

Maths - Volume - II

Time: 3 hrs.  
Marks: - 90

XII Std

Part - A

(20 x 1 = 20)

1. A stone is thrown up vertically. The height it reaches at time  $t$  seconds is given by  $x = 30t - 16t^2$ . The stone reaches the maximum height in time  $t$  seconds is given by  
 (1) 2      (2) 2.5      (3) 3      (4) 3.5.
2. Angle between  $y^2 = x$  and  $x^2 = y$  at the origin is (1)  $\tan^{-1}(\frac{3}{4})$  (2)  $\tan^{-1}(\frac{4}{3})$   
 (3)  $\frac{\pi}{2}$       (4)  $\frac{\pi}{4}$ .
3.  $\lim_{x \rightarrow 0} \frac{e^{3x} - e^{-2x}}{5x} =$  (1) 5      (2) 3      (3) -2      (4) 1.
4. The point of inflection of the curve  $y = (x-1)^3$  is  
 (1) (0, 0)      (2) (0, 1)      (3) (1, 0)      (4) (1, 1).
5. If we measure the side of a cube is 4 cm with an error of 0.1 cm, then the error in our calculation of the volume is  
 (1) 0.4 cu. cm      (2) 0.45 cu. cm      (3) 2 cu. cm      (4) 4.8 cu. cm.
6. If  $u = x \sin y + y \sin x$ , then  $u_{xy} =$  (1)  $\cos x - \cos y$       (2)  $\cos x + \cos y$   
 (3)  $\sin x + \cos y$       (4)  $\cos x + \sin y$ .
7. If  $f(x) = \frac{x}{x+1}$ ; then  $df =$  (1)  $\frac{1}{(x+1)^2} dx$       (2)  $\frac{1}{(x+1)^2} dx$       (3)  $\frac{1}{x+1} dx$       (4)  $\frac{-1}{x+1} dx$
8. For any value of  $n \in \mathbb{Z}$ ;  $\int_0^{\pi} e^{\cos^2 x} e^{\sin^2 x} ((2n+1)x) dx$  is  
 (1)  $\frac{\pi}{2}$       (2)  $\pi$       (3) 0      (4) 2.
9. If  $\frac{\Gamma(n+2)}{\Gamma(n)} = 90$ ; then  $n =$  (1) 10      (2) 5      (3) 8      (4) 9
10. The area between  $y^2 = 4x$  and its L.R is (1)  $\frac{2}{3}$       (2)  $\frac{4}{3}$       (3)  $\frac{8}{3}$       (4)  $\frac{5}{3}$ .
11. The volume of the solid that result when the ellipse  $\frac{x^2}{9} + \frac{y^2}{4} = 1$  is revolved about major axis is  
 (1)  $16\pi$       (2)  $24\pi$       (3)  $32\pi$       (4)  $48\pi$ .
12. The order and degree of  $\frac{d^2y}{dx^2} + (\frac{dy}{dx})^3 + x^4y = 0$  is  
 (1) 2; 3      (2) 3; 3      (3) 2; 6      (4) 2; 4.
13. The D.E of the family of the curves  $y = Ae^x + Be^{-x}$ ; A, B are constants is  
 (1)  $\frac{d^2y}{dx^2} + y = 0$       (2)  $\frac{d^2y}{dx^2} - y = 0$       (3)  $\frac{dy}{dx} + y = c$       (4)  $\frac{dy}{dx} - y = 0$ .

14. The I.F of  $\frac{dy}{dx} - 2x \cot y = e^y \sin y$  is  
 (1)  $\sec^2 y$  (2)  $\cos^2 y$  (3)  $\sin^2 y$  (4)  $\cot^2 y$ .
15. ~~Let x represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times.~~ Let x represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. Then the possible value of x are  
 (1)  $i+2n; i=0, 1, \dots, n$  (2)  $2i-n; i=0, 1, 2, \dots, n$  (3)  $ni; i=0, 1, 2, \dots, n$   
 (4)  $2i+2n; i=0, 1, \dots, n$
16. X has p.d.f  $f(x) = \begin{cases} ax+6 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$  and  $E(x) = \frac{7}{12}$ ; then  $(a, b) =$   
 (1)  $1; \frac{1}{2}$  (2)  $\frac{1}{2}; 1$  (3)  $2; 1$  (4)  $1; 2$ .
17. In a binomial trials; X is a binomial variate which follows the relation  $9P(X=4) = P(X=2)$ ; then  $p =$  (1) 0.125 (2) 0.25 (3) 0.375 (4) 0.75.
18. \* is defined as  $a * b = \sqrt{a^2 + b^2}$ ;  $a, b \in \mathbb{R}$ ; then  $(3 * 4) * 5 =$   
 (1) 5 (2)  $5\sqrt{2}$  (3) 25 (4)  $25\sqrt{2}$ .
19. Which one of the following statements has truth value F?  
 (1) Chennai is in India or  $\sqrt{2}$  is an integer (2) Chennai is in India or  $\sqrt{2}$  is an irrational no.  
 (3) Chennai is in China or  $\sqrt{2}$  is an integer (4) Chennai is in China or  $\sqrt{2}$  is an irrational no.
20. Subtraction is not a binary operation on  
 (1)  $\mathbb{R}$  (2)  $\mathbb{Z}$  (3)  $\mathbb{N}$  (4)  $\emptyset$ .
- Part-B (Answer any 7 Qns) (7 \* 2 = 14)
21. Determine whether \* is defined by  $a * b = a\sqrt{b}$  is a binary on  $\mathbb{R}$ .
22. If  $f(x) = \begin{cases} Ax(2-x) & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$  is a p.d.f of X; Find A.
23. Solve  $\frac{dy}{dx} = e^{x+y} + x^3 e^y$ .
24. Evaluate  $\int_0^4 x dx$  as the limit of a sum.
25. Evaluate  $\int_3^7 \frac{\sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}} dx$ .
26. Assuming  $\log_{10} 2 = 0.4343$ ; Find the approximate value of  $\log_e 1003$ .
27. If  $W(x, y, z) = x^2 y + y^2 z + z^2 x$ ;  $x, y, z \in \mathbb{R}$ ; Find the differential  $dW$ .
28. Evaluate  $\lim_{x \rightarrow 0} \left( \frac{1}{x} - \frac{1}{e^x - 1} \right)$ .



29) If  $s(t) = a \cos 2t + b \sin 3t$  is the distance moved by a particle in time 't' sec. Find its acceleration

30) Show that  $y = e^{-x} + mx + n$  is a solution of D.E  $e^x \left( \frac{d^2y}{dx^2} \right) - 1 = 0$ .

Part-c (Answer any 7 Qns) (7x3=21)

31) Find the value of 'c' in (1, 2) of L.M.V.T for  $f(x) = x - x^2$ ;  $1 \leq x \leq 2$ .

32) Find the absolute maxima and absolute minima for  $f(x) = x^2 - 2x$   
 $f(x) = 3 \cos x$  on  $[0, 2\pi]$ .

33) Show that the Percentage error in the  $n^{\text{th}}$  root of a number is approximately  $\frac{1}{n}$  times the Percentage error in the number.

Find

34) Let  $u(x, y, z) = xyz$ ;  $x = e^{-t}$ ;  $y = e^{-t} \cos t$ ;  $z = \sin t$ ;  $t \in \mathbb{R}$ .  $\frac{du}{dt}$ .

35) Find an approximate value of  $\int_1^{1.5} x^2 dx$  by applying the right-end rule with Partition  $\{1.1, 1.2, 1.3, 1.4, 1.5\}$ .

36) Evaluate  $\int_0^{2\pi} x^2 \sin nx dx$  where  $n$  is a +ve integer.

37) Find the Particular solution of  $(1+x^3)dy - x^2y dx = 0$  satisfying  $y(1) = 2$ .

38) Solve  $x \frac{dy}{dx} = y - x \cos^2\left(\frac{y}{x}\right)$ .

39) Two balls are chosen randomly from an urn containing 8 white and 4 black balls. He wins Rs 20 for each black ball and he loses Rs 10 for each white ball. Find the expected winning amount.

40) Prove  $\Phi \rightarrow p \equiv \neg p \rightarrow \neg \Phi$ .

(7x5=35)

Part-D (Answer all Qns)

41) Find the equations of the tangents to the curve  $y = \frac{x+1}{x-1}$  which are  $\parallel$  to the line  $x+2y = 6$  (OR)

Prove that among all the rectangles of the given area square has the least Perimeter.

42) Consider  $f(x, y) = \frac{xy}{x^2+y^2}$ ;  $f(x, y) \neq 0$  and  $f(0, 0) = 0$ . Show that  $f$  is not continuous at  $(0, 0)$  and continuous at all other points of  $\mathbb{R}^2$  (OR)

If  $u = \tan^{-1}\left(\frac{x^2+y^3}{x-y}\right)$ ; Prove by using Euler's theorem

$$x u_x + y u_y = \sin 2u.$$

43) Find the area of the region common to the circle  $x^2 + y^2 = 16$  and the parabola  $y^2 = 6x$ . (OR)

Find the volume of the solid formed by revolving the region bounded by the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ;  $a > b$  about the major axis.

44) Solve  $x \log x \frac{dy}{dx} + y = x \sin 2x$ . (OR)

A tank contains 1000 litres of water in which 100 grams of salt is dissolved. Brine runs in a rate of 10 litres per minute and each litre contains 5 grams of dissolved salt. The mixture of the tank is kept uniform by stirring. Brine runs out at 10 litres per minute. Find the amount of salt at any time  $t$ .

45) Find the average and standard deviation of  $x$  whose P.d.f is

$$f(x) = \begin{cases} 4e^{-4x} & x \geq 0 \\ 0 & \text{otherwise.} \end{cases} \quad \text{(OR)}$$

A multiple choice Exam has 10 questions. Each question has 4 choice with one correct answer. Suppose a student answers by guessing and if  $x$  denote the number of correct answer. Find the Probability that the student will answer (i) exactly 7 correct answers (ii) at least one correct answer.

46) Let  $M = \left\{ \begin{pmatrix} x & y \\ x & y \end{pmatrix} \mid x, y \in \mathbb{R} \right\}$  and  $*$  is the matrix multiplication. Verify (i) closure (ii) commutative (iii) Associative (iv) Identity (v) Inverse Property. (OR) Prove  $P \cup (Q \cap R) = (P \cup Q) \cap (P \cup R)$ .

47) Solve  $(1 + 3e^{\frac{y}{x}})dy + 3e^{\frac{y}{x}}(1 - \frac{y}{x}) = 0$  given  $y = 0$  when  $x = 1$ . (OR)

A conical water tank with vertex down of 12 m high has a radius of 5 m at the top. If water flows into the tank at a rate of  $10 \text{ cm}^3/\text{min}$ , how fast is the depth of the water increases when the water is 8 m deep?

All the Best

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