10th standard - Half yearly Exam –2016

Time Allowed: 2½ Hours **Maximum Marks: 100**

SECTION I (MARKS: 15)

Note : (i) Answer all questions. **15x1=15**

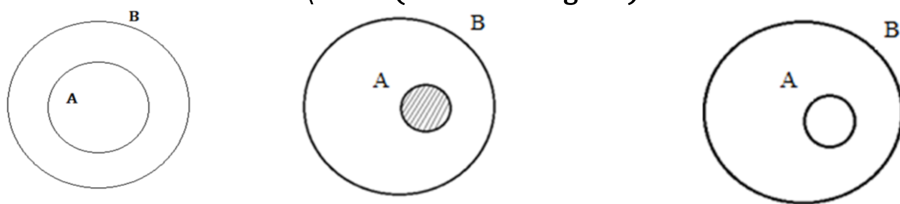
1. Given $f(x) = (-1)^x$ is a function from ℕ to ℤ . Then the range of f is
   A) {1} B) ℕ **C) {1,-1}** D) ℤ
2. If a, b, c are in G.P, then $\frac{a-b}{b-c}$ is equal to
   A) $\boldsymbol{\frac{a}{b}}$ B) $\frac{b}{a}$ C) $\frac{a}{c}$ D) $\frac{c}{b}$
3. The sequence -3, -3, -3, ……… is
   A) an A.P. only B) a G.P. only C) neither A.P. nor G.P. **D) both A.P. and G.P.**
4. A system of two linear equation $x - 4y = 8, 3x - 12y = 24$
   A) has infinitely many solutions B) has no solution
   C) has a unique solution D) may or may not have a solution
5. If $ax^2 + bx + c = 0$ has equal roots, then c is equal
   A) $\frac{b^2}{2a}$ **B)** $\boldsymbol{\frac{b^2}{4a}}$ C) $-\frac{b^2}{2a}$ D) $-\frac{b^2}{4a}$
6. If $\begin{pmatrix} 8 & 4 \\ x & 8 \end{pmatrix} = 4 \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ then the value of x is
   A) 1 B) 2 C) $\frac{1}{4}$ **D) 4**
7. The angle of inclination of a straight line parallel to x-axis is equal to
   A) 0^0 B) 60^0 C) 45^0 D) 90^0
8. The point of intersection of the straight lines $y=0$ and $x=-4$ is
   A) (0, -4) **B) (-4, 0)** C) (0, 4) D) (4, 0)
9. A point P is 26 cm away from the centre O of a circle and PT is the tangent drawn from P to the circle is 10 cm, then OT is equal to
   A) 36 cm B) 20 cm C) 18 cm **D) 24 cm**
10. The perimeter of two similar triangles ΔABC and ΔDEF are 36 cm and 24 cm respectively. If DE =10cm, then AB is
    A) 12 cm B) 20 cm **C) 15 cm** D) 18 cm
11. If $\tan\theta = \frac{a}{x}$, then the value of $\frac{x}{\sqrt{a^2+x^2}} =$
    A) cos $\boldsymbol{\theta}$ B) sin θ C) cosec θ D) sec θ
12. $9\tan^2\theta - 9\sec^2\theta =$
    A) 1 B) 0 C) 9 **D) -9**
13. The total surface area of a solid right circular cylinder whose radius is half of its height h is equal to
    A) $\frac{3}{2}\pi h$ sq.untis B) $\frac{2}{3}\pi h^2$ sq.untis **C) $\frac{3}{2}\pi h^2$ sq.untis** D) $\frac{2}{3}\pi h$ sq.untis
14. For any collection of n items, $\Sigma(x-\bar{x})$=
    A) $\Sigma\, x$ B) $\bar{x}$ C) $n\bar{x}$ **D) 0**
15. Probability of sure event is
    A) 1 B) 0 C) 100 D) 0.1

SECTION – II (MARKS: 20)

Note : (i) Answer 10 questions. 10x2=20

(ii) Question number 30 is compulsory. Select any questions from the first 14 questions.

16. If $A \subset B$, then find $A \cap B$ and $A \backslash B$ (Use Venn diagram).



$A \subset B$ $A \cap B = A$ $A \backslash B = \emptyset$

17. If $\mathbf{R} = \{(\mathbf{a}, -\mathbf{2}), (-\mathbf{5}, \mathbf{b}), (\mathbf{8}, \mathbf{c}), (\mathbf{d}, -\mathbf{1})\}$ represents the identity function, find the values of $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ **and** $\boldsymbol{d}$.

$$a = -2, b = -5, c = 8, d = -1$$

18. Which term of the geometric sequence. 1, 2, 4, 8, ……… is 1024?

1, 2, 4, 8, ………

$a = 1, r = \frac{t_2}{t_1} = \frac{2}{1} = 2$

$t_n = 1024$ $\boxed{t_n = ar^{n-1}}$

$ar^{n-1} = 1024$

$1 \times 2^{n-1} = 2^{10}$

$2^{n-1} = 2^{10}$

$n - 1 = 10$

$n = 10 + 1 = 11$

1, 2, 4, 8, ………… 1024 .

Then, the 11th term of the given geometric sequence is 1024.

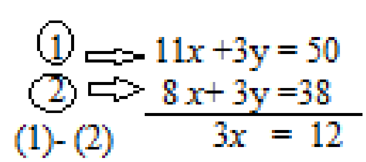
19. The cost of 11 pencils and 3 erasers is Rs.50 and the cost of 8 pencils and 3 erasers is Rs.38 Find the cost of each pencil and each eraser. *(Oct-2013)*

Let x denote the cost of a pencil in rupees and y denote the cost of an eraser in rupees.

Then according to the given information we have

$11x + 3y = 50$……………………………….(1)

$8x + 3y = 38$…………………………………..(2)



Subtracting (2) from (1) we get, $3x = 12$ which gives $x = 4$

Now substitute $x = 4$ in (1) to find the value of y, we get,

$11(4) + 3y = 50$

$y = 2$

Therefore, $x = 4$ and $y = 2$ is the solution of the given pair of equations. Thus, the cost of a pencil is Rs.4 and that of an eraser is Rs.2

20. If $(x+3)$ is a factor of x^3+ax^2-bx+6 and $a+b=7$ find the values of a and b

$x+3$ is a factor of $x^3+ax^2-bx+6=0$

∴ Substitute $x=-3$ in above polynomial

$(-3)^3+a(-3)^2-b(-3)+6=0$

$-27+9a+3b+6=0$

$9a+3b-21=0$

$\div 3 \qquad 3a+b-7=0$

$3a+b=7$ …………..(1)

Given $a+b=7$……………………………………(2)

Simplifying by elimination method $2a=0 \Rightarrow a=0$

$a+b=7 \Rightarrow 0+b=7 \Rightarrow b=7$

$$\begin{array}{r} 3a+b=7 \\ a+b=7 \\ (-)\ (-)\quad (-) \\ \hline 2a=0 \end{array}$$

21. Construct a 2 x 3 matrix A = $[a_{ij}]$ whose elements are given by $a_{ij}=|2i-3j|$.

A $=\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}=|2i-3j|$

$a_{11}=|2(1)-3(1)|=|2-3|=|-1|=1$

$a_{12}=|2(1)-3(2)|=|2-6|=|-4|=4$

$a_{13}=|2(1)-3(3)|=|2-9|=|-7|=7$

$a_{21}=|2(2)-3(1)|=|4-3|=|1|=1$

$a_{22}=|2(2)-3(2)|=|4-6|=|-2|=2$

$a_{23}=|2(2)-3(3)|=|4-9|=|-5|=5$

A $=\begin{pmatrix} 1 & 4 & 7 \\ 1 & 2 & 5 \end{pmatrix}$

22. If A = $\begin{pmatrix} 4 & -2 \\ 5 & -9 \end{pmatrix}$ and B = $\begin{pmatrix} 8 & 2 \\ -1 & -3 \end{pmatrix}$ find 6A - 3B.

$6A-3B=6\begin{pmatrix} 4 & -2 \\ 5 & -9 \end{pmatrix}-3\begin{pmatrix} 8 & 2 \\ -1 & -3 \end{pmatrix}=\begin{pmatrix} 24 & -12 \\ 30 & -54 \end{pmatrix}+\begin{pmatrix} -24 & -6 \\ 3 & 9 \end{pmatrix}$

$=\begin{pmatrix} 0 & -18 \\ 33 & -45 \end{pmatrix}$

23. If the x- intercept and y- intercept of a straight line are $\frac{2}{3}$ and $\frac{3}{4}$ respectively, then find the equation of the straight line.

Given that x-intercept of the straight line $a=\frac{2}{3}$

and the y-intercept of the straight line $b=\frac{3}{4}$

Using intercept form, the equation of the straight line is $\frac{x}{a}+\frac{y}{b}=1$

$\frac{x}{\frac{2}{3}}+\frac{y}{\frac{3}{4}}=1$

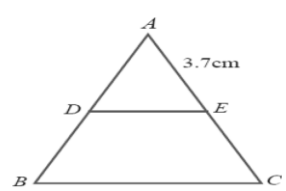
$\frac{3x}{2}+\frac{4y}{3}=1$

Hence $9x+8y-6=0$ is the required equation

24. In ΔABC, $DE \parallel BC$ and $\frac{AD}{DB}=\frac{2}{3}$. If AE= 3.7 cm, find EC.

By Thales theorem, we have $\frac{AD}{DB}=\frac{AE}{EC}$

$\frac{2}{3}=\frac{3.7}{EC}$



$$EC = \frac{3.7 \times 3}{2} = \frac{11.1}{2}$$
$$= 5.55 \text{ cm}$$

25. Prove the following identity $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \cot\theta$

We consider $\frac{1+\cos\theta-\sin^2\theta}{\sin\theta(1+\cos\theta)} = \frac{1-\sin^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{\cos^2\theta+\cos\theta}{\sin\theta(1+\cos\theta)} = \frac{\cos\theta(\cos\theta+1)}{\sin\theta(1+\cos\theta)} = \frac{\cos\theta}{\sin\theta} = \cot\theta$

26. A girl of height 150 cm stands in front of a lamp-post and casts a shadow of length $150\sqrt{3}$ cm on the ground. Find the angle of elevation of the top of the lamp-post.

In the right ΔABC

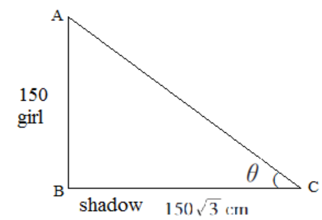
$\tan\theta = \frac{AB}{BC}$

$= \frac{150}{150\sqrt{3}}$

$= \frac{1}{\sqrt{3}}$

$\tan 30^0 = \frac{1}{\sqrt{3}}$

∴The angle of elevation $\theta = 30^0$



27. If the vertical angle and the radius of a right circular cone are $60°$ and 15cm respectively, then find its height and slant height.

In the figure, OAB is the cone.

Draw $OC \perp CB$.

Given that the vertical angle $\angle AOB = 60°$ and $AC = 15$cm

$\angle AOC = \frac{\angle AOB}{2} = \frac{60°}{2} = 30°$

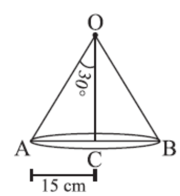
Consider the right angled ΔOAC we have

$\tan 30° = \frac{AC}{OC}$

$\frac{1}{\sqrt{3}} = \frac{15}{OC}$

$OC = 15\sqrt{3}$

Hence, the height of the cone is $15\sqrt{3}$ cm



28. If $n = 10, \bar{x} = 12$ and $\sum x^2 = 1530$, then calculate the coefficient of variation .

n= 10, $\bar{x}$ = 12, $\sum x^2$= 1530

S.D= $\sqrt{\frac{\sum x^2}{n} - \left[\frac{\sum x}{n}\right]^2} = \sqrt{\frac{1530}{10} - 12^2} = \sqrt{153-144} = \sqrt{9} = 3$

C.V. = $\frac{\sigma}{\bar{x}} \times 100 = \frac{3}{12} \times 100 = \frac{1}{4} \times 100 = 25$

C.V. = 25

29. A bag contains 6 red balls and some blue balls. If probability of drawing a blue ball from the bag is twice that of drawing a red ball, then find the number of blue balls in the bag.

Let the number of blue balls be x

Total number of balls n(S) = 6+ x

B be the event of drawing a blue ball and R be the event of drawing a red ball P(B) = 3P(R)

$$\frac{n(B)}{n(S)} = 2\ \frac{n(R)}{n(S)}$$

$$\frac{x}{6+x} = 2\left[\frac{6}{6+x}\right] \Rightarrow x = 2 \times 6 = 12$$

The number of blue balls $x = 12$

30.(a) Find the volume of the largest right circular cone that can be cut out of a cube whose edge is 14 cm.

Diameter of the cone= 14 cm

Radius(r) = 7 cm

Height(h) = 14 cm

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \text{x} \frac{22}{\cancel{7}} \text{x} \cancel{7} \text{x } 7\text{x}14$$

$$= 718.67 \text{ cm}^3$$

(OR)

30.(b) If the three points $(\boldsymbol{h}, \mathbf{0}), (\boldsymbol{a}, \boldsymbol{b}), (\mathbf{0}, \boldsymbol{k})$ lie on a straight line, then using the area of the triangle formula show that that $\frac{\boldsymbol{a}}{\boldsymbol{h}} + \frac{\boldsymbol{b}}{\boldsymbol{k}} = \mathbf{1}$ where $\boldsymbol{h}, \boldsymbol{k} \neq \mathbf{0}$

The three points $(h, 0), (a, b), (0, k)$ lie on a straight lines

Thus $\frac{1}{2}\left\{\begin{matrix} h & a & 0 & h \\ 0 & b & k & 0 \end{matrix}\right\} = 0$

$\Rightarrow (hb + ak + 0) - (0 + 0 + kh) = 0$

$\Rightarrow hb + ak = kh$

Since $h, k \neq 0$ Divided by hk on both sides, we get

$\frac{a}{h} + \frac{b}{k} = 1$

SECTION – III (MARKS: 45)

Note: (i) Answer 9 questions: 9x5=45

(ii) Question number 45 is compulsory. Select any 8 questions from the first 14 questions.

31. For $\boldsymbol{A} = \{\boldsymbol{x}\ |{-3} \le \boldsymbol{x} < 4, x \in \mathbb{R}\}, \boldsymbol{B} = \{\boldsymbol{x}\ |\boldsymbol{x} < 5, x \in \mathbb{N}\}$ and $= \{-\mathbf{5}, -\mathbf{3}, -\mathbf{1}, \mathbf{0}, \mathbf{1}, \mathbf{3}\}$, Show that $\boldsymbol{A} \cap (\boldsymbol{B} \cup \boldsymbol{C}) = (\boldsymbol{A} \cap \boldsymbol{B}) \cup (\boldsymbol{A} \cap \boldsymbol{C})$

First note that the set A contains all the real numbers (not just integers) that are greater than or equal to -3 and less than 4.

On the other hand the set B contains all the positive integers that are less than 5. So,

$A = \{x\ |{-3} \le x < 4, x \in \mathbb{R}\} = \{-3, -2, -1,0,1,2,3\}$, that is A consists of all real numbers from -3 upto 4 but 4 is not included.

Also $B = \{x \mid x < 5, x \in \mathbb{N}\} = \{1,2,3,4\}$. Now we find

LHS: $A \cap (B \cup C) \Rightarrow$

$$B \cup C = \{1,2,3,4\} \cup \{-5,-3,-1,0,1,3\} = \{1,2,3,4,-5,-3,-1,0\}$$

$$A \cap (B \cup C) = \{-\mathbf{3},-2,-\mathbf{1},\mathbf{0},\mathbf{1},\mathbf{2},\mathbf{3}\} \cap \{\mathbf{1},\mathbf{2},\mathbf{3},4,-5,-\mathbf{3},-\mathbf{1},\mathbf{0}\}$$

$$= \{-3,-1,0,1,2,3\} \ldots\ldots\ldots\ldots(1)$$

Next, to find $(A \cap B) \cup (A \cap C)$, we consider

RHS: $(A \cap B) \cup (A \cap C) \Rightarrow$

$$A \cap B = \{-3,-2,-1,0,\mathbf{1},\mathbf{2},\mathbf{3}\} \cap \{\mathbf{1},\mathbf{2},\mathbf{3},4\} = \{1,2,3\}$$

$$A \cap C = \{-\mathbf{3},-2,-\mathbf{1},\mathbf{0},\mathbf{1},2,\mathbf{3}\} \cap \{-5,-\mathbf{3},-\mathbf{1},\mathbf{0},\mathbf{1},\mathbf{3}\} = \{-3,-1,0,1,3\}$$

Hence, $(A \cap B) \cup (A \cap C) = \{1,2,3\} \cup \{-3,-1,0,1,3\} = \{-3,-1,0,1,2,3\} \ldots\ldots\ldots\ldots(2)$

From (1) and (2), $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

32. Let $A = \{6,9,15,18,21\}$; $B = \{1,2,4,5,6\}$ and $f: A \to B$ be defined by $f(x) = \frac{x-3}{3}$. Represent f by (i) an arrow diagram, (ii) a set of ordered pairs, (iii) a table , (iv) a graph

Given $A = \{6,9,15,18,21\}$; $B = \{1,2,4,5,6\}$

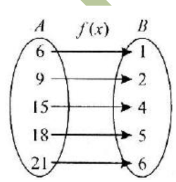
Let us find the images under f

Now, $f(x) = \frac{x-3}{3}, x \in A$

$f(6) = \frac{6-3}{3} = \frac{3}{3} = 1,$ $\quad f(9) = \frac{9-3}{3} = \frac{6}{3} = 2,$ $\quad f(15) = \frac{15-3}{3} = \frac{12}{3} = 4$

$f(18) = \frac{18-3}{3} = \frac{15}{3} = 5$ $\quad f(21) = \frac{21-3}{3} = \frac{18}{3} = 6$

(i) An arrow diagram

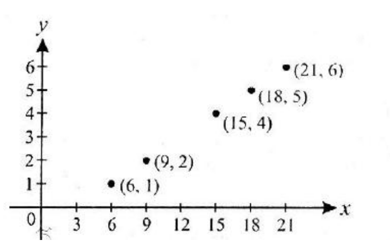


(ii) Set of ordered pairs $f = \{(6,1),(9,2),(15,4),(18,5),(21,6)\}$

(iii) Table

x	6	9	15	18	21
$f(x)$	1	2	4	5	6

(iv) Graph



Here, the graph is the collection of all points $(6,1),(9,2),(15,4),(18,5),(21,6)$ in xy plane.

33. In an AP 6th term is half the 4th term, and the 3rd term is 15. If $S_n = 66$ then find the values of n

Given $t_3 = 15$, $t_6 = \frac{1}{2}t_4$

$t_3 = 15 \Rightarrow a + 2d = 15$……………………….(1)

$t_6 = \frac{1}{2}t_4 \Rightarrow a + 5d = \frac{1}{2}(a + 3d)$

$2a + 10d = a + 3d$

$a + 7d = 0$ ……………………….(2)

Simplifying (1)& (2) by elimination method we get

$d = -3$

$a + 2d = 15$
$a + 7d = 0$
(-) (-) (-)
$-5d = 15$
$d = -3$

Sub in (2) $a + 7d = 0 \Rightarrow a + 7(-3) = 0 \Rightarrow a = 21$

$S_n = \frac{n}{2}[2a + (n-1)d]$ ………………….(3)

Sub the values in (3) $a = 21, d = -3, S_n = 66$

$66 = \frac{n}{2}[2(21) + (n-1)(-3)]$

$132 = n[42 - 3n + 3]$

$132 = 45n - 3n^2$

$3n^2 - 45n + 132 = 0$

$\div 3$ $n^2 - 15n + 44 = 0$

$(n - 11)(n - 4) = 0$

$n = 11$ or $n = 4$

34. Factorize $x^3 - 23x^2 + 142x - 120$

Let $p(x) = x^3 - 23x^2 + 142x - 120$

When $x = 1$, $p(1) = (1)^3 - 23(1)^2 + 142(1) - 120$

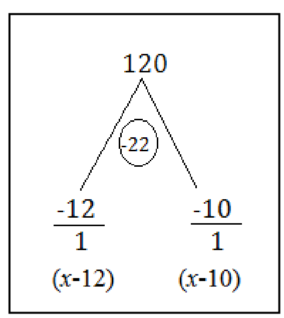
$= 1 - 23 + 142 - 120 = 0$

$p(1) = 0$ thus $(x - 1)$ is a factor of $p(x)$

We shall use synthetic division to find the other factors

1	1	−23	142	−120
	0	1	−22	120
	1	−22	120	0 → Remainder,

$(x - 1)$ is a factor



The other factor is $x^2 - 22x + 120$

Thus, $p(x) = (x - 1)(x^2 - 22x + 120)$

$x^2 - 22x + 120 = x^2 - 12x - 10x + 120 = (x - 12)(x - 10)$

$x^3 - 23x^2 + 142x - 120 = (x - 1)(x - 12)(x - 10)$

Factors are $(x - 1)(x - 12)(x - 10)$

35. If α and β are the roots of the equation $3x^2 - 5x + 2 = 0$, then find the values of

(i) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ (i) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}$

$3x^2 - 5x + 2 = 0$, here $a = 3, b = -5, c = 2$

$\alpha+\beta = \frac{-b}{a} = \frac{-(-5)}{3} = \frac{5}{3}$

$\alpha\beta = \frac{c}{a} = \frac{2}{3}$

(i) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta} = \frac{\left(\frac{5}{3}\right)^2-2\left(\frac{2}{3}\right)}{\frac{2}{3}} = \frac{\left(\frac{25}{9}\right)-\left(\frac{6}{3}\right)}{\frac{2}{3}} = \left(\frac{25-12}{9}\right)\frac{3}{2} = \frac{13}{6}$

(ii) $\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha} = \frac{\alpha^3+\beta^3}{\alpha\beta} = \frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta} = \frac{\left(\frac{5}{3}\right)^3-3\left(\frac{2}{3}\right)\left(\frac{5}{3}\right)}{\left(\frac{2}{3}\right)} = \frac{\frac{125}{27}-\frac{30}{9}}{\frac{2}{3}} = \frac{125-90}{27}\times\frac{3}{2} = \frac{35}{18}$

36. If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then show that $A^2 - (a+d)A = (bc-ad)I_2$

Consider $A^2 = A \times A$

$$= \begin{pmatrix} a & b \\ c & d \end{pmatrix} \times \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix} \ldots\ldots\ldots\ldots(1)$$

Now $(a+d)A = (a+d)\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a^2+ad & ab+bd \\ ac+cd & ad+d^2 \end{pmatrix} \ldots\ldots\ldots\ldots(2)$

From (1) and (2) we get ,

$$A^2-(a+d)A = \begin{pmatrix} a^2+bc & ab+bd \\ ac+cd & bc+d^2 \end{pmatrix} - \begin{pmatrix} a^2+ad & ab+bd \\ ac+cd & ad+d^2 \end{pmatrix}$$

$$= \begin{pmatrix} bc-ad & 0 \\ 0 & bc-ad \end{pmatrix} = (bc-ad)\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = (bc-ad)I_2$$

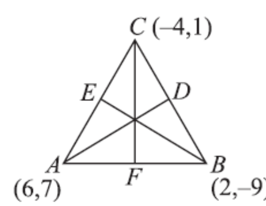
Thus, $A^2-(a+d)A = (bc-ad)I_2$

37. A triangle has vertices at $(6,7)$, $(2,-9)$ and $(-4,1)$. Find the slopes of its medians.

Let the vertices be $A(6,7), B(2,-9)$ and $C(-4,1)$

Let D, E, F be the midpoints of BC, CA, AB respectively.

Then AD, BE and CF are the medians of the ΔABC



The midpoint of BC is $D\left(\frac{2-4}{2}, \frac{-9+1}{2}\right) = D(-1,-4)$

The midpoint of CA is $E\left(\frac{-4+6}{2}, \frac{1+7}{2}\right) = E(1,4)$

The midpoint of AB is $F\left(\frac{6+2}{2}, \frac{7-9}{2}\right) = F(4,-1)$

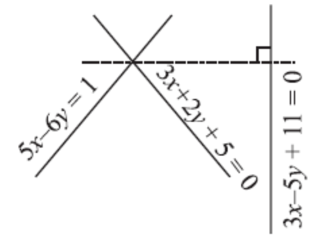
Slope of $AD = \frac{-4-7}{-1-6} = \frac{-11}{-7} = \frac{11}{7}$

Slope of $BE = \frac{4+9}{1-2} = \frac{13}{-1} = -13$

Slope of $CF = \frac{-1-1}{4+4} = -\frac{2}{8} = -\frac{1}{4}$

Hence, the slopes of the medians are $\frac{11}{7}$, -13 and $-\frac{1}{4}$

38. Find the equation of the straight line which passes through the point of intersection of the straight lines $5x - 6y = 1$ and $3x + 2y + 5 = 0$ and is perpendicular to the straight line $3x - 5y + 11 = 0$



The given equation are $5x - 6y = 1$ ....(1)

$3x + 2y = -5$.........(2)

Solve (1) and (2), we get the point of intersection $(-1, -1)$

Slope of the line $3x - 5y + 11 = 0$ is $m = \frac{-3}{-5} = \frac{3}{5}$

Thus, the slope of the required line (Perpendicular line) is $-\frac{5}{3}$

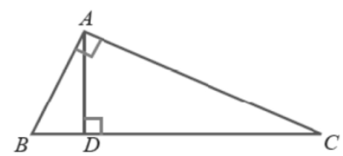
Hence the equation of the line passing through $(-1, -1)$ with slope $-\frac{5}{3}$ is

$$y + 1 = -\frac{5}{3}(x + 1)$$

$$5x + 3y + 8 = 0$$

39. State and Prove Pythagoras theorem

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Given : In a right angled $\Delta ABC, \angle A = 90°$

To prove: $BC^2 = AB^2 + AC^2$

Construction: Draw $AD \perp BC$

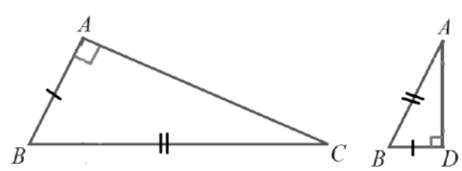
Proof:

In triangles ABC and DBA, $\angle B$ is the common angle.

Also, we have $\angle BAC = \angle ADB = 90°$

$\Delta ABC \sim \Delta DBA$ (AA similarity criterion)

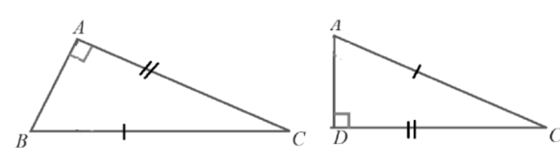
Thus, their corresponding sides are proportional.



Hence, $\frac{AB}{DB} = \frac{BC}{BA}$

$\therefore AB^2 = DB \times BC$...(1)

Similarly, we have $\Delta ABC \sim \Delta DAC$



Thus, $\frac{BC}{AC} = \frac{AC}{DC}$

$\therefore AC^2 = BC \times DC$...(2)

Adding (1)& (2) we get,

$AB^2 + AC^2 = (DB \times BC) + (BC \times DC)$

$= BC(BD + DC)$

$= BC \times BC = BC^2$

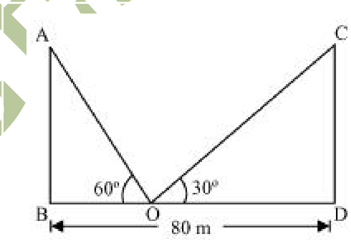
Thus, $BC^2 = AB^2 + AC^2$

Hence, the Phythagoras theorem.

40. Two poles of equal heights are standing opposite each other on either side of a road. Which is 80m wide. From a point between them on the road, the angle of elevation of the top of the poles are $60°$ and $30°$ respectively. Find the height of the poles and the distance of the point from the poles.

AB and CD be the poles

O is the point from where the elevation angles are measured



In ΔABO

$\frac{AB}{BO} = \tan 60°$

$\frac{AB}{BO} = \sqrt{3}$

$BO = \frac{AB}{\sqrt{3}}$

In ΔCDO

$\frac{CD}{DO} = \tan 30°$

$\frac{CD}{80-BO} = \frac{1}{\sqrt{3}}$

$\sqrt{3}\, CD = 80 - BO$

$\sqrt{3}\, CD = 80 - \frac{AB}{\sqrt{3}}$

$\sqrt{3}\, CD + \frac{AB}{\sqrt{3}} = 80$

Since the poles are equal height $(CD = AB)$

$CD\left[\sqrt{3} + \frac{1}{\sqrt{3}}\right] = 80$

$CD\left[\frac{3+1}{\sqrt{3}}\right] = 80$

$CD\left[\frac{4}{\sqrt{3}}\right] = 80$

$CD = 80 \times \frac{\sqrt{3}}{4} = 20\sqrt{3}$ m

$DO = BD - BO = 80 - 20 = 60$m

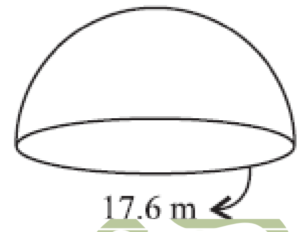
∴ The height of the poles is $20\sqrt{3}$ m

The point is 20m and 60m far from the poles

41. The inner curved surface area of a hemispherical dome of a building needs to be painted. If the circumference of the base is 17.6 m, find the cost of painting it at the rate of Rs. 5 per sq.m

Let r be the radius of the hemispherical dome.

Given that base circumference of the dome $2\pi r = 17.6$ m



$$r = \frac{17.6 \times 7}{2 \times 22} = 2.8$$

CSA , $2\pi r^2 = 2 \times \frac{22}{7} \times 2.8 \times 2.8 = 49.28\ m^2$

Cost of painting for $1m^2 = \text{Rs. }5$

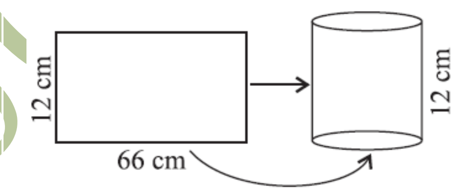
Hence, the total cost of painting for the dome $= 49.28 \times 5 = Rs. 246.40$

42. A rectangular sheet of metal foil with dimension 66 cm × 12 cm is rolled to form a cylinder of height 12 cm. Find the volume of the cylinder

Let r and h be the radius and height of the right circular cylinder respectively.

Given that the dimension of the sheet is 66 cm × 12 cm

$l = 66$ cm , $b = 12$ cm



Thus, base circumference $2\pi r = l$

$$2 \times \frac{22}{7} \times r = 66$$

$$r = \frac{66 \times 7}{2 \times 22} = \frac{21}{2}$$

Height of the cylinder $=$ length of the rectangular sheet

$$h = b = 12\text{cm}$$

Thus, the volume of the cylinder $= \pi r^2 h = \frac{22}{7} \times \left(\frac{21}{2}\right)^2 \times 12 = 4158 \text{ cm}^3$

43. Find the standard deviation of the numbers 62, 58, 53, 50, 63, 52, 55.

Let us take $A = 55$ as the assumed mean and form the following table

x	$d = x - A$ $= x - 55$	d^2
50	-5	25
52	-3	9
53	-2	4
55	0	0
58	3	9
62	7	49
63	8	64
	$\sum d = 8$	$\sum d^2 = 160$

$$\sigma = \sqrt{\frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2}$$

$$= \sqrt{\frac{160}{7} - \left(\frac{8}{7}\right)^2} = \sqrt{\frac{160}{7} - \frac{64}{49}}$$

$$= \sqrt{\frac{1056}{49}} = \frac{32.49}{7}$$

Standard deviation $\sigma \cong 4.64$

44. If a die is rolled twice, find the probability of getting an even number in the first time or a total of 8 .

Let S be the sample space consisting of all possible outcomes.

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),\\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\ (3,1),(3,2),(3,3),(3,4),(3,5),(3,6),\\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\ (5,1),(5,2),(5,3),(5,4),(5,5),(5,6),\\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

$$n(S) = 36$$

Let A be the event of getting an even number in the first time.

$$A = \{(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),\\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6),\\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}. \quad \text{So, } n(A) = 18$$

Thus, $P(A) = \frac{n(A)}{n(s)} = \frac{18}{36} = \frac{1}{2}$

Let B be the event of getting a total 8.

$$B = \{(2,6),(3,5),(4,4),(5,3),(6,2)\}. \text{ So, } n(B) = 5$$

Thus, $P(B) = \frac{n(B)}{n(s)} = \frac{5}{36}$

Also, $A \cap B = \{(2,6),(4,4),(6,2)\}$ and $P(A \cap B) = \frac{3}{36}$

Hence, the required probability

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$

45. (a) If S_1, S_2 and S_3 are the sum of first $n, 2n$ and $3n$ terms of a geometric series respectively, then prove that $S_1(S_3 - S_2) = (S_2 - S_1)^2$

Let $S_1 = \frac{a(1-r^n)}{1-r}$, $S_2 = \frac{a(1-r^{2n})}{1-r}$ and $S_3 = \frac{a(1-r^{3n})}{1-r}$

LHS $S_1(S_3 - S_2) = \left(\frac{a(1-r^n)}{1-r}\right)\left(\frac{a(1-r^{3n})}{1-r} - \frac{a(1-r^{2n})}{1-r}\right)$

$= \left(\frac{a^2(1-r^n)}{(1-r)^2}\right)[\cancel{1} - r^{3n} - \cancel{1} + r^{2n}] = \left(\frac{a^2(1-r^n)}{(1-r)^2}\right)[r^{2n} - r^{3n}]$

$= \frac{a^2r^{2n}(1-r^n)(1-r^n)}{(1-r)^2} = \frac{a^2r^{2n}(1-r^n)^2}{(1-r)^2}$ ....(1)

RHS $S_2 - S_1 = \frac{a(1-r^{2n})}{1-r} - \frac{a(1-r^n)}{1-r}$

$= \frac{a}{1-r}(1 - r^{2n} - 1 + r^n)$

$= \frac{ar^n}{1-r}(1 - r^n)$

$(S_2 - S_1)^2 = \frac{a^2 r^{2n}}{(1-r)^2}(1 - r^n)^2$ ………………………………………………………………………(2)

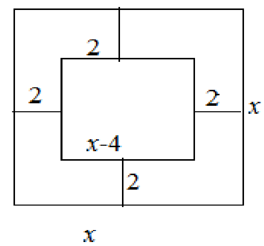
From (1) and (2) $S_1(S_3 - S_2) = (S_2 - S_1)^2$ **(OR)**

45.(b) A chess board contains 64 equal squares and the area of each square is 6.25 sq,cm. A brother around the board is 2cm wide. Find the length of the side of the chess board

Let the length of side of the chess board $= x$ cm.

Area of the each square $= 6.25 \text{ cm}^2$

Thus, the area of 64 squares $= 64 \times 6.25$



$(x-4)^2 = 400$

$x - 4 = \pm 20$

$x = 20 + 4 \text{ (or) } -20 + 4$

$x = 24 \text{ or } -16$

Since, length of the sides of a board can't be negative, so $x = 24cm$

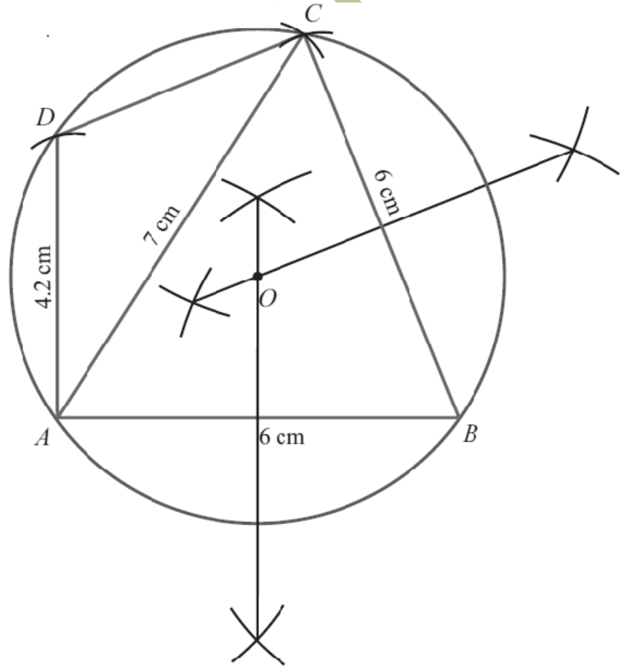
SECTION –IV (MARKS: 20)

Note: Answer both the questions choosing either of the alternatives. **2 x 10=20**

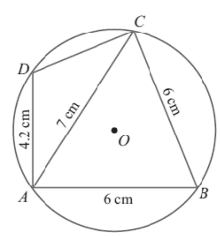
46. (a) Construct a cyclic quadrilateral $ABCD$ in which $AB = 6$ cm, $AC = 7$ cm, $BC = 6$cm and $AD = 4.2$cm

Given cyclic quadrilateral $ABCD$, $AB = 6$ cm, $AC = 7$ cm, $BC = 6$cm and $AD = 4.2$cm

Fair diagram:



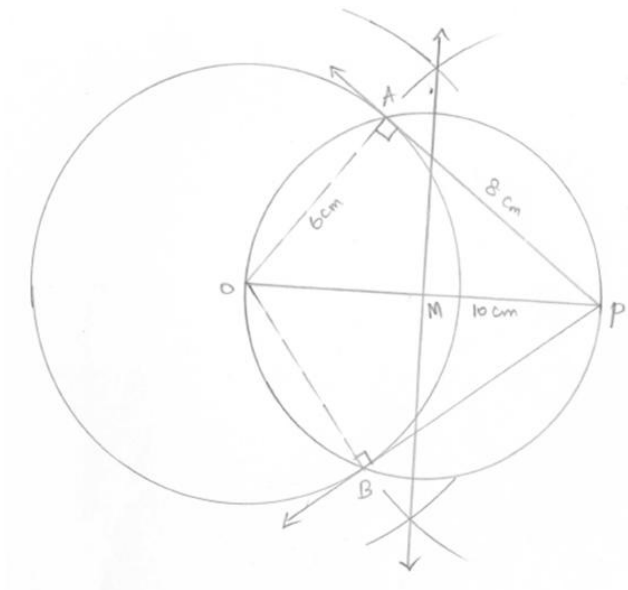
Rough diagram



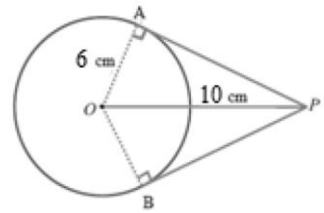
(OR)

46. (b) Draw the two tangents from a point which is 10cm away from the centre of a circle of radius 6cm . Also, measure the lengths of the tangents.

Fair diagram:



Rough diagram



Verification:

$PA = \sqrt{10^2 - 6^2} = \sqrt{100 - 36} = \sqrt{64} = 8$ cm

47. (a) Draw the graph of $y = x^2$ and hence solve $x^2 - 4x - 5 = 0$

Table for $y = x^2$

x	-2	-1	0	1	2	3	4	5	6
$y = x^2$	4	1	0	1	4	9	16	25	36

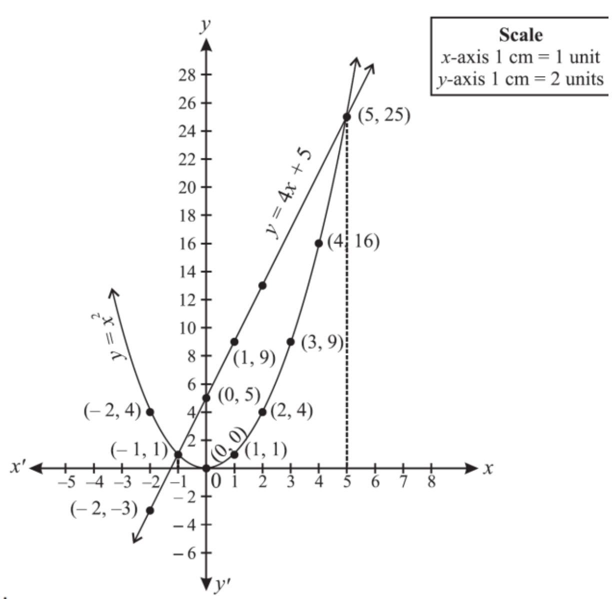
Plot the points $(-2,4), (-1,1), (0,0), (1,1), (2,4),$
$(3,9), (4,16), (5,25), (6,36)$

Solve $y = x^2 + 0x + 0$
$0 = x^2 - 4x - 5$
$y = 4x + 5$

Let us draw the graph of the straight line $y = 4x + 5$

Now, form the table for the line $y = 4x + 5$

x	-2	-1	0	1	2
$y = 4x + 5$	-3	1	5	9	13



Points
$(-2,-3), (-1,1), (0,5), (1,9), (2,13)$

(OR)

47. (b)A cyclist travels from a place A to a place B along the same route at a uniform speed on different days. The following table gives the speed of his travel and corresponding time he took to cover the distance.

Speed in km /hr x	2	4	6	10	12
Time in hrs y	60	30	20	12	10

Draw the speed-time graph and use it to find

(i) the number of hours he will take if he travels at a speed of 5 km/hr

(ii) the speed with which he should travel if he has to cover the distance in 40 hrs.

Points: (2,60) (4,30) (6,20) (10,12) (12,10)

Thus, the variation is called indirect variation.

$xy = k$

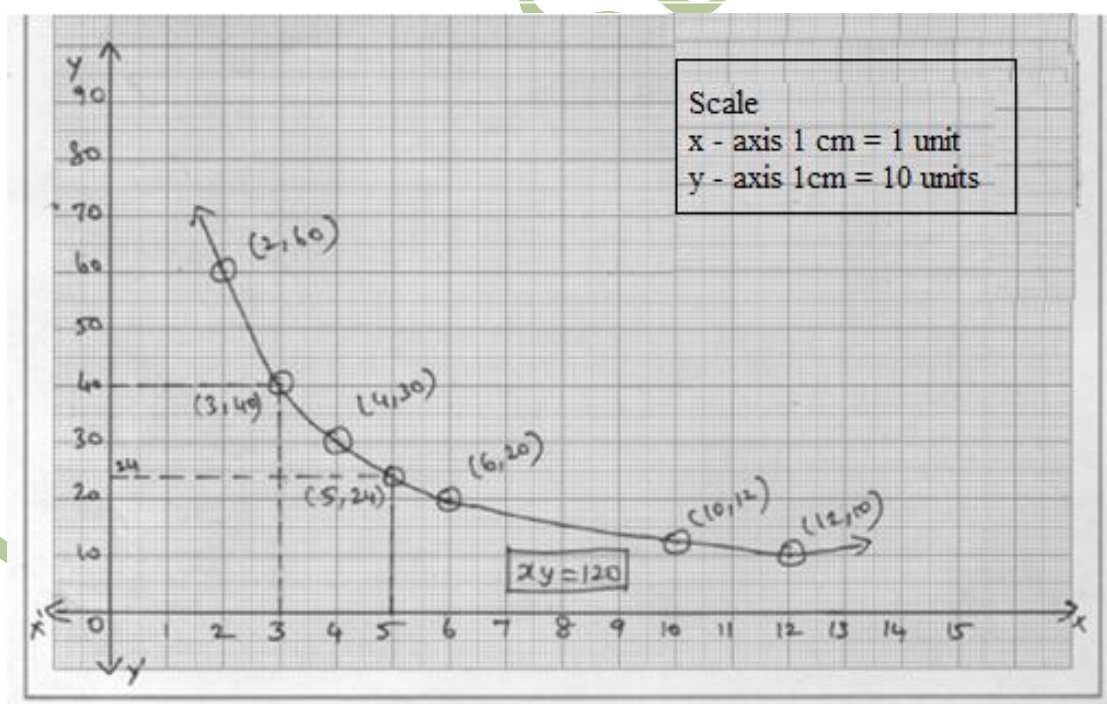
$k = (2 \times 60) = (4 \times 30) = (6 \times 20) = (10 \times 12) = (12 \times 10)$

$k = 120$

i) When $x = 5$, $5 \times y = 120$

$y = \frac{120}{5} = 24$ hours

ii) When $y = 40$, $x \times 40 = 120 \Rightarrow x = \frac{120}{40} = 3$ km / hour



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