12th Maths –Quarterly Exam-2017-Answer Key

Part - A

Q.No	Option	Q.No	Option	Q.No	Option	Q.No	Option
1	3	11	3	21	3	31	1
2	4	12	3	22	4	32	3
3	1	13	1	23	1	33	3
4	3	14	3	24	3	34	2
5	2	15	2	25	3	35	3
6	2	16	3	26	2	36	2
7	3	17	3	27	1	37	2
8	2	18	3	28	2	38	2
9	2	19	2	29	2	39	4
10	2	20	2	30	1	40	4

Part – B

41.

$|A| = -11$

$adj\ A = \begin{bmatrix} -5 & -2 \\ -3 & 1 \end{bmatrix}$

$A(adjA) = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$........(1)

$(adjA)A = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$......(2)

$|A|I_2 = \begin{bmatrix} -11 & 0 \\ 0 & -11 \end{bmatrix}$......(3)

From (1), (2),(3) we get

$A(adjA) = (adjA)A = |A|I_2$

42.

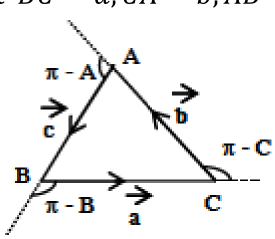
$$A = \begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 3 & 4 \\ -2 & 4 & -1 & -3 \\ -1 & 2 & 7 & 6 \end{bmatrix} \begin{matrix} R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 + R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 3 & 4 \\ 0 & 0 & 5 & 5 \\ 0 & 0 & 10 & 10 \end{bmatrix} R_3 \to R_3 - 2R_2$$

$\rho(A) = 2$

43. Let $\overrightarrow{BC} = \vec{a}, \overrightarrow{CA} = \vec{b}, \overrightarrow{AB} = \vec{c}$



By the area property of triangles

$$\frac{1}{2}|\vec{a} \times \vec{b}| = \frac{1}{2}|\vec{b} \times \vec{c}| = \frac{1}{2}|\vec{c} \times \vec{a}|$$

$$|\vec{a} \times \vec{b}| = |\vec{b} \times \vec{c}| = |\vec{c} \times \vec{a}|$$

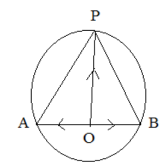
$$ab \sin(\pi - C) = bc \sin(\pi - A) = ca \sin(\pi - B)$$

$$ab \sin C = bc \sin A = ca \sin B$$

Divide by abc $\quad \frac{\sin C}{c} = \frac{\sin A}{a} = \frac{\sin B}{b}$

Take the reciprocals $\quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

44. Let P be a point on the surface of the sphere and AB be a diameter. Consider the great circle on the sphere passing through the points P, A and B. Take the centre O as the point of reference.



$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP}$

$\overrightarrow{AP} = \overrightarrow{OP} - \overrightarrow{OA} = \overrightarrow{OP} + \overrightarrow{OB}$

$\overrightarrow{AP}.\overrightarrow{PB} = (\overrightarrow{OP} + \overrightarrow{OB}).(\overrightarrow{OB} - \overrightarrow{OP})$

$= |\overrightarrow{OP}|^2 - |\overrightarrow{OB}|^2 = 0$

Since $|\overrightarrow{OP}| = |\overrightarrow{OB}|$

AB subtends a right angle at P on the surface.

45. (i)

$$\vec{a} = a_1\vec{\imath} + a_2\vec{\jmath} + a_3\vec{k}$$

$\vec{\imath} \times (\vec{a} \times \vec{\imath}) = (\vec{\imath}.\vec{\imath})\vec{a} - (\vec{\imath}.\vec{a})\vec{\imath} = \vec{a} - a_1\vec{\imath}$

$\vec{\jmath} \times (\vec{a} \times \vec{\jmath}) = (\vec{\jmath}.\vec{\jmath})\vec{a} - (\vec{\jmath}.\vec{a})\vec{\jmath} = \vec{a} - a_2\vec{\jmath}$

$\vec{k} \times (\vec{a} \times \vec{k}) = (\vec{k}.\vec{k})\vec{a} - (\vec{k}.\vec{a})\vec{k} = \vec{a} - a_3\vec{k}$

L.H.S. $= 3\vec{a} - (a_1\vec{\imath} + a_2\vec{\jmath} + a_3\vec{k}) = 2\vec{a}$ =R.H.S

45. (ii)

$$\vec{b} = 3\vec{\imath} - \vec{\jmath} - 2\vec{k}$$

$\sin\theta = \frac{\vec{b}.\vec{n}}{|\vec{b}||\vec{n}|} = \frac{3}{\sqrt{14}\sqrt{26}} = \frac{3}{2\sqrt{91}}$

$\theta = \sin^{-1}\left(\frac{3}{2\sqrt{91}}\right)$

46.

Let $z = x + iy, \bar{z} = x - iy$

$\bar{z} + 1 = x - iy + 1 = (x+1) - iy$

$\bar{z} - 1 = x - iy - 1 = x - 1 - iy$

$\frac{\bar{z}+1}{\bar{z}-1} = \frac{(x+1)-iy}{(x-1)-iy} \times \frac{(x-1)+iy}{(x-1)+iy}$

$= \frac{(x+1)(x-1)+i(x+1)y-iy(x-1)+y^2}{x^2+(y+1)^2}$

$Re\left(\frac{\bar{z}+1}{\bar{z}-1}\right) = 0$

$\frac{(x+1)(x-1)+y^2}{x^2+(y+1)^2} = 0$

$x^2 + 1 + y^2 = 0$

$x^2 + y^2 + 1 = 0$

47. The modules of sum of two complex numbers is always less than or equal to the sum of their moduli.

$$|z_1 + z_2| \le |z_1| + |z_2|$$

Let z_1 and z_2 be two complex numbers

We know that

$|z_1 + z_2|^2 = (z_1 + z_2)\overline{(z_1 + z_2)}$

$(\because |z|^2 = z\bar{z})$

$= (z_1 + z_2)(\bar{z_1} + \bar{z_2})$

$= z_1\bar{z_1} + z_1\bar{z_2} + z_2\bar{z_1} + z_2\bar{z_2}$

$= z_1\bar{z_1} + z_2\bar{z_2} + z_1\bar{z_2} + \overline{z_1\bar{z_2}}$

$= |z_1|^2 + |z_2|^2 + 2Re\,(z_1\bar{z_2})$

$\le |z_1|^2 + |z_2|^2 + 2|z_1\bar{z_2}| \quad (Re(z) \le |z|)$

$= |z_1|^2 + |z_2|^2 + 2|z_1||z_2|$

$= [|z_1| + |z_2|]^2$

$|z_1 + z_2|^2 \le [|z_1| + |z_2|]^2$

Thus taking positive square root we get $|z_1 + z_2| \le |z_1| + |z_2|$

48. Since $2 - i$ is a root, $2 + i$ is also a root.

Sum of the roots $= 4$

Products of the roots $=5$

The corresponding factor is

$= x^2 - 4x + 5$

$6x^4 - 25x^3 + 32x^2 + 3x - 10$

$\equiv (x^2 - 4x + 5)(x^2 + px - 2)$

Equating x term, we get $5p + 8$

$p = -1$

The other factor is $6x^2 - x - 2 = 0$

$x = \frac{2}{3}, -\frac{1}{2}$

Thus the roots are $2 \pm i, \frac{2}{3}, -\frac{1}{2}$

49. $3x - y - 5 = 0$

$x + 3y + k = 0$

Centre $(2,1)$

$k = 5$

Other asymptotes $x + 3y + 5 = 0$

Combined equation

$(3x - y - 5)(x + 3y - 5) = 0$

Equation of R.H is

$(3x - y - 5)(x + 3y - 5) + c = 0$

Passes through $(1, -1)$

$c = -7$

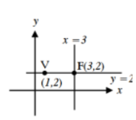
Required equation

$(3x - y - 5)(x + 3y - 5) - 7 = 0$

50. (i) $(y - k)^2 = 4a(x - h)$

$V(1, 2)$,

$(y - 2)^2 = 4a(x - 1)$



Focus $(3,2)$

Required equation is $(y - 2)^2 = 8(x - 1)$

50. (ii) $2xx_1 + 5yy_1 - 20 = 0$

Point $(2, 4)$

$x + 5y - 5 = 0$

51. $F = \frac{5}{x} + 100x$

$\frac{dF}{dx} = -\frac{5}{x^2} + 100$

$\frac{d^2F}{dx^2} = \frac{10}{x^3}$

$\Rightarrow +ve \Rightarrow$minimum

For maximum or minimum $\frac{dF}{dx} = 0$

$-\frac{5}{x^2} + 100 = 0 \Rightarrow -\frac{5}{x^2} = -100$

$x^2 = \frac{5}{100} \Rightarrow x = \pm\frac{1}{2\sqrt{5}}$

$x = -\frac{1}{2\sqrt{5}}$ is not admissible

The resistance is minimum

When $x = \frac{1}{2\sqrt{5}}$, the resistance

$F = \frac{5}{\frac{1}{2\sqrt{5}}} + \frac{100}{2\sqrt{5}}$

Minimum value of resistance $= 20\sqrt{5}$

52. f is a continuous function on $[0, 2\pi]$

f is a differentiable on $(0, 2\pi)$

$f(a) = f(0) = 0, f(b) = f(2\pi) = 0$

$f'(c) = 0$

$f'(x) = -\sin x$

$f'(c) = -\sin c$

$-\sin c = 0$

$c = \pi \in (0, 2\pi)$

When $x = \pi, y = -2$

x axis at $(\pi, -2)$

53. Consider the difference

$f(x) = (1 + x)^n - (1 + nx)$

$f'(x) = n(1 + x)^{n-1} - n$

$= n[(1 + x)^{n-1} - 1]$

Since $x > 0$ and $n - 1 > 0$ we have

$(1 + x)^{n-1} > 1$, so $f'(x) > 0$

Therefore f is strictly increasing on $[0, \infty)$

$x > 0 \Rightarrow f(x) > f(0)$

$(1 + x)^n - (1 + nx) > (1 + 0) - (1 + 0)$

$(1 + x)^n - (1 + nx) > 0$

$(1 + x)^n > (1 + nx)$

54. If $T = k\sqrt{l} = kl^{\frac{1}{2}}$

Then $\frac{dT}{dl} = k\left(\frac{1}{2} \times l^{\frac{-1}{2}}\right) = \left(\frac{k}{2\sqrt{l}}\right)$

$dl = 32.0 - 32.1 = -0.1$ cm

Error in $T =$ Approximate change in T

$$\Delta T \approx dT = \left(\frac{dT}{dl}\right)dl = \left(\frac{k}{2\sqrt{l}}\right)(-0.1)$$

Percentage error $= \left(\frac{\Delta T}{T}\right) \times 100\%$

$$= \frac{\left(\frac{k}{2\sqrt{l}}\right)(-0.1)}{k\sqrt{l}} \times 100\% = \frac{-0.1}{2l} \times 100\%$$

$$= \frac{-0.1}{2(32.1)} \times 100\% = \frac{-0.1}{64.2} \times 100\%$$

$$= -0.156\%$$

Hence the percentage error in the time of swing is a decrease of 0.156%

55. a

$2x + 3y = 8$.......................................(1)

$4x + 6y = 16$.....................................(2)

$$\Delta = \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} = 12 - 12 = 0$$

$$\Delta_x = \begin{vmatrix} 8 & 3 \\ 16 & 6 \end{vmatrix} = 48 - 48 = 0$$

$$\Delta_y = \begin{vmatrix} 2 & 8 \\ 4 & 16 \end{vmatrix} = 32 - 32 = 0$$

Since $\Delta = 0$, and $\Delta_x = \Delta_y = 0$ and atleast one of the coefficients a_{ij} of $\Delta \neq 0$ the system is consistent and has infinitely many solutions.

All 2×2 minor are zero and atleast 1×1 minor is non zero. The system is reduced to a single equation. We assign arbitrary value to x $(or\ y)$ and solve for $y(or\ x)$

Suppose we assign $x = t$, from equation (1)

We get $y = \frac{1}{3}(8 - 2t)$

The solution set is $(x, y) = \left(t, \frac{1}{3}(8 - 2t)\right)\ t \in R$

55. b

$$u = xy^2 \sin\left(\frac{x}{y}\right)$$

$$u(tx, ty) = (tx)(ty)^2 \sin\left(\frac{tx}{ty}\right)$$

$$= t^3 xy^2 \sin\left(\frac{x}{y}\right)$$

u is a homogeneous function of degree 3

By Euler's theorem $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$

Part – C

56.

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 1 & -5 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 13 \\ 5 \end{bmatrix}$$

$$A\,X = B$$

$$|A| = 22 \neq 0$$

A is a non-singular matrix and hence A^{-1} exists.The cofactors are

$A_{11} = 6, A_{12} = -8, A_{13} = 2$

$A_{21} = 2, A_{22} = 1, A_{23} = -3$

$A_{31} = 4, A_{32} = 13, A_{33} = 5$

The matrix formed by the cofactors is

$$[A_{ij}] = \begin{bmatrix} 6 & -8 & 2 \\ 2 & 1 & -3 \\ 4 & 13 & 5 \end{bmatrix}$$

$$\text{adj } A = \begin{bmatrix} 6 & 2 & 4 \\ -8 & 1 & 13 \\ 2 & -3 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{|A|}(adj\ A)$$

$$\therefore x = 4, y = 1, z = 0$$

57. The system of equations can be written as $A\,X = B$

$$\begin{bmatrix} 1 & 1 & 3 \\ 4 & 3 & \mu \\ 2 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The augmented matrix is

$$[A, B] = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 4 & 3 & \mu & 0 \\ 2 & 1 & 2 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \mu - 12 & 0 \\ 0 & -1 & -4 & 0 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 4R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 3 & 0 \\ 0 & -1 & \mu - 12 & 0 \\ 0 & 0 & 8 - \mu & 0 \end{bmatrix} R_3 \to R_3 - R_2$$

Case(1): If $\mu \neq 8$ then $8 - \mu \neq 0$ and hence there are three non-zero rows.

$\therefore \rho(A) = \rho(A, B) = 3$ =number of unknowns.

$\therefore$ The system has the trivial solution

$x = 0, y = 0, z = 0$

Case(2): If $\mu = 8$ then

$\therefore \rho(A, B) = 2$ and $\rho(A) = 2$

$\therefore \rho(A) = \rho(A, B) = 2 <$ number of unknowns

The given system is equivalent to

$$x + y + 3z = 0; y + 4z = 0; y = -4z$$
$$x = z$$

Taking $z = k$, we get

$x = k, y = -4k, z = k, k \in R - \{0\}$

Which are non-trivial solutions.

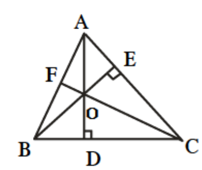
Thus the system is consistent and has infinitely many non-trivial solutions.

58. Let ABC be a triangle and let AD, BE be its two altitudes intersecting at O. In order to prove that the altitudes are concurrent it is sufficient to prove that CO is perpendicular to AB.

Taking O as the origin, let the position vectors of A, B, C be $\vec{a}$, $\vec{b}$, $\vec{c}$ respectively.

Then $\overrightarrow{OA} = \vec{a}, \overrightarrow{OB} = \vec{b}, \overrightarrow{OC} = \vec{c}$

Now $AD \perp BC$



$\overrightarrow{OA} \perp \overrightarrow{BC}$

$\Rightarrow \overrightarrow{OA} . \overrightarrow{BC} = 0$

$\Rightarrow \vec{a}.(\vec{c} - \vec{b}) = 0$

$\Rightarrow \vec{a}.\vec{c} - \vec{a}.\vec{b} = 0$...............(1)

$BE \perp CA$

$\overrightarrow{OB} \perp \overrightarrow{CA}$

$\Rightarrow \overrightarrow{OB} . \overrightarrow{CA} = 0$

$\Rightarrow \vec{b}.(\vec{a} - \vec{c}) = 0$

$\Rightarrow \vec{b}.\vec{a} - \vec{b}.\vec{c} = 0$................(2)

Adding (1) and (2), we get,

$$\vec{a}.\vec{c}-\vec{a}.\vec{b}+\vec{b}.\vec{a}-\vec{b}.\vec{c}=0$$
$$\vec{a}.\vec{c}-\vec{b}.\vec{c}=0$$
$$(\vec{a}-\vec{b}).\vec{c}=0$$
$$\Rightarrow \overrightarrow{BA}.\overrightarrow{OC}=0$$
$$\Rightarrow \overrightarrow{OC}\perp\overrightarrow{AB}$$

Hence, the three altitudes are concurrent.

59. The condition for intersecting is

$$\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}=0$$

Compare with $\frac{x-x_1}{l_1}=\frac{y-y_1}{m_1}=\frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2}=\frac{y-y_2}{m_2}=\frac{z-z_2}{n_2}$

The determinant becomes

$$\begin{vmatrix} 2-1 & 1+1 & -1+0 \\ 1 & -1 & 3 \\ 1 & 2 & -1 \end{vmatrix}=0$$

$\therefore$ The lines are intersecting lines

Point of intersection:

Take $\frac{x-1}{1}=\frac{y+1}{-1}=\frac{z}{3}=\lambda$

Any point on the line is of the form

$$(\lambda+1,-\lambda-1,3\lambda)$$

Take $\frac{x-2}{1}=\frac{y-1}{2}=\frac{z+1}{-1}=\mu$

Any point on the line is of the form

$$(\mu+2,2\mu+1,-\mu-1)$$

Since they are intersecting, for some λ,μ

$$(\lambda+1,-\lambda-1,3\lambda)=(\mu+2,2\mu+1,-\mu-1)$$

$\mu=-1$, $\lambda=0$

The point of intersection $(1-1,0)$

60. $$x^2-2px+(p^2+q^2)=0$$
$$x=p\pm iq, \text{Let } \alpha=p+iq, \beta=p-iq$$
$$\alpha-\beta=2qi$$

Given $\tan\theta=\frac{q}{y+p}$

$$y+p=\frac{q}{\tan\theta}$$
$$y+p=q\cot\theta$$
$$y=q\cot\theta-p$$
$$y+\alpha=q\frac{\cos\theta+i\sin\theta}{\sin\theta}$$

$(y+\alpha)^n=\frac{q^n}{\sin^n\theta}[\cos n\theta+i\sin n\theta]$.....(1)

Similarly

$(y+\beta)^n=\frac{q^n}{\sin^n\theta}[\cos n\theta-i\sin n\theta]$......(2)

(1)−(2) gives

$$(y+\alpha)^n-(y+\beta)^n=q^{n-1}\frac{\sin n\theta}{\sin^n\theta}$$

61. $$x^9+x^5-x^4-1=0$$
$$x^5(x^4+1)-1(x^4+1)=0$$
$$(x^5-1)(x^4+1)=0$$
$$x^5-1=0;\ x^4+1=0$$

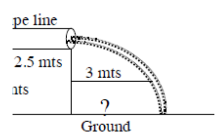
(i) $x=(1)^{\frac{1}{5}}=(\cos 0+i\sin 0)^{\frac{1}{5}}$

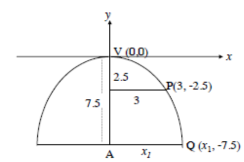
$$=(\cos 2k\pi+i\sin 2k\pi)^{\frac{1}{5}}$$
$$=\cos\frac{2k\pi}{5}+i\sin\frac{2k\pi}{5}$$
$$k=0,1,2,3,4$$

(ii) $x=(-1)^{\frac{1}{4}}=(\cos\pi+i\sin\pi)^{\frac{1}{4}}$

$$=(\cos(2k+1)\pi+i\sin(2k+1)\pi)^{\frac{1}{4}}$$
$$=\cos\frac{(2k+1)\pi}{4}+i\sin\frac{(2k+1)\pi}{4}$$
$$k=0,1,2,3$$

Thus we have 9 roots.

62. 



$$x^2=-4ay$$

Let P be the point on the flow path, 2.5m below the line of the pipe and 3m beyond the vertical line through the end of the pipe.

$$P(3,-2.5)$$

Thus, $9=-4a(-2.5)$

$$a=\frac{9}{10}$$

The equation of the parabola is

$$x^2=-4\times\frac{9}{10}y$$

Let x_1 be the distance between the bottom of the vertical line on the ground from the pipe end and the point on which the water touches the ground. But the height of the pipe from the ground is 7.5 m

The point $(x_1,-7.5)$ lies on the parabola

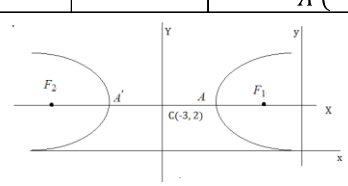
$$x_1{}^2=-4\times\frac{9}{10}\times(-7.5)=27$$
$$x_1=3\sqrt{3}$$

The water strikes the ground $3\sqrt{3}$m beyond the vertical line.

63. $$x^2-4y^2+6x+16y-11=0$$
$$x^2+6x-4y^2+16y=11$$
$$(x^2+6x+3^2-3^2)-4(y^2-4y+2^2-2^2)=11$$
$$\{(x+3)^2-9\}-4\{(y-2)^2-4\}=11$$
$$(x+3)^2-4(y-2)^2=4$$
$$\frac{(x+3)^2}{4}-\frac{(y-2)^2}{1}=4$$

	$\frac{X^2}{4}-\frac{Y^2}{1}=1$ where $X=x+3, Y=y-2$ The transverse axis is along X axis $a^2=4, b^2=1, a=2, b=1$ $e=\sqrt{1+\frac{b^2}{a^2}}=\frac{\sqrt{5}}{2}$ $ae=\sqrt{5}$

	Referred to X, Y	Referred to x, y $X=x+3,$ $Y=y-2$
Centre	$(0,0)$	$X=0, Y=0$ $x+3=0, y-2=0$ $C(-3,2)$
Foci	$(\pm ae, 0)$ $(\pm\sqrt{5}, 0)$	$(\sqrt{5}, 0)$ $X=\sqrt{5}, Y=0$ $x+3=\sqrt{5}, y-2=0$ $x=-3+\sqrt{5}, y=2$ $F_1(-3+\sqrt{5}, 2)$
		$(-\sqrt{5}, 0)$ $X=-\sqrt{5}, Y=0$ $x+3=-\sqrt{5}, y-2=0$ $x=-3-\sqrt{5}, y=2$ $F_2(-3-\sqrt{5}, -2)$
Vertices	$(\pm a, 0)$ $(\pm 2, 0)$	$(2,0)$ $X=2, Y=0$ $x+3=2, y-2=0$ $x=-1, y=2$ $A(-1,2)$
		$(-2,0)$ $X=-2, Y=0$ $x+3=-2, y-2=0$ $x=-5, y=2$ $A'(-5,2)$



64 The condition for $y=mx+c$ to be a tangent to a hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$c^2=a^2m^2-b^2$

$3x-y-5=0$

$-y=-3x+5$

$y=3x-5$

$m=3, c=-5$

$2x^2-3y^2=6$

$\frac{x^2}{3}-\frac{y^2}{2}=1$

$a^2=3, b^2=2$

$c^2=25$;

$a^2m^2-b^2=3(9)-2=25$

$\Rightarrow c^2=a^2m^2-b^2$

Thus the line $3x-y=0$is a tangent to the hyperbola. It touches the hyperbola

The point of contact is $\left(\frac{-a^2m}{c}, \frac{-b^2}{c}\right)$

$\frac{-a^2m}{c}=\frac{-3(3)}{-5}=\frac{9}{5}$

$\frac{-b^2}{c}=\frac{-2}{-5}=\frac{2}{5}$

The point of contact is $\left(\frac{9}{5}, \frac{2}{5}\right)$

65 Let r, h respectively denote the base radius and height of the cone of volume V, at time t min. Then we are given that $2r=h$

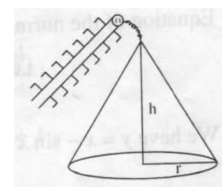
To find $\frac{dh}{dt}$ when $h=10$ ft and

$\frac{dv}{dt}=30\, ft^3/min$

Volume of the cone $V=\frac{1}{3}\pi r^2 h$

$=\frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$=\frac{\pi}{12}(h^3)$



$\frac{dv}{dt}=\frac{\pi}{12}\times 3h^2\frac{dh}{dt}$

$\frac{dh}{dt}=\frac{4\frac{dv}{dt}}{\pi h^2}$

$\left(\frac{dh}{dt}\right)=\frac{4\times 30}{\pi\times 100}=\frac{12}{10\pi}=\frac{6}{5\pi}$ ft/ min

$\therefore$ The height of the cone is increasing at the rate of $\frac{6}{5\pi}$ ft/min.

66 $f(x)=x-2\sin x$, is continuous in $[0,2\pi]$

$f'(x)=1-2\cos x$

$f'(x)=0$

$1-2\cos x=0$

$\cos x=\frac{1}{2}$

$x=\frac{\pi}{3}$ or $\frac{5\pi}{3}$

The values of f at these critical points are

$f\left(\frac{\pi}{3}\right)=\frac{\pi}{3}-2\sin\frac{\pi}{3}=\frac{\pi}{3}-\sqrt{3}$

$f\left(\frac{5\pi}{3}\right)=\frac{5\pi}{3}-2\sin\frac{5\pi}{3}$

$=\frac{5\pi}{3}+\sqrt{3}\approx 6.968039$

The values of f at the end points are

$f(0)=0$ and $f(2\pi)=2\pi\approx 6.28$

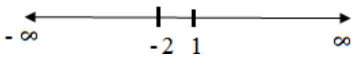
Comparing these four numbers, the absolute minimum is $f\left(\frac{\pi}{3}\right)=\frac{\pi}{3}-\sqrt{3}$ and the absolute maximum is $f\left(\frac{5\pi}{3}\right)=\frac{5\pi}{3}+\sqrt{3}$

67 $y=12x^2-2x^3-x^4$

$\frac{dy}{dx}=24x-6x^2-4x^3$

$\frac{d^2y}{dx^2}=24-12x-12x^2=-12(x-1)(x+2)$

$\frac{d^2y}{dx^2}=0\Rightarrow x=-2, x=1$



$f''(x) < 0$ for $x \in (-\infty, -2)$ and $x \in (1, \infty)$ and $f''(x) > 0$ for $x \in (-2, 1)$

f is concave downward on $(-\infty, -2) \cup (1, \infty)$ and f is concave upward on $(-2, 1)$

And the points of inflections are$(-2, f(-2)), (1, f(1))$

That is $(-2, 48)$ and $(1, 9)$

68.

$w = u^2 e^v, \quad u = \frac{x}{y}, v = y \log x$

$\frac{\partial w}{\partial u} = 2ue^v \quad \frac{\partial w}{\partial v} = u^2 e^v \quad \frac{\partial u}{\partial x} = \frac{1}{y}$

$\frac{\partial u}{\partial y} = -\frac{x}{y^2} \quad \frac{\partial v}{\partial x} = \frac{y}{x} \quad \frac{\partial v}{\partial y} = \log x$

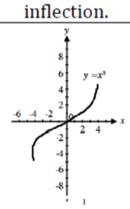
$$\frac{\partial w}{\partial x} = \frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial w}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\frac{\partial w}{\partial x} = x^y \frac{x}{y^2}(2 + y)$$

$$\frac{\partial w}{\partial y} = \frac{\partial w}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial w}{\partial v} \cdot \frac{\partial v}{\partial y}$$

$$\frac{\partial w}{\partial y} = \frac{x^2}{y^3} x^y [y \log x - 2]$$

69.

Domain	The function is defined for all real values of x and hence the domain is the entire interval $(-\infty, \infty)$
Extent	Horizontal extent is $-\infty < x < \infty$ Vertical extent is $-\infty < y < \infty$
Intercepts	$x = 0$, then $y = 0$ $y = 0$, then $x = 0$
Origin	The curve passes through $(0, 0)$
Symmetry Test	It is symmetrical about the origin
Asymptotes	The curve does not admit any asymptote.
Monotonicity	Since $y' \geq 0$ for all x The curve is increasing in $(-\infty, \infty)$
Special points	Since $y'' = 6x$, the curve is concave upward in $(0, \infty)$ and convex downward $(-\infty, 0)$ $y'' = 0$ for $x = 0$ yields $(0, 0)$ as the point of inflection.
Diagram	

70. a

Cartesian form:

Let a, b and c be the x, y and z intercepts of the plane respectively.

The plane passes through the points $(a, 0, 0), (0, b, 0), (0, 0, c)$

Here The equation is

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x - a & y - 0 & z - 0 \\ -a & b - 0 & 0 \\ -a & 0 & c - 0 \end{vmatrix} = 0$$

$$(x - a)(bc) - (y - 0)(-ac) + (z - 0)(0 + ab) = 0$$

$$xbc + yac + zab = abc$$

$$\div abc \qquad \frac{xbc}{abc} + \frac{yac}{abc} + \frac{zab}{abc} = \frac{abc}{abc}$$

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

Vector form:

The equation of the plane passing through three given points is

$$\vec{r} = (1 - s - t)\vec{a} + s\vec{b} + t\vec{c}$$

$$\vec{r} = (1 - s - t)a\vec{i} + sb\vec{j} + tc\vec{k}$$

$$x\vec{i} + y\vec{j} + z\vec{k} = (1 - s - t)a\vec{i} + sb\vec{j} + tc\vec{k}$$

$$x = (1 - s - t)a; y = sb; z = tc$$

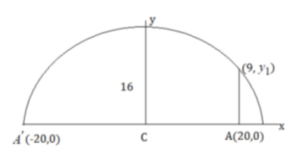
$$\frac{x}{a} = 1 - s - t, \frac{y}{b} = s, \frac{z}{c} = t$$

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 - s - t + s + t$$

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

70. b

Take the midpoint of the base as the centre $C(0,0)$. Since the base is 40 ft, the vertices



A and A' are $(20,0)$ and $(-20,0)$

Clearly $2a = 40 \Rightarrow a = 20, b = 16$

The corresponding equation

$$\frac{x^2}{400} + \frac{y^2}{256} = 1$$

$(9, y_1)$ is a point on the equation

$$\frac{9^2}{400} + \frac{{y_1}^2}{256} = 1$$

$${y_1}^2 = 256\left(\frac{319}{400}\right) \Rightarrow y_1 = \frac{4\sqrt{319}}{5}$$

The height of the arch 9ft from the right of the centre is $\frac{4\sqrt{319}}{5}$ feet.

Prepared by K.Dinesh M.Sc., M.Phil., PGDCA.,(Ph.D)