Assignment

Class-12 **Subject:Mathematics**

Unit 2

Complex Numbers

Part - A

I. One mark questions

Fill in the blanks:

1. $i^n + i^{n+1} + i^{n+2} + i^{n+3}$ is ________

2. The conjugate of a complex number is $\frac{1}{i-2}$ then, the complex number is ________

3. If $z = \frac{(\sqrt{3}+i)^3(3i+4)^2}{(8+6i)^2}$, then $|z|$ is equal to ________

4. If $|z - 2 + i| \leq 2$, then the greatest value of $|z|$ is ________

5. If $|z| = 1$, then the value of $\frac{1+z}{1+\bar{z}}$ is ______

Choose the best answers:

6. If $|z_1| = 1, |z_2| = 2, |z_3| = 3$ and $|9z_1z_2 + 4z_1z_3 + z_2z_3| = 12$, then the value of $|z_1 + z_2 + z_3|$ is

(i) 1 (ii) 2 (iii) 3 (iv) 4

7. z_1, z_3 and z_3 are complex numbers such that $z_1 + z_2 + z_3 = 0$ and $|z_1| = |z_2| = |z_3| = 1$then $z_1^2 + z_2^2 + z_3^2$ is

(i) 3 (ii) 2 (iii) 1 (iv) 0

8. If $z = x + iy$ is a complex number such that $|z + 2| = |z - 2|$, then the locus of z is

(i) real axis (ii) imaginary axis (iii) ellipse (iv) circle

9. The principle of argument of $\frac{3}{-1+i}$ is

(i) $\frac{-5\pi}{6}$ (ii) $\frac{-2\pi}{3}$ (iii) $\frac{-3\pi}{4}$ (iv) $\frac{-\pi}{2}$

10. If α and β are the roots of $x^2 + x + 1 = 0$ then $\alpha^{2020} + \beta^{2020}$ is

(i) -2 (ii) -1 (iii) 1 (iv) 2

Part – B

II.Very Short Answer.

1. Simplify the following $i^{59} + \frac{1}{i^{59}}$

2. Write $\frac{3+4i}{5-12i}$ in the $x + iy$ form, hence find its real and imaginary parts

3. If $|z| = 3$, show that $7 \leq |z + 6 - 8i| \leq 13$

4. Find the square root of $6 - 8i$

5. Simplify $\left(\sin\frac{\pi}{6} + i\cos\frac{\pi}{6}\right)^{18}$

Part – C

III. Short Answer.

1. Simplify $\left(\frac{1+i}{1-i}\right)^3 - \left(\frac{1-i}{1+i}\right)^3$ into rectangular form
2. Show that the equation $z^3 + 2\bar{z} = 0$ has five solutions
3. If $z = x + iy$ is a complex number such that $\left|\frac{z-4i}{z+4i}\right| = 1$ show that the locus number of z is real axis
4. Write in polar form: $-2 - i2$
5. If $\omega \neq 1$ is a cube root of unity, show that $(1 - \omega + \omega^2)^6 + (1 + \omega - \omega^2)^6 = 128$

Part – D

IV.Write in detail.

1. If $z = x + iy$ is a complex number such that $\text{Im}\left(\frac{2z+1}{iz+1}\right) = 0$, show that the locus of z is $2x^2 + 2y^2 + x - 2y = 0$
2. If $z = x + iy$ and $\arg\left(\frac{z-i}{z+2}\right) = \frac{\pi}{4}$, show that $x^2 + y^2 + 3x - 3y + 2 = 0$

Unit – 3 - Theory of Equations

Part - A

I. One mark questions

1.A zero of $x^2 + 64$ is _____

2. If f and g are polynomials of degrees m and n respectively, and if $h(x) = (f\ o\ g)(x)$, then the degree of h is _____

3. A polynomial equation in x of degree n always has ______

4. If α, β and γ are the zeroes of $x^2 + px^2 + qx + r$, then $\sum \frac{1}{\alpha}$ is ______

5. According to the rational root theorem, which number is not possible rational zero of $4x^7 + 2x^4 - 10x^3 - 5$ _______

6. The polynomial $x^3 - kx^2 + 9x$ has three real zero if and only if, k satisfies

   (ii) $|k| \leq 6$ (ii) $k = 0$ (iii) $|k| > 6$ (iv) $|k| \geq 6$

7. The number of real number in $(0,2\pi)$ satisfying $\sin^4 x - 2\sin^2 x + 1$ is

   (i) 2 (ii) 4 (iii) 1 (iv) ∞

8. If $x^2 + 12x^2 + 10ax + 1999$ definitely has a positive zero, if and only if

(i) $a \geq 0$ (ii) $a > 0$ (iii) $a < 0$ (iv) $a \leq 0$

9. The polynomial $x^3 + 2x + 3$ has

(i) one negative and two imaginary zeroes

(ii) one positive and two imaginary zeroes

(iii) three real zeroes

(iv) no zeroes

10. The number of positive zeroes of the polynomial $\sum_{j=0}^{n} {}^{n}C_r(-1)^r x^r$ is

(i) 0 (ii) n (iii) $< n$ (iv) r

Part - B

II. Very short answer

1. Form a polynomial equation with integer coefficients with $\sqrt{\frac{\sqrt{2}}{\sqrt{3}}}$ as a root

2. If α, β and γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{1}{\beta\gamma}$ in terms of the coefficients.

3. Find a polynomial equation of minimum degree with rational coefficients, having $2 - \sqrt{3}$ as a root.

4. Find a polynomial equation of minimum degree with rational coefficients, having $2i + 3$ as a root.

5. Show the equation $2x^2 - 6x + 7 = 0$ cannot be satisfied by any real values of x.

Part - C

III. Short answer

1. If p and q are the roots of the equation $lx^2 + nx + n = 0$, show that

$$\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{1}} = 0$$

2. Find the roots of $2x^2 + 3x^2 + 2x + 3 = 0$

3. Find all real numbers satisfying $4^x - 3(2^{x+2}) + 2^5 = 0$

4. If α, β and γ are the roots of the polynomial equation $ax^3 + bx^2 + cx + d = 0$, find the value of $\sum \frac{\alpha}{\beta\gamma}$ in terms of the coefficients.

5. Solve the cubic equations: (i) $2x^3 - 9x^2 + 10X = 3$ (ii) $8x^3 - 2x^2 - 7x + 3 = 0$

Part - D

IV. Answer in Detail

1. If $2 + i$ and $3, \sqrt{2}$ are roots of the equation $x^6 - 1x^5 + 62x^4 - 126x^3 + 65x^2 + 127x - 140 = 0$, find all roots

2. Solve the equation $6x^4 - 5x^3 - 38x^2 - 5x + 6 = 0$ if it is known that $\frac{1}{3}$ is a solution.