : 90

I All questions are compulsory. 20 X 1 = 20

1. If $A = \begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$, then adj (adj A) is

a) $\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$ b) $\begin{pmatrix} 6 & -6 & 8 \\ 4 & -6 & 8 \\ 0 & -2 & 2 \end{pmatrix}$ c) $\begin{pmatrix} -3 & 3 & -4 \\ -2 & 3 & -4 \\ 0 & 1 & -1 \end{pmatrix}$ d) $\begin{pmatrix} 3 & -3 & 4 \\ 0 & -1 & 1 \\ 2 & -3 & 4 \end{pmatrix}$

2. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is a) 1 b) -1 c) i d) 0

3. A zero of $x^3 + 64$ is a) 0 b) 4 c) 4i d) - 4

4. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to

a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

5. The domain of the function defined by $f(x) = \sin^{-1}\sqrt{x-1}$ is

a) [-1, 0] b) [0,1] c) [-1, 1] d) [1,2]

6. If x + y = k is a normal to the parabola $y^2 - 12x$, then the value of k is

a) 1 b) 9 c) 3 d) - 1

7. If the coordinates at one end of a diameter of the circle $x^2+y^2-8x-4y + c = 0$ are (11,2) the cordinates of the other end are

a) (-5, 2) b) (-3, 2) c) (5,-2) d) (-2,5)

8. Distance from the origin to the plane 3x - 6y +2z + 7 = 0 is

a) 0 b) 1 c) 2 d) 3

9. The value of $(1+i)^4 + (1-i)^4$ is

a) 8 b) 4 c) -8 d) -4

10. If A is a square matrix of order n, then |adj A| =

a) $|A|^{n-1}$ b) $|A|^{n-2}$ c) $|A|^n$ d) None

11. The point of inflection of the curve $y = (x-1)^3$ is

a) (0,0) b) (0,1) c) (1,0) d) (1,1)

12. If $u(x,y) = e^{x^2+y^2}$, then $\frac{\partial u}{\partial x}$ is equal to

a) $e^{x^2+y^2}$ b) 2xu c) x^2u d) y^2u

13. The value of $\int_0^{\pi} \sin^4 x\,dx$ is a) $\frac{3\pi}{10}$ b) $\frac{3\pi}{8}$ c) $\frac{3\pi}{4}$ d) $\frac{3\pi}{2}$

14. The value of $\int_0^1 x(1-x)^{99}\,dx$ is

a) $\frac{1}{11000}$ b) $\frac{1}{10100}$ c) $\frac{1}{10010}$ d) $\frac{1}{10001}$

15. The solution of the differential equation $\frac{dy}{dx} = 2xy$ is

a) $y = Ce^{x^2}$ b) $y = 2^{x^2} + C$ c) $y = Ce^{-x^2} + C$ d) $y = x^2 + C$

16. The integrating factor of the differential equation $\frac{dy}{dx} + p(x)y = Q(x)$ is x, then p(x)

a) x b) $\frac{x^2}{2}$ c) $\frac{1}{x}$ d) $\frac{1}{x^2}$

17. A random variable X has binomial distribution with n= 25 and p = 0.8 then standard deviation X is

a) 6 b) 4 c) 3 d) 2

18. If p(x=0) = 1 - p(x = 1), if E(x) = 3 var (x) then p(x = 0) is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

19. Subtraction is not a binary operation in

a) R b) Z c) N d) Q

20. If x + y = 8, then the maximum value of xy is ....

a) 8 b) 16 c) 20 d) 24

II Answer any seven question. Q.No. 30 Compulsory. 7 X 2 = 14

21. Prove that $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$ is orthogonal.

22. Prove that $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3 = 2i$.

23. Form a polynomial equation with integer coefficient with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root.

24. Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$.

25. Find the acute angle between the straight line $\frac{x-4}{2} = \frac{y}{1} = \frac{z+1}{-2}$ and $\frac{x-1}{4} = \frac{y+1}{-4} = \frac{z-2}{2}$.

26. Prove that : $\int_0^{\pi/2} \frac{f(\sin x)}{f(\sin x)+\cos(x)}\,dx = \frac{\pi}{4}$.

27. Solve : $\frac{dy}{dx} = \frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$.

28. Find the points on the curve $y = x^3 - 3x^2 + x - 2$ at which the tangent is parallel to the line $y = x$.

29. Prove that the identity is unique if it exists.

30. Find the equation of tangent to the curve $y = x^2 - x^4$ at (1,0).

III Answer any seven question. Q.No. 40 is compulsory. 7 X 3 = 21

31. If $A = \begin{pmatrix} 0 & -3 \\ 1 & 4 \end{pmatrix}$ $B = \begin{pmatrix} -2 & -3 \\ 0 & -1 \end{pmatrix}$ verify that $(AB)^{-1} = B^{-1}A^{-1}$.

32. Find the square root of 6 - 8i.

33. Solve the equation $x^4 - 9x^2 + 20 = 0$.

34. With usual notation, in any triangle ABC prove by vector method that $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

35. Evaluate : $\lim_{x \to \infty} \frac{2x^2 - 3}{x^2 - 5x + 3}$.

36. If $u(x,y) = \frac{x^2 + y^2}{\sqrt{x+y}}$ prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = \frac{3}{2}u$.

37. Let $*$ be defined on R by $(a*b) = a + b + ab - 7$. If $*$ binary an R? If so find $3 * \left(\frac{-7}{15}\right)$.

38. Show that $\int_0^{\pi/3} \frac{\sec x \tan x}{1+\sec^2 x}\,dx = \tan^{-1}(2)\frac{-\pi}{4}$.

39. The mean and variance of a binaryial variate x are respectively 2 and 1.5. Find p(x=0).

40. Obtain the equation of circle for which (3,4) and (2,-7) are the end of a diameter.

IV Answer all the questions. $7 \times 5 = 35$

41. a) Solve $\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-1=0$, $\frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0$, $\frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$ by Cramer's rule. ***(OR)*** b) If 2 cos $\alpha = x + \frac{1}{x}$ and 2 cos $\beta = y + \frac{1}{y}$ show that

i) $\frac{x^m}{y^n}-\frac{y^n}{x^m} = 2i \sin(m\alpha - n\beta)$ ii) $x^m y^n + \frac{1}{x^m y^n} = 2\cos(m\alpha + n\beta)$

42. a) Find the equation of the circle passing through the points (1,1), (2,-1) and (3,2). ***(OR)***

b) By vector method, prove that $\cos(\alpha - \beta) = \cos\alpha \cos\beta + \sin\alpha \sin\beta$.

43. a) Solve : $6x^4 - 35x^3 + 6x^2 - 35x + 6 = 0$. ***(OR)***

b) Evaluate : $\int_{-\pi}^{\pi} \frac{\cos^2 x}{1+a^x} dx$

44. a) Find vector and Cartesion equation of the plane passing through the point (0,1,5) and parallel to the straight line $\bar{r}$ = (i + 2j - 4k) + s (2i + 3j + 6k) and $\bar{r}$ = (i - 3j + 5k) + t(i + j - k) ***(OR)***

b) If u = $\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\tan u$.

45. a) Find the value of $\cot^{-1}(1)+\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)-\sec^{-1}(-2)$ ***(OR)***

b) Show that the equation of the parabola with focus $(-\sqrt{2},0)$ and directrix and $x = \sqrt{2}$ is $y^2 = -4\sqrt{2}x$.

46. a) Show that the area of the region bounded by $3x - 2y + 6 = 0$, $x = -3$ $x = 1$ and x axis, is $\frac{15}{2}$. ***(OR)***

b) Show that the solution of the differential equation $(1+x^2)\frac{dy}{dx}=1+y^2$ is $\tan^{-1}y = \tan^{-1}x + c$ or $\tan^{-1}x = \tan^{-1}y + c$.

47. a) Prove $p \rightarrow (7qvr) = 7pr(7qvr)$ using truth table. ***(OR)***

b) The distribution function of a continuous random variable is

$$p(x)=\begin{cases} 0 & x<1 \\ \frac{x-1}{4} & 1\le x\le 5 \\ 1 & x>0 \end{cases}$$ find i) $p(x < 3)$ ii) $p(2 < x < 4)$ ii) $p(3 \le x)$.

THIRUVALLUR

COMMON FIRST REVISION TEST - 2023

Standard XII Reg.No.

MATHEMATICS

Time: 3.00 hours **Part - I** *Marks: 90*

I. Choose the correct answer: **20 x 1 = 20**

1. If $A=\begin{bmatrix}2 & 0\\1 & 5\end{bmatrix}$ and $B=\begin{bmatrix}1 & 4\\2 & 0\end{bmatrix}$ then $|\text{adj}(AB)|=$
   a) −40 b) −80 c) −60 d) −20

2. If $A=\begin{bmatrix}2 & 3\\5 & -2\end{bmatrix}$ be such that $\lambda A^{-1}=A$, then λ is
   a) 17 b) 14 c) 19 d) 21

3. The principal argument of $(\sin 40^\circ + i\cos 40^\circ)^5$ is
   a) -110° b) -70° c) 70° d) 110°

4. If α and β are the roots of $x^2+x+1=0$, then $\alpha^{2020}+\beta^{2020}$ is
   a) −2 b) −1 c) 1 d) 2

5. The minimum number of imaginary roots for the polynomial $9x^9+2x^5-x^4-7x^2+2$ is
   a) 3 b) 6 c) 4 d) 9

6. The domain of the function defined by $f(x)=\sin^{-1}\sqrt{x-1}$ is
   a) [1,2] b) [−1,1] c) [0,1] d) [−1,0]

7. If $\sin^{-1}\frac{x}{5}+\operatorname{cosec}^{-1}\frac{5}{4}=\frac{\pi}{2}$, then the value of x is
   a) 4 b) 5 c) 2 d) 3

8. If the coordinates at one end of a diameter of the circle $x^2+y^2-8x-4y+c=0$ are (11,2), the coordinates of the other end are
   a) (−5,2) b) (2,−5) c) (5,−2) d) (−2,5)

9. Identify the type of the conic for the equation $x^2-2y=x+3$
   a) ellipse b) circle c) parabola d) hyperbola

10. The distance between the planes $x+2y+3z+7=0$ and $2x+4y+6z+7=0$ is
    a) $\frac{\sqrt{7}}{2\sqrt{2}}$ b) $\frac{7}{2}$ c) $\frac{\sqrt{7}}{2}$ d) $\frac{7}{2\sqrt{2}}$

11. The shortest distance between the two given straight lines
    $\vec{r}=(2\hat{i}+3\hat{j}+4\hat{k})+t(-2\hat{i}+\hat{j}-2\hat{k})$ and $\frac{x-3}{2}=\frac{y}{-1}=\frac{z+2}{2}$ is
    a) $\frac{365}{3}$ b) $\frac{\sqrt{365}}{3}$ c) $\frac{365}{4}$ d) $\frac{365}{\sqrt{3}}$

12. The position of a particle moving along a horizontal line of any time t is given by $s(t)=3t^2-2t-8$. The time at which the particle is at rest is
    a) $t=0$ b) $t=\frac{1}{3}$ c) $t=1$ d) $t=3$

13. The point of inflection of the curve $y=(x-1)^3$ is
    a) (0,0) b) (0,1) c) (1,0) d) (1,1)

14. The percentage error of the fifth root of 31 is approximately how many times the percentage error in 31?

a) $1/31$ b) $1/5$ c) 5 d) 31

15. The value of $\int_0^{\infty} e^{-3x}x^2\,dx$ is

a) $7/27$ b) $5/27$ c) $4/27$ d) $2/27$

16. The integrating factor of the differential equation $\frac{dy}{dx}+y=\frac{1+y}{\lambda}$ is

a) $\frac{x}{e^{\lambda}}$ b) $\frac{e^{\lambda}}{x}$ c) λe^{x} d) e^{x}

17. The order and degree of the differential equation $\left(\frac{d^4y}{dx^4}\right)^3+4\left(\frac{dy}{dx}\right)^7+6y=5\cos 3x$ are respectively

a) 2,3 b) 3,3 c) 2,6 d) 4,3

18. If in 6 trials, X is a binomial variable which follows the relation 9P(X=4) = P(X=2), then the probability of success is

a) 0.125 b) 0.25 c) 0.375 d) 0.75

19. If a compound statement involves 3 simple statements, then the number of rows in the truth table is

a) 9 b) 8 c) 6 d) 3

20. The operation * defined by $a*b=\frac{ab}{3}$ is not a binary operation on

a) Q^{+} b) Z c) R d) C

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. If $adj(A)=\begin{bmatrix}0 & -2 & 0\\ 6 & 2 & -6\\ -3 & 0 & 6\end{bmatrix}$, find A^{-1}.

22. Simplify : $i\, i^2\, i^3 \ldots\ldots i^{2000}$

23. If α, β and γ are the roots of the equation $x^3+px^2+qx+r=0$, find the value of $\Sigma\frac{1}{\beta r}$ in terms of the coefficients.

24. Simplify : $\cos^{-1}\left(\cos\left(\frac{13\pi}{3}\right)\right)$

25. If $2\hat{i}-\hat{j}+3\hat{k}$, $3\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+m\hat{j}+4\hat{k}$ are coplanar, find the value of m.

26. Find the value in the interval $\left(\frac{1}{2}, 2\right)$ satisfied by the Rolle's theorem for the function $f(x) = x + \frac{1}{x},\ x \in \left[\frac{1}{2}, 2\right]$

27. A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9.8 cm. Find the approximate change in the volume.

28. Evaluate : $\int_{-\pi/2}^{\pi/2} x \cos x \, dx$

29. Show that $x^2 + y^2 = r^2$ where r is a constant, is a solution of the differential equation $\frac{dy}{dx} = \frac{-x}{y}$

30. The probability density function of X is given by $f(x) = \begin{cases} k x e^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \le 0 \end{cases}$. Find the value of k

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. if $A = \begin{bmatrix} 8 & -4 \\ -5 & 3 \end{bmatrix}$, verify that $A(\text{adj} A) = (\text{adj} A)A = |A| I_2$.

32. Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$ and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

33. Solve the equation : $x^3 - 5x^2 - 4x + 20 = 0$

34. Find the value of $\sin^{-1}(-1) + \cos^{-1}\left(\frac{1}{2}\right) + \cot^{-1}(2)$

35. Find the equation of the ellipse with foci $(0, \pm 4)$ and end points of major axis $(0, \pm 5)$

36. Find the torque of the resultant of the three forces represented by $-3\hat{i} + 6\hat{j} - 3\hat{k}$, $4\hat{i} - 10\hat{j} + 12\hat{k}$ and $4\hat{i} + 7\hat{j}$ acting at the point with position vector $8\hat{i} - 6\hat{j} - 4\hat{k}$, about the point with position vector $18\hat{i} + 3\hat{j} - 9\hat{k}$

37. Find the intervals of monotonicity and hence find the local extremum for the function $f(x) = 2x^3 + 3x^2 - 12x$

38. Evaluate : $\int_2^3 \frac{\sqrt{x}}{\sqrt{5-x} + \sqrt{x}} dx$

39. Define an operation * on Q as follows $a * b = \left(\frac{a+b}{2}\right)$; $a, b \in Q$. Examine the closure, commutative and associative properties satisfied by * on Q.

40. If μ and σ^2 are the mean and variance of the discrete random variable X, and $E(X + 3) = 10$ and $E(X + 3)^2 = 116$, find μ and σ^2.

Part - IV

IV. Answer all the questions. **$7 \times 5 = 35$**

41. a) Solve the following system of linear equations of Gaussian elimination method $2x - 2y + 3z = 2$, $x + 2y - z = 3$, $3x - y + 2z = 1$ ***(OR)***

b) Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.

42. a) Solve the equation $Z^3 + 27 = 0$ ***(OR)***

b) Solve : $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$, if $6x^2 < 1$

43. a) Assume that water issuing from the end of a horizontal pipe 7.5 m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below the line of the pipe, the flow of water has curved outward 3 m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground? ***(OR)***

b) By Vector method, prove that $\cos(\alpha + \beta) = \cos\alpha \cos\beta - \sin\alpha \sin\beta$

44. a) Find the non-parametrics form of vector equation, and cartesian equations of the plane $r = (6\hat{i} - \hat{j} + \hat{k}) + s(-\hat{i} + 2\hat{j} + \hat{k}) + t(-5\hat{i} - 4\hat{j} - 5\hat{k})$ ***(OR)***

b) Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

45. a) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile 10 metre high?

(OR)

b) If $V(x, y) = \log\left(\frac{x^2 + y^2}{x + y}\right)$, prove that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1$

46. a) Prove that among all the rectangles of the given perimeter, the square has the maximum area. ***(OR)***

b) Find the area of the region bounded between the curves $y = \sin x$ and $y = \cos x$ and the lines $x = 0$ and $x = \pi$.

47. a) The mean and standard deviation of a binomial variate X are respectively 6 and 2. Find (i) the probability mass function (ii) $P(X = 3)$ (iii) $P(X \geq 2)$ ***(OR)***

b) Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

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KRISHNAGIRI

Krishnagiri

COMMON FIRST REVISION TEST – 2023

Reg.No. :

Standard XII

MATHEMATICS

Time: 3.00 hrs. Part - I Marks: 90

I. Choose the correct answer: $20 \times 1 = 20$

1. $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$ be such that $\lambda A^{-1} = A$, then λ is

a) 19 b) 17 c) 21 d) 14

2. If $|z - 2 + i| \le 2$, then the greatest value of $|z|$ is

a) $\sqrt{3} - 2$ b) $\sqrt{3} + 2$ c) $\sqrt{5} - 2$ d) $\sqrt{5} + 2$

3. If $A^T A^{-1}$ is symmetric, then $A^2 = ?$

a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

4. $\cos^{-1}\left(\sqrt{3}/2\right)$ - The principal value

a) $\pi/2$ b) $\pi/3$ c) $5\pi/6$ d) $\pi/6$

5. If α, β and γ are the zero's of $x^3 + px^2 + qx + r$ then $\sum \frac{1}{\alpha}$ is

a) $-q/r$ b) $-p/r$ c) q/r d) $-q/p$

6. If $(1 + i)(1 + 2i)(1 + 3i) \ldots\ldots\ldots (1 + ni) = x + iy$ then the value of $2.5.10\ldots.(1 + n^2)$ is

a) 1 b) i c) $x^2 + y^2$ d) $1 + n^2$

7. The radius of the circle $3x^2 + by^2 + 4bx - 6by + b^2 = 0$ is

a) 1 b) 3 c) $\sqrt{10}$ d) $\sqrt{11}$

8. The general equation of a circle with centre $(-3,-4)$ and radius 3 units is

a) $x^2 + y^2 - 6x + 8y + 16 = 0$ b) $x^2 + y^2 - 6x - 8y + 16 = 0$

c) $x^2 + y^2 + 6x - 8y + 16 = 0$ d) $x^2 + y^2 + 6x + 8y + 16 = 0$

9. If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$, then the value of $[\vec{a}, \vec{b}, \vec{c}]$

a) $|\vec{a}||\vec{b}||\vec{c}|$ b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ c) 1 d) -1

10. The value of $\int_0^1 x(1-x)^{99}\,dx$ is

a) $\frac{1}{10010}$ b) $\frac{1}{11000}$ c) $\frac{1}{10001}$ d) $\frac{1}{10100}$

11. The minimum value of the function $|3x - x| + 9$ is

a) 0 b) 3 c) 6 d) 9

12. The angle between the lines $\frac{x-2}{3} = \frac{y+1}{-2}$ $z = 2$ and $\frac{x-1}{1} = \frac{2y+3}{3} = \frac{z+5}{2}$ is

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

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13. If $f(x) = \frac{x}{x+1}$, then its differential is

a) $\frac{1}{x+1}dx$ b) $\frac{-1}{(x+1)^2}dx$ c) $\frac{-1}{x+1}dx$ d) $\frac{-1}{(x+1)^2}dx$

14. The value of $\int_0^{\infty} e^{-3x}x^2\,dx$ is

a) $\frac{7}{27}$ b) $\frac{5}{27}$ c) $\frac{4}{27}$ d) $\frac{2}{27}$

15. The solution of $\frac{dy}{dx} + P(x)y = 0$ is

a) $y = ce^{\int Pdx}$ b) $y = ce^{-\int Pdx}$ c) $x = ce^{-\int Pdy}$ d) $x = ce^{\int Pdy}$

16. Angle between $y^2 = x$ and $x^2 = y$ at the orgin is

a) $\tan^{-1}\left(\frac{3}{4}\right)$ b) $\tan^{-1}\left(\frac{4}{3}\right)$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{4}$

17. The solution of the differential equation $\frac{dy}{dx} = 2xy$ is

a) $y = ce^{x^2}$ b) $y = 2x^2 + c$ c) $x = ce^{-x^2}$ d) $x = x^2 + c$

18. If $P(X = 0) = 1 - P(X = 1)$ if $E(X) = 3\,var(X)$, then $P(X = 0)$ is

a) $\frac{2}{3}$ b) $\frac{2}{5}$ c) $\frac{1}{5}$ d) $\frac{1}{3}$

19. The operation * defined by $a * b = \frac{ab}{7}$ is not a binary operation on

a) Q^+ b) Z c) R d) C

20. The value of the limit $\lim_{x\to 0}\left(\cot x - \frac{1}{x}\right)$ is

a) 0 b) 1 c) 2 d) ∞

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. If $\text{adj}A = \begin{bmatrix} -1 & 2 & 2 \\ 1 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, find A^{-1}.

22. Find a polynomial equation of minimum degree with rational co-efficients, having $2 - \sqrt{3}$ as a root.

23. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form.

24. Find the principal value of $\tan^{-1}(\sqrt{3})$

25. Obtain the equation of the circle for which (3,4) and (2,−7) are the ends of a diameter.

26. If $\vec{a}, \vec{b}, \vec{c}$ are the three vectors, prove that $[\vec{a}+\vec{c}, \vec{a}+\vec{b}, \vec{a}+\vec{b}+\vec{c}] = [\vec{a}, \vec{b}, \vec{c}]$

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27. Find df for $f(x) = x^2 + 3x$ and evaluate it for $x = 2$ and $dx = 0.1$

28. Evaluate : $\int_{-\pi/2}^{\pi/2} x\cos x\,dx$

29. Write the Maclaurin series expansion of the following function : e^x

30. Show that $\sim(p \wedge q) \equiv \sim p \vee \sim q$

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. Solve the following system of linear equation using Matrix inversion method : $5x + 2y = 3$, $3x + 2y = 5$

32. If z_1, z_2 and z_3 are three complex numbers such that $|z_1| = 1$, $|z_2| = 2$, $|z_3| = 3$ and $|z_1 + z_2 + z_3| = 1$, show that $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 6$

33. Show that the equation $x^9 - 5x^5 + 4x^4 + 2x^2 + 1 = 0$ has atleast 6 imaginary solutions.

34. Prove that $\tan(\sin^{-1} x) = \frac{x}{\sqrt{1-x^2}}$, $-1 < x < 1$

35. The equation $y = \frac{1}{32}x^2$ models cross sections of parabolic mirrors that are used for solar energy. there is heating tube located at the focus of each parabola : How high is this tube located above the vertex of the parabola?

36. Evaluate : $\lim_{x\to 1}\left(\frac{x^2 - 3x + 2}{x^2 - 4x + 3}\right)$

37. Use the linear approximation to find approximate value of $(123)^{\frac{2}{3}}$

38. Let $A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$ be any three boolean matrices of the same type, find (i) $A \vee B$ ii) $A \wedge B$

39. Find the mean and variance of a random variable X, whose probability density function is $f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

40. Prove that $[\vec{a} \times \vec{b},\ \vec{b} \times \vec{c},\ \vec{c} \times \vec{a}] = [\vec{a},\ \vec{b},\ \vec{c}]^2$

Part - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) Solve the following systems of linear equations by Cramer's rule :

$$\frac{3}{x} - \frac{4}{y} - \frac{2}{z} - 1 = 0,\quad \frac{1}{x} + \frac{2}{y} + \frac{1}{z} - 2 = 0,\quad \frac{2}{x} - \frac{5}{y} - \frac{4}{z} + 1 = 0$$

(OR)

b) Prove by vector method that $\sin(\alpha + \beta) = \sin\alpha \cos\beta + \cos\alpha \sin\beta$

42. a) If $z = x + iy$ is a complex number such that $Im\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$

(OR)

b) Prove that $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\left[\frac{x+y+z-xyz}{1-xy-yz-zx}\right]$

43 a) If $2 + i$ and $3 - \sqrt{2}$ are roots of equation $x^6 - 13x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots.

(OR)

b) . Find the vertex, focus, equation of directrix and length of the latus rectum of the following and draw the graph $y^2 - 4y - 8x + 12 = 0$

44. a) A bridge has a parabolic arch that is 10 m high in the centre and 30 m wide at the bottom. Find the height of the arch 6 m from the centre, on either sides.

(OR)

b) Show that the angle between the curves $y = x^2$ and $x = y^2$ at (0,0) and (1,1) is $\pi/2$ and $\tan^{-1}(3/4)$

45. a) Find the local extrema of the function $f(x) = 4x^6 - 6x^4$

(OR)

b) Find the parametric vector, non parametric vector and cartesian form of the equations of the plane passing through the three non-collinear points (3,6,–2) (–1,–2,6) and (6,4,–2)

46. a) Prove that $g(x, y) = x\log\left(y/x\right)$ is homogeneous, what is the degree? Verify Euler's theorem for g.

(OR)

b) The growth of population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

47. a) Find the area of the region bounded between the parables $y^2 = 4x$ and $x^2 = 4y$

(OR)

b) Let $M = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let * be the matrix multiplication. Determine whether M closed under *. If so, examine the commutative, associate, identity and inverse properties for the operation * on M.

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SIVAGANGA

COMMON FIRST REVISION TEST – 2023

Standard XII Reg No. : ☐☐☐☐☐☐

MATHEMATICS

Time: 3.00 hrs. **Part - I** **Marks: 90**

I. Choose the correct answer: **20 x 1 = 20**

1. If $A^T A^{-1} A^2$ is symmetric then A^2 =
   a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

2. If $x^a y^b = e^m$, $x^c y^d = e^n$, $\Delta_1 = \begin{vmatrix} m & b \\ n & d \end{vmatrix}$, $\Delta_2 = \begin{vmatrix} a & m \\ c & n \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, the value of x & y respectively
   a) $e^{(\Delta_2/\Delta_1)}, e^{(\Delta_3/\Delta_1)}$ b) $\log(\Delta_1/\Delta_3), \log(\Delta_2/\Delta_1)$
   c) $\log(\Delta_2/\Delta_1), \log(\Delta_3/\Delta_1)$ d) $e^{(\Delta_1/\Delta_3)}, e^{(\Delta_2/\Delta_1)}$

3. If $|z - 2 + i| \le 2$, then the greatest value of $|z|$ is
   a) $\sqrt{3} - 2$ b) $\sqrt{3} = 2$ c) $\sqrt{5} - 2$ d) $\sqrt{5} + 2$

4. If z is a complex number such that $z \in C \setminus R$ and $z + \frac{1}{z} \in R$, then $|z|$ is
   a) 0 b) 1 c) 2 d) 3

5. The polynomial $x^3 - kx^2 + 9x$ has three real zeros if and only if, k satisfies
   a) $|k| \le 6$ b) $k = 0$ c) $|k| > 6$ d) $|k| \ge 6$

6. $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right)$ is equal to
   a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

7. The equation $\tan^{-1}x - \cot^{-1}x = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$ has
   a) no solution b) unique solution
   c) two solutions d) infinite number of solutions

8. Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is
   a) 2ab b) ab c) $\sqrt{ab}$ d) a/b

9. The circle passing through (1 –2) and touching the axis of x at (3,0) passing through the points
   a) (–5, 2) b) (2, –5) c) (5, –2) d) (–2,5)

10. If $\vec{a}.\vec{b} = \vec{b}.\vec{c} = \vec{c}.\vec{a} = 0$ then $[\vec{a}, \vec{b}, \vec{c}]$ is
    a) $|\vec{a}||\vec{b}||\vec{c}|$ b) $\frac{1}{3}|\vec{a}||\vec{b}||\vec{c}|$ c) 1 d) –1

11. The distance between the planes $x + 2y + 3z + 7 = 0$ and $2x + 4y + 6z + 7 = 0$ is
    a) $\frac{\sqrt{7}}{2\sqrt{2}}$ b) $\frac{7}{2}$ c) $\frac{\sqrt{7}}{2}$ d) $\frac{7}{2\sqrt{2}}$

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12. The abscissa of the point on the curve f($f(x) = \sqrt{8-2x}$ at which the slope of the tangent is –0.25?

a) –8 b) –4 c) –2 d) 0

13. One of the closed points on the curve $x^2 - y^2 = 4$ to the point (6,0) is

a) (2,0) b) $((\sqrt{5},1)$ c) $(3,\sqrt{5})$ d) $(\sqrt{13} - \sqrt{3})$

14. If $u(x,y) = e^{x^2-y^2}$ then $\frac{\partial u}{\partial x}$ is equal to

a) $e^{x^2+y^2}$ b) 2xu c) x^2u d) y^2u

15. If f(x,y,z) = xy + yz + zx then $f_x - f_y$ is equal to

a) z – x b) y – z c) x – z d) y – x

16. If $f(x) = \int_0^x t\cos t\,dt$ then $\frac{df}{dx} =$

a) cos x – x sin x b) sin x + x cos x c) x cos x d) x sin x

17. The value of $\int_0^a \left(\sqrt{a^2 - x^2}\right)^3 dx$ is

a) $\frac{\pi a^3}{16}$ b) $\frac{3\pi a^4}{16}$ c) $\frac{3\pi a^2}{8}$ d) $\frac{3\pi a^4}{8}$

18. The order and degree of the differential equation $\log\left(\frac{dy}{dx}\right) + \frac{d^3y}{dx^3} + 4x = 0$ are respectively

a) 3,1 b) 1,3 c) 3, not defined d) not defined, 3

19. In a Binomial Distribution the variance $\sigma^2 =$

a) 1 b) 0 c) np d) npq

20. The truth table value of $p \vee q$ is F if

a) p is false b) q is false

c) both p and q are false d) either p or q is false

Part - II

II. Answer any 7 questions. (Q.No.30 is compulsory) **7 x 2 = 14**

21. Find the rank of the matrix $\begin{bmatrix} 1 & -2 & -1 & 0 \\ 3 & -6 & -3 & 1 \end{bmatrix}$ by minor method:

22. Find the square root of 6 – 8i

23. State the reason for $\cos^{-1}\left[\cos\left(-\frac{\pi}{6}\right)\right] \neq -\frac{\pi}{6}$

24. The line 3x + 4y – 12 = 0 meets the coordinate axes at A and B. Find the equation of the circle drawn on AB as diameter.

25. Find the values in the interval $(1/2, 2)$ satisfied by the Rolle's theorem for the function $f(x) = x + \frac{1}{x},\ x \in \left[\frac{1}{2}, 2\right]$

26. Let $g(x) = x^2 + \sin x$. Calculate the differential dg.

27. Evaluate : $\int_0^{\infty} x^5 e^{-3x}\, dx$

28. Two fair coins are tossed simultaneously (equivalent to a fair coin is tossed twice). Find the probability mass function for number of heads occurred.

29. Let $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ be any two boolean matrices of the same type. Find $A \vee B$ and $A \wedge B$

30. Find a polynomial equation of minimum degree with rational coefficients having $2 - \sqrt{3}$ as a root.

Part - III

III. Answer any 7 questions. (Q.No.40 is compulsory) **7 x 3 = 21**

31. Solve the following system of linear equations, using matrix inversion method: $5x + 2y = 3,\ 3x + 2y = 5$

32. Show that the points $1, \frac{-1}{2} + i\frac{\sqrt{3}}{2}$ and $\frac{-1}{2} - i\frac{\sqrt{3}}{2}$ are the vertices of an equilateral triangle.

33. Solve the equation: $x^4 - 14x^2 + 45 = 0$

34. Prove that the point of intersection of the tangents at 't_1' and 't_2' on the parabola $y^2 = 4ax$ is $[at_1t_2, a(t_1 + t_2)]$

35. Evaluate: $\lim_{x \to 0}\left(\frac{1}{x} - \frac{1}{e^x - 1}\right)$

36. Evaluate: $\int_0^a \frac{f(x)}{f(x) + f(a - x)} dx$

37. Solve the differential equations: $(e^y + 1)\cos x\, dx + e^y \sin x\, dy = 0$

38. Find the mean and variance of a random variable X, whose probability density function is $f(x) = \begin{cases} \lambda e^{-\lambda x} & \text{for } x \ge 0 \\ 0 & \text{otherwise} \end{cases}$

39. Prove that $p \to (\neg q \vee r) \equiv \neg p \vee (\neg q \vee r)$ using truth table.

40. Determine whether the pair of straight lines $\vec{r} = (2\hat{i} + 6\hat{j} + 3\hat{k}) + t(2\hat{i} + 3\hat{j} + 4\hat{k})$, $\vec{r} = (2\hat{j} - 3\hat{k}) + s(\hat{i} + 2\hat{j} + 3\hat{k})$ are parallel. Find the shortest distance between them.

Part - IV

IV. Answer all the questions. **7 x 5 = 35**

41. a) A boy is walking along the path $y = ax^2 + bx + c$ through the points $(-6,8)$, $(-2,-12)$ and $(3,8)$. He wants to meet his friend at $P(7,60)$. Will he meet his friend? (Use Gaussian elimination method)

(OR)

b) If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$ then show that $x^2 + y^2 + z^2 + 2xyz = 1$

42. a) If $\cos\alpha + \cos\beta + \cos\gamma = \sin\alpha + \sin\beta + \sin\gamma = 0$, show that
   (i) $\cos3\alpha + \cos3\beta + \cos3\gamma = 3\cos(\alpha + \beta + \gamma)$ and
   (ii) $\sin3\alpha + \sin3\beta + \sin3\gamma = 3\sin(\alpha + \beta + \gamma)$

(OR)

b) The growth of a population is proportional to the number present. If the population of a colony doubles in 50 years, in how many years will the population become triple?

43 a) Solve the following equation: $x^4 - 10x^3 + 26x^2 - 10x + 1 = 0$

(OR)

b) Find the equations of the tangents to the curve $y = 1 + x^3$ for which the tangent is orthogonal with the line $x + 12y = 12$

44 a) Show that the line $x - y + 4 = 0$ is a tangent to the ellipse $x^2 + 3y^2 = 12$. Also find the coordinates of the point of contact.

(OR)

b) Prove by vector method that the perpendiculars (altitudes) from the vertices to the opposite sides of a triangle are concurrent.

45. a) Find the vector equation and Cartesian equation of the plane passing through the point $(1,-2,4)$ and perpendicular to the plane $x + 2y - 3z = 11$ and parallel to the line $\frac{x+7}{3} = \frac{y+3}{-1} = \frac{z}{1}$

(OR)

b) Find the area of the region bounded between the parabolas $y^2 = 4x$ and $x^2 = 4y$

46. a) If $w(x,y,z) = \log\left(\frac{5x^3y^4 + 7y^2xz^4 - 75y^3z^4}{x^2 + y^2}\right)$,

find $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$

(OR)

b) Two balls are chosen randomly from an urn containing 8 white and 4 black balls. Suppose that we win Rs.20 for each black ball selected and we lose Rs.10 for each white ball selected. Find the expected winning amount and variance.

47. a) Solve : $\frac{dy}{dx} = \frac{x - y + 5}{2(x - y) + 7}$

(OR)

b) Verify (i) closure property (ii) commutative property (iii) associative property (iv) existence of identity and (v) existence of inverse for the operation $+_5$ on Z_5 using table corresponding to addition modulo 5.

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V12M Virudhunagar District Common Examinations

First Revision Examination - January 2023

Standard 12

Time allowed: 3 hours **MATHEMATICS** Maximum Marks: 90

PART - A

Answer all the questions. Choose the correct or most suitable answer: **20×1=20**

1) If $A^{+}A^{-1}$ is symmetric, then $A^2 =$

   a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

2) If $A = \begin{bmatrix} 3/5 & 4/5 \\ x & 3/5 \end{bmatrix}$ and $A^T = A^{-1}$, then the value of x is

   a) $-4/5$ b) $-3/5$ c) $4/5$ d) $3/5$

3) If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is

   a) $|z|$ b) $\bar{z}$ c) $1/z$ d) 1

4) If z is a complex number such that $z \in C\backslash R$ and $z + \frac{1}{z} \in R$, then $|z|$ is

   a) 0 b) 1 c) 2 d) 3

5) The polynomial $x^3 - kx^2 + 9x$ has three zeros if and only if, k satisfies

   a) $|k| \le 6$ b) $k = 0$ c) $|k| > 6$ d) $|k| \ge 6$

6) If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = 3\pi/2$ then the value of $x^{2017} + y^{2018} + z^{2019} - \frac{9}{x^{101} + y^{101} + z^{101}}$ is

   a) 0 b) 1 c) 2 d) 3

7) $\sin(\tan^{-1}x)$, $|x| < 1$ is equal to

   a) $\frac{x}{\sqrt{1-x^2}}$ b) $\frac{1}{\sqrt{1-x^2}}$ c) $\frac{1}{\sqrt{1+x^2}}$ d) $\frac{x}{\sqrt{1+x^2}}$

8) Area of the greatest rectangle inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

   a) 2ab b) ab c) $\sqrt{ab}$ d) a/b

9) The focus of the parabola $y^2 - 8x - 2y + 17 = 0$ is

   a) (1, 4) b) (3, 1) c) (4, 1) d) (1, 3)

10) Which of the complex number is nearer to origin?

   a) 1+4i b) −3+2i c) 4−3i d) 1+2i

11) The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

   a) $y = 0$ b) $y = \pm\sqrt{3}$ c) $y = 1/2$ d) $y = \pm 3$

12) The maximum product of two positive numbers when their sum of the squares is 200, is

   a) 100 b) $25\sqrt{7}$ c) 28 d) $24\sqrt{14}$

13) If $u = x^y y^x$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} =$

a) $(x+y)u$ b) $(x+y+\log u)u$ c) $x+y+\log u$ d) $u(x+y+\log u)u$

14) If we measure the side of a cube to be 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is

a) 0.4 cu.cm b) 0.45 cu. cm c) 2 cu. cm d) 4.8 cu. cm

15) The value of $\int_{\pi/2}^{\pi/2} \sin^2 x \cos x \, dx$

a) 3/2 b) 1/2 c) 0 d) 2/3

16) If $\int_0^a \frac{1}{4+x^2} dx = \frac{\pi}{8}$ then a is

a) 4 b) 1 c) 3 c) 2

17) $A = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ then $A \cap B$

a) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

18) The solution of the differential equation $\frac{\partial y}{\partial x} = 2xy$ is

a) $y = ce^{x^2}$ b) $y = 2x^2+c$ c) $y = ce^{-x^2} + c$ d) $y = x^2+c$

19) Which of the following is a discrete random variable?

I] The number of cars crossing a particular signal in a day

II] The number of customers in a queue to buy train tickets at a moment

III] The time taken to complete a telephone call

a) I and II b) II only c) III only d) II and III

20) The operation * defined by $a*b = ab/7$ is not a binary operation

a) Q^+ b) Z c) R d) C

Part - B

i) Answer any seven questions. ii) Q.No. 30 is compulsory. 7×2=14

21) Find rank of the matrix by using minor method $\begin{bmatrix} 1 & -2 & -1 & b \\ 3 & -6 & -3 & 1 \end{bmatrix}$

22) Construct the cubic equation with roots 2, $\frac{1}{2}$ and 1.

23) Find the domain of the $\tan^{-1}\sqrt{a-x^2}$

24) If $|\vec{a}+\vec{b}| = 60$, $|\vec{a}-\vec{b}| = 40$ and $|\vec{a}| = 22$ then find $|\vec{b}|$

25) Suppose of (x) is differentiable function for all x with $f'(x) \le 29$ and $f(2) = 17$. What is the maximum value of f(7)?

26) If $w(x,y) = x^3-3xy+2y^2$, $x, y \in R$ find the linear approximation for w at (1, -1)

27) Evaluate: $\int_0^\infty e^{-ax} x^n \, dx$

28) Find the differential equation for the family of all straight lines passing through the origin.

29) Four coins are tossed once find the probability mass function for number of heads.

30) Prove De Morgan's law by using Truth table.

Part - C

Note: i) Answer any seven questions only. ii) Q.No. 40 is compulsory. $7 \times 3 = 21$

31) If $A = \frac{1}{9}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ 1 & -8 & 4 \end{bmatrix}$, Prove that $A^{-1} = A^{T}$

32) Prove by Vector method that the area of the quadrilateral ABCD having diagonal AC and BD is $\frac{1}{2}\left|\overline{AC} \times \overline{BD}\right|$

33) Represent the complex number $1 + i\sqrt{3}$ in polar form.

34) If p and q are the roots of the equation $x^2+nx+n = 0$, show that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$

35) Find the value of $2\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

36) If the equation $3x^2+(3-p)xy+qy^2-2px = 8pq$ represents a circle, find p and q. Also determine the centre and radius of the circle.

37) If the radius of the sphere, with radius 10 cm, has to decrease by 0.1 cm approximately how much with it's volume decrease?

38) Evaluate: $\int_0^{\pi/2} \frac{dx}{1 + 5\cos^2 x}$

39) Find the mean and variance of random variable, x whose probability density function is $f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}$

40) Find the local extrema of the function $f(x) = x^4+32x$.

Part - D

Note: Answer all the questions: **7×5=35**

41) a] Solve by Cramer's rule, the system of equations $x_1-x_2 = 3$, $2x_1+3x_2+4x_3 = 17$, $x_2+2x_3 = 7$.

(OR)

b] If $z = (\cos\theta + i\sin\theta)$ show that $z^n + \frac{1}{z^n} = 2\cos n\theta$ and $z^n - \frac{1}{z^n} = 2i\sin n\theta$

42) a] Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta$, $y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

(OR)

b] Evaluate: $\int_0^{4} \frac{1}{\sin x + \cos x}\,dx$

43) a] Assume that water issuing from the end of a horizontal pipe, 7.5 m above the ground, describes a parabola path. The vertex of the parabolic path is at the end of the pipe. At a position 2.5 m below due line of the pipe, the flow of water has curved outward 3 m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

(OR)

b] Find the non parametric form of vector equation and Cartesian equation of the plane passing through the point (2, 3, 6) and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$

44) a] If the roots of $x^3+px^2+qx+r=0$ are in H.P. prove that $9pqr = 27rq + 2p$

(OR)

b] Evaluate $\sin\left(\sin^{-1}\left(\frac{3}{5}\right)+\sin^{-1}\left(\frac{5}{4}\right)\right)$

45) a] For the function $f(x) = 4x^3+3x^2-6x+1$ find point of inflection.

(OR)

b] Find the volume of a sphere when rotating a circle with radius a.

46) a] Find the area of the region bounded by the curve $y = \sin x$ and $y = \cos x$ between $x = \pi/4$ and $x = 5\pi/4$

(OR)

b] Solve: $\frac{\partial y}{\partial x}=\sqrt{\frac{1-y^2}{1-x^2}}$

47) a] If the probability that a fluorescent light has a useful life of atleast 600 hours is 0.9, find the probabilities that among 12 such lights.

(i) exactly 10 will have a useful life of atleast 600 hours.

(ii) at least 11 will have a useful life of at least 600 hours.

(iii) at least 2 will not have a useful life of atleast 600 hours.

(OR)

b] Using the equivalence property, show that $p \leftrightarrow q \equiv (p \cap q) \cup (\neg p \cap \neg q)$

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THIRUNELVELI

Tsi12M

Tenkasi District Common Examinations
First Revision Examination - January 2023

06-01-2023

Standard 12

MATHEMATICS

Part - I

Time: 3.00 hrs | Marks: 90

Note: i) All questions are compulsory.
ii) Choose the correct or most suitable answer from the given four alternatives. Write the options code and the corresponding answer. **20×1=20**

1) If A, B and C are invertible matrices of some order, then which one of the following is not true?
a) adj $A = |A|A^{-1}$
b) adj $(AB) = (\text{adj } A)(\text{adj } B)$
c) det $A^{-1} = (\det A)^{-1}$
d) $(ABC)^{-1} = C^{-1}B^{-1}A^{-1}$

2) If $z = x+iy$ is a complex number such that $|z+2| = |z-2|$, then the locus of z is
a) real axis b) imaginary axis c) ellipse d) circle

3) A zero of x^3+64 is
a) 0 b) 4 c) 4i d) -4

4) The product of all four values of $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^{\frac{3}{4}}$ is
a) -2 b) -1 c) 1 d) 2

5) If $\sin^{-1}x = 2\sin^{-1}\alpha$ has a solution, then
a) $|\alpha| \le \frac{1}{\sqrt{2}}$ b) $|\alpha| \ge \frac{1}{\sqrt{2}}$ c) $|\alpha| < \frac{1}{\sqrt{2}}$ d) $|\alpha| > \frac{1}{\sqrt{2}}$

6) Which one is the inverse of the statement $(p \vee q) \rightarrow (p \wedge q)$?
a) $(p \wedge q) \rightarrow (p \vee q)$
b) $\neg(p \vee q) \rightarrow (p \wedge q)$
c) $(\neg p \vee \neg q) \rightarrow (\neg p \wedge \neg q)$
d) $(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)$

7) If $x+y=k$ is a normal to the parabola $y^2 = 12x$, then the value of k is
a) 3 b) -1 c) 1 d) 9

8) Distance from the origin to the lane $3x-6y+2z+7 = 0$ is
a) 0 b) 1 c) 2 d) 3

9) Evaluate: $\int_0^1 [2x]\,dx$ where [.] is the greatest integer function.
a) 0 b) 2 c) $\frac{1}{2}$ d) 1

10) The minimum value of the function $|3-x|+9$ is
a) 0 b) 3 c) 6 d) 9

11) If $f(x, y) = e^{xy}$, then $\frac{\partial^2 f}{\partial x \partial y}$ is equal to
a) xye^{xy} b) $(1+xy)e^{xy}$ c) $(1+y)e^{xy}$ d) $(1+x)e^{xy}$

12) The differential equation representing the family of curves $y = A\cos(x+B)$, where A and B are parameters is
a) $\frac{d^2y}{dx^2} - y = 0$ b) $\frac{d^2y}{dx^2} + y = 0$ c) $\frac{d^2y}{dx^2} = 0$ d) $\frac{d^2x}{dy^2} = 0$

13) A baloon rises straight up at 10 m/s. An observer is 40 m away from the spot where the balloon left the ground. The rate of change of the balloon's angle of elevation in radian per second when the balloon is 30 meters above the ground.
a) $\frac{3}{25}$ radians/sec b) $\frac{4}{25}$ radians/sec c) $\frac{1}{5}$ radians/sec d) $\frac{1}{3}$ radians/sec

14) The area between $y^2 = 4x$ and its latus rectum is

a) $\frac{2}{3}$ b) $\frac{4}{3}$ c) $\frac{8}{3}$ d) $\frac{5}{3}$

15) If $2\vec{i} - \vec{j} + 3\vec{k}$, $3\vec{i} + 2\vec{j} + \vec{k}$, $\vec{i} + m\vec{j} + 4\vec{k}$ are coplanar, find the vlue of m.

a) 3 b) $-\frac{1}{3}$ c) -3 d) $\frac{1}{3}$

16) The solution of the differential equation $\frac{dy}{dx} + \frac{1}{\sqrt{1-x^2}} = 0$ is

a) $y+\sin^{-1}x = c$ b) $x+\sin^{-1}y = 0$ c) $y^2+2\sin^{-1}x = c$ d) $x^2+2\sin^{-1}y=0$

17) If $y = 4x+c$ is a tangent to the circle $x^2+y^2 = 9$ find c

a) $\pm 3\sqrt{17}$ b) $\pm 17\sqrt{4}$ c) $\pm 17\sqrt{3}$ d) $\pm 4\sqrt{17}$

18) If A^TA^{-1} is symmetric, then $A^2 =$

a) A^{-1} b) $(A^T)^2$ c) A^T d) $(A^{-1})^2$

19) On a multiple-choice exam with 3 possible destructives for each of the 5 questions, the probability that a student will get 4 or more correct answers just by guessing is

a) $\frac{11}{243}$ b) $\frac{3}{8}$ c) $\frac{1}{243}$ d) $\frac{5}{243}$

20) Subtraction is not a binary operation in

a) $\mathbb{R}$ b) $\mathbb{Z}$ c) $\mathbb{N}$ d) $\mathbb{Q}$

Part - II

Note: Answer any seven questions. Question No. 30 is compulsory **7×2=14**

21) Prove that $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$ is orthogonal.

22) Find the square root of $-6+8i$

23) Find the value of $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$

24) Find the angle between the planes $\vec{r}.(\vec{i} + \vec{j} - 2\vec{k}) = 3$ and $2x-2y+z = 2$.

25) If $f(x, y) = x^3-3x^2+y^2+5x+6$ then find f_x at $(1, -2)$

26) Evaluate: $\int_0^{\pi/2} (\sin^2 x + \cos^4 x)\,dx$

27) Determine the order and degree (if exists) of the differential equation

$$\left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^2 = x\sin\left(\frac{d^2y}{dx^2}\right)$$

28) Find the mean of the distribution $f(x) = \begin{cases} 3e^{-3x}, & 0 < x < \infty \\ 0, & \text{elsewhere} \end{cases}$

29) Find the equation of the parabola whose vertex is $(-2, 5)$ and focus $(-2, 2)$

30) On $\mathbb{Z}$, define $\otimes$ by $(m \otimes n) = m^n + n^m : \forall m,n \in \mathbb{Z}$. Is $\otimes$ binary on $\mathbb{Z}$?

Part - III

7×3=21

Note: Answer any seven questions. Question No. 40 is compulsory.

31) Solve by matrix inversion method: $5x+2y=4$, $7x+3y=5$.

32) Solve the equation $2x^3+11x^2-9x-18=0$

33) State and prove that triangle inequality.

34) Evaluate: $\int_0^{\pi/3} \frac{\sec x \tan x}{1+\sec^2 x}dx$

35) Prove that the length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is $\frac{2b^2}{a}$

36) Solve the differential equation $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

37) If $u(x, y)=\frac{x^2+y^2}{\sqrt{x+y}}$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{3}{2}u$

38) Verify whether the compound proposition $(p \to q) \leftrightarrow (\neg p \to q)$ is a tautology or contradiction or contingency.

39) The mean and variance of a binomial variate X are respectively 2 and 1.5. Find $P(X=0)$

40) If $\vec{a}, \vec{b}, \vec{c}$ are three vectors prove that $[\vec{a}+\vec{b}, \vec{b}+\vec{c}, \vec{c}+\vec{a}] = 2[\vec{a}, \vec{b}, \vec{c}]$

Part - IV

7×5=35

Note: Answer all the questions.

41) a] Investigate for what values of λ and μ the system of linear equations $x+2y+z=7$, $x+y+\lambda z=\mu$, $x+3y-5z=5$ has (i) no solution (ii) a unique solution.

(OR)

b] If $z=x+iy$ and $\arg\left(\frac{z-1}{z+2}\right)=\frac{\pi}{4}$, then show that $x^2+y^2+3x-3y+2=0$

42) a] If the curves $ax^2+by^2=1$ and $cx^2+dy^2=1$ intersect each other orthogonally then, $\frac{1}{a}-\frac{1}{b}=\frac{1}{c}-\frac{1}{d}$

(OR)

b] At a water fountain, water attains a maximum height of 4 m at horizontal distance of 0.5 m from its origin. The flow is from the origin and the path of water is a parabola open upwards, find the height of water at a horizontal distance of 0.75 m from the point of origin.

43) a] If $\vec{a}=\hat{i}-\hat{j}$, $\vec{b}=\hat{i}-\hat{j}-4\hat{k}$, $\vec{c}=3\hat{j}-\hat{k}$ and $\vec{d}=2\hat{i}+5\hat{j}+\hat{k}$ verify that $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a}, \vec{b}, \vec{d}]\vec{c}-[\vec{a}, \vec{b}, \vec{c}]\vec{d}$

(OR)

b] The equation of electromotive force for an electric circuit containing resistance an self-inductance is $E = Ri+L\frac{di}{dt}$, where E is the electromotive force given to the circuit, the resistance and L, the coefficient of induction. Find the current i at time t when $E=0$.

44) a] Find intervals of concavity and points of inflexion for the function $f(x) = \frac{1}{2}(e^x - e^{-x})$

(OR)

b] Verify (i) closure property (ii) commutative property (iii) associative property, (iv) existence of identity and (v) existence of inverse for the operation $\times_{11}$ on a subset A = {1, 3, 4, 5, 9} of the set of remainder {0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

45) a] Find the area of the region bounded between the parabola $x^2 = y$ and the curve $y = |x|$

(OR)

b] Find the non-parametric form of vector equation, and Cartesian equations of the plane $\vec{r} = (6\hat{i} - \hat{j} - \hat{k}) + s(-\hat{i} + 2\hat{j} + \hat{k}) + t(-5\hat{i} - 4\hat{j} - 5\hat{k})$

46) a] For the ellipse $4x^2+y^2+24x-2y+21 = 0$, find the centre, vertices, foci and the length of latus rectum.

(OR)

b] If $U = \sec^{-1}\left(\frac{x^3 - y^3}{x + y}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2 \cot u$

47) a] The cumulative distribution function of a discrete random variable is given by $f(x) = \begin{cases} 0 & -\infty < x < -1 \\ 0.15 & -1 \le x < 0 \\ 0.35 & 0 \le x < 1 \\ 0.60 & 1 \le x < 2 \\ 0.85 & 2 \le x < 3 \\ 1 & 3 \le x < \infty \end{cases}$ Find (i) the probability mass function (ii) $P(X < 1)$ and (iii) $P(X \ge 2)$

(OR)

b] If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$ and $0 < x, y, z < 1$ then show that $x^2+y^2+z^2+2xyz = 1$.

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